Chapter 1
Measurement and Vectors

Conceptual Problems

1 • [SSM] Which of the following is not one of the base quantities in the SI system? (a) mass, (b) length, (c) energy, (d) time, (e) All of the above are base quantities.

Determine the Concept The base quantities in the SI system include mass, length, and time. Force is not a base quantity. (c) is correct.

2 • In doing a calculation, you end up with m/s in the numerator and m/s² in the denominator. What are your final units? (a) m²/s³, (b) 1/s, (c) s³/m², (d) s, (e) m/s.

Picture the Problem We can express and simplify the ratio of m/s to m/s² to determine the final units.

Express and simplify the ratio of
m/s to m/s²:
\[
\frac{\text{m}}{\text{s}} \cdot \frac{\text{m}}{\text{s}^2} = \frac{\text{m} \cdot \text{s}}{\text{m} \cdot \text{s}^2} = \text{s}
\]
and (d) is correct.

3 • The prefix giga means (a) 10³, (b) 10⁶, (c) 10⁹, (d) 10¹², (e) 10¹⁵.

Determine the Concept Consulting Table 1-1 we note that the prefix giga means 10⁹. (c) is correct.

4 • The prefix mega means (a) 10⁻⁹, (b) 10⁻⁶, (c) 10⁻³, (d) 10⁶, (e) 10⁹.

Determine the Concept Consulting Table 1-1 we note that the prefix mega means 10⁶. (d) is correct.

5 • [SSM] Show that there are 30.48 cm per foot. How many centimeters are there in one mile?

Picture the Problem We can use the facts that there are 2.540 centimeters in 1 inch and 12 inches in 1 foot to show that there are 30.48 cm per ft. We can then use the fact that there are 5280 feet in 1 mile to find the number of centimeters in one mile.
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Multiply 2.540 cm/in by 12 in/ft to find the number of cm per ft:

\[
\left(\frac{2.540 \text{ cm}}{\text{in}}\right) \left(\frac{12 \text{ in}}{\text{ft}}\right) = \boxed{30.48 \text{ cm/ft}}
\]

Multiply 30.48 cm/ft by 5280 ft/mi to find the number of centimeters in one mile:

\[
\left(\frac{30.48 \text{ cm}}{\text{ft}}\right) \left(\frac{5280 \text{ ft}}{\text{mi}}\right) = \boxed{1.609 \times 10^5 \text{ cm/mi}}
\]

Remarks: Because there are exactly 2.54 cm in 1 in and exactly 12 inches in 1 ft, we are justified in reporting four significant figures in these results.

The number 0.000 513 0 has ___ significant figures. (a) one, (b) three, (c) four, (d) seven, (e) eight.

Determine the Concept Counting from left to right and ignoring zeros to the left of the first nonzero digit, the last significant figure is the first digit that is in doubt. Applying this criterion, the three zeros after the decimal point are not significant figures, but the last zero is significant. Hence, there are four significant figures in this number. \(c\) is correct.

The number 23.0040 has ___ significant figures. (a) two, (b) three, (c) four, (d) five, (e) six.

Determine the Concept Counting from left to right, the last significant figure is the first digit that is in doubt. Applying this criterion, there are six significant figures in this number. \(e\) is correct.

Force has dimensions of mass times acceleration. Acceleration has dimensions of speed divided by time. Pressure is defined as force divided by area. What are the dimensions of pressure? Express pressure in terms of the SI base units kilogram, meter and second.

Determine the Concept We can use the definitions of force and pressure, together with the dimensions of mass, acceleration, and length, to find the dimensions of pressure. We can express pressure in terms of the SI base units by substituting the base units for mass, acceleration, and length in the definition of pressure.
Use the definition of pressure and the dimensions of force and area to obtain:

\[
[p] = \frac{[F]}{[A]} = \frac{ML}{T^2} = \frac{M}{LT^2}
\]

Express pressure in terms of the SI base units to obtain:

\[
\frac{N}{m^2} = \frac{kg \cdot m}{m^2 \cdot s^2} = \frac{kg}{m \cdot s^2}
\]

9  •  True or false: Two quantities must have the same dimensions in order to be multiplied.

False. For example, the distance traveled by an object is the product of its speed (length/time) multiplied by its time of travel (time).

10  •  A vector has a negative x component and a positive y component. Its angle measured counterclockwise from the positive x axis is (a) between zero and 90 degrees. (b) between 90 and 180 degrees. (c) More than 180 degrees.

**Determine the Concept** Because a vector with a negative x-component and a positive y-component is in the second quadrant, its angle is between 90 and 180 degrees. **(b) is correct.**

11  •  [SSM] A vector \( \vec{A} \) points in the +x direction. Show graphically at least three choices for a vector \( \vec{B} \) such that \( \vec{B} + \vec{A} \) points in the +y direction.

**Determine the Concept** The figure shows a vector \( \vec{A} \) pointing in the positive x direction and three unlabeled possibilities for vector \( \vec{B} \). Note that the choices for \( \vec{B} \) start at the end of vector \( \vec{A} \) rather than at its initial point. Note further that this configuration could be in any quadrant of the reference system shown.

12  •  A vector \( \vec{A} \) points in the +y direction. Show graphically at least three choices for a vector \( \vec{B} \) such that \( \vec{B} - \vec{A} \) points in the +x direction.
Determine the Concept Let the $+x$ direction be to the right and the $+y$ direction be upward. The figure shows the vector $-\vec{A}$ pointing in the $-y$ direction and three unlabeled possibilities for vector $\vec{B}$. Note that the choices for $\vec{B}$ start at the end of vector $-\vec{A}$ rather than at its initial point.

13 • [SSM] Is it possible for three equal magnitude vectors to add to zero? If so, sketch a graphical answer. If not, explain why not.

Determine the Concept In order for the three equal magnitude vectors to add to zero, the sum of the three vectors must form a triangle. The equilateral triangle shown to the right satisfies this condition for the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ for which it is true that $A = B = C$, whereas $\vec{A} + \vec{B} + \vec{C} = 0$.

Estimation and Approximation

14 • The angle subtended by the moon’s diameter at a point on Earth is about 0.524° (Fig. 1-2). Use this and the fact that the moon is about 384 Mm away to find the diameter of the moon. HINT: The angle can be determined from the diameter of the moon and the distance to the moon.

Picture the Problem Let $\theta$ represent the angle subtended by the moon’s diameter, $D$ represent the diameter of the moon, and $r_m$ the distance to the moon. Because $\theta$ is small, we can approximate it by $\theta \approx D/r_m$ where $\theta$ is in radian measure. We can solve this relationship for the diameter of the moon.

Express the moon’s diameter $D$ in terms of the angle it subtends at Earth $\theta$ and the Earth-moon distance $r_m$:

$$D = \theta r_m$$

Substitute numerical values and evaluate $D$:

$$D = \left(0.524^\circ \times \frac{2\pi \text{ rad}}{360^\circ}\right)(384 \text{ Mm})$$

$$= 3.51 \times 10^6 \text{ m}$$
15 • [SSM] Some good estimates about the human body can be made if it is assumed that we are made mostly of water. The mass of a water molecule is $29.9 \times 10^{-27}$ kg. If the mass of a person is 60 kg, estimate the number of water molecules in that person.

**Picture the Problem** We can estimate the number of water molecules in a person whose mass is 60 kg by dividing this mass by the mass of a single water molecule.

Letting $N$ represent the number of water molecules in a person of mass $m_{\text{human body}}$, express $N$ in terms of $m_{\text{human body}}$ and the mass of a water molecule $m_{\text{water molecule}}$:

$$N = \frac{m_{\text{human body}}}{m_{\text{water molecule}}}$$

Substitute numerical values and evaluate $N$:

$$N = \frac{60 \text{ kg}}{29.9 \times 10^{-27} \text{ kg}} = 2.0 \times 10^{27} \text{ molecules}$$

16 • In 1989, IBM scientists figured out how to move atoms with a scanning tunneling microscope (STM). One of the first STM pictures seen by the general public was of the letters IBM spelled with xenon atoms on a nickel surface. The letters IBM were 15 xenon atoms across. If the space between the centers of adjacent xenon atoms is 5 nm ($5 \times 10^{-9}$ m), estimate how many times could "IBM" could be written across this 8.5 inch page.

**Picture the Problem** We can estimate the number of times $N$ that "IBM" could be written across this 8.5-inch page by dividing the width $w$ of the page by the distance $d$ required by each writing of "IBM."

Express $N$ in terms of the width $w$ of the page and the distance $d$ required by each writing of "IBM."

$$N = \frac{w}{d}$$

Express $d$ in terms of the separation $s$ of the centers of adjacent xenon atoms and the number $n$ of xenon atoms in each writing of "IBM."

$$d = sn$$

Substitute for $d$ in the expression for $N$ to obtain:

$$N = \frac{w}{sn}$$
Substitute numerical values and evaluate \( N \):

\[
N = \frac{(8.5 \text{ in})(2.540 \frac{\text{cm}}{\text{in}})}{(5 \text{ nm} \times 10^{-9} \frac{\text{m}}{\text{nm}})(15)} = 3 \times 10^6
\]

17 There is an environmental debate over the use of cloth versus disposable diapers. (a) If we assume that between birth and 2.5 y of age, a child uses 3 diapers per day, estimate the total number of disposable diapers used in the United States per year. (b) Estimate the total landfill volume due to these diapers, assuming that 1000 kg of waste fills about 1 m\(^3\) of landfill volume. (c) How many square miles of landfill area at an average height of 10 m is needed for the disposal of diapers each year?

**Picture the Problem** We’ll assume a population of 300 million and a life expectancy of 76 y. We’ll also assume that a diaper has a volume of about half a liter. In (c) we’ll assume the disposal site is a rectangular hole in the ground and use the formula for the volume of such an opening to estimate the surface area required.

(a) Express the total number \( N \) of disposable diapers used in the United States per year in terms of the number of children \( n \) in diapers and the number of diapers \( D \) used by each child per year:

\[
N = nD \quad (1)
\]

Use the estimated daily consumption and the number of days in a year to estimate the number of diapers \( D \) required per child per year:

\[
D = \frac{3 \text{ diapers}}{\text{child} \cdot \text{d}} \times \frac{365.24 \text{ d}}{\text{y}}
\approx 1.1 \times 10^3 \text{ diapers/child} \cdot \text{y}
\]

Use the assumed life expectancy to estimate the number of children \( n \) in diapers yearly:

\[
n = \left(\frac{2.5 \text{ y}}{76 \text{ y}}\right)(300 \times 10^6 \text{ children})
\approx 10^7 \text{ children}
\]

Substitute numerical values in equation (1) to obtain:

\[
N = (10^7 \text{ children}) \left(3 \times 10^3 \frac{\text{diapers}}{\text{y}}\right)
\approx 1.1 \times 10^{10} \text{ diapers}
\]
(b) Express the required landfill volume \( V \) in terms of the volume of diapers to be buried:

Substitute numerical values and evaluate \( V \):

\[
V = \left( 1.1 \times 10^{10} \text{ diapers} \right) \left( \frac{0.5 \text{ L}}{\text{ diaper}} \times 10^{-3} \text{ m}^3 \right) = 5.5 \times 10^6 \text{ m}^3
\]

(c) Express the required volume in terms of the volume of a rectangular parallelepiped:

\[
V = Ah \Rightarrow A = \frac{V}{h}
\]

Substitute numerical values evaluate \( A \):

\[
A = \frac{5.5 \times 10^6 \text{ m}^3}{10 \text{ m}} = 5.5 \times 10^5 \text{ m}^2
\]

Use a conversion factor (see Appendix A) to express this area in square miles:

\[
A = 5.5 \times 10^5 \text{ m}^2 \times \frac{0.3861 \text{ mi}^2}{1 \text{ km}^2} \approx 2.1 \text{ mi}^2
\]

18  ••  (a) Estimate the number of gallons of gasoline used per day by automobiles in the United States and the total amount of money spent on it. (b) If 19.4 gal of gasoline can be made from one barrel of crude oil, estimate the total number of barrels of oil imported into the United States per year to make gasoline. How many barrels per day is this?

**Picture the Problem** The population of the United States is roughly \( 3 \times 10^8 \) people. Assuming that the average family has four people, with an average of two cars per family, there are about \( 1.5 \times 10^8 \) cars in the United States. If we double that number to include trucks, cabs, etc., we have \( 3 \times 10^8 \) vehicles. Let’s assume that each vehicle uses, on average, 14 gallons of gasoline per week and that the United States imports half its oil.

\[
(a) \text{ Find the daily consumption of gasoline } G: \\
G = (3 \times 10^8 \text{ vehicles}) (2 \text{ gal/d}) = 6 \times 10^8 \text{ gal/d}
\]

Assuming a price per gallon \( P = $3.00 \), find the daily cost \( C \) of gasoline:

\[
C = GP = (6 \times 10^8 \text{ gal/d}) ($3.00 / \text{ gal}) = 18 \times 10^8 / \text{ d} \approx 2 \text{ billion dollars/d}
\]
(b) Relate the number of barrels $N$ of crude oil imported annually to the yearly consumption of gasoline $Y$ and the number of gallons of gasoline $n$ that can be made from one barrel of crude oil:

$$N = \frac{fY}{n} = \frac{fG\Delta t}{n}$$

where $f$ is the fraction of the oil that is imported.

Substitute numerical values and estimate $N$:

$$N = \left(0.5\right)\left(6\times10^8 \frac{\text{gallons}}{\text{d}}\right)\left(365.24 \frac{\text{d}}{y}\right)$$

$$\approx \frac{19.4 \text{ gallons}}{\text{barrel}}$$

$$\approx 6\times10^9 \frac{\text{barrels}}{y}$$

Convert barrels/y to barrels/d to obtain:

$$N = 6\times10^9 \frac{\text{barrels}}{y} \times \frac{1 \text{ y}}{365.24 \text{ d}}$$

$$\approx 2\times10^7 \frac{\text{barrels}}{d}$$

19 ** [SSM] A megabyte (MB) is a unit of computer memory storage. A CD has a storage capacity of 700 MB and can store approximately 70 min of high-quality music. (a) If a typical song is 5 min long, how many megabytes are required for each song? (b) If a page of printed text takes approximately 5 kilobytes, estimate the number of novels that could be saved on a CD.

**Picture the Problem** We can set up a proportion to relate the storage capacity of a CD to its playing time, the length of a typical song, and the storage capacity required for each song. In (b) we can relate the number of novels that can be stored on a CD to the number of megabytes required per novel and the storage capacity of the CD.

(a) Set up a proportion relating the ratio of the number of megabytes on a CD to its playing time to the ratio of the number of megabytes $N$ required for each song:

$$\frac{700 \text{ MB}}{70 \text{ min}} = \frac{N}{5 \text{ min}}$$

Solve this proportion for $N$ to obtain:

$$N = \left(\frac{700 \text{ MB}}{70 \text{ min}}\right)(5 \text{ min}) = 50 \text{ MB}$$
(b) Letting $n$ represent the number of megabytes per novel, express the number of novels $N_{\text{novels}}$ that can be stored on a CD in terms of the storage capacity of the CD:

$$N_{\text{novels}} = \frac{700 \text{ MB}}{n}$$

Assuming that a typical page in a novel requires 5 kB of memory, express $n$ in terms of the number of pages $p$ in a typical novel:

$$n = \left(\frac{5 \text{ kB}}{\text{page}}\right)p$$

Substitute for $n$ in the expression for $N_{\text{novels}}$ to obtain:

$$N_{\text{novels}} = \frac{700 \text{ MB}}{\left(\frac{5 \text{ kB}}{\text{page}}\right)p}$$

Assuming that a typical novel has 200 pages:

$$N_{\text{novels}} = \frac{700 \text{ MB} \times 10^3 \text{ kB}}{\left(\frac{5 \text{ kB}}{\text{page}}\right)\left(\frac{200 \text{ pages}}{\text{novel}}\right)} = 7 \times 10^2 \text{ novels}$$

**Units**

20 • Express the following quantities using the prefixes listed in Table 1-1 and the unit abbreviations listed in the table Abbreviations for Units. For example, 10,000 meters = 10 km. (a) 1,000,000 watts, (b) 0.002 gram, (c) $3 \times 10^{-6}$ meter, (d) 30,000 seconds.

**Picture the Problem** We can use the metric prefixes listed in Table 1-1 and the abbreviations on page EP-1 to express each of these quantities.

(a)

1,000,000 watts = $10^6$ watts = $1\text{MW}$

(c)

$3 \times 10^{-6}$ meter = $3\mu\text{m}$

(b)

0.002 gram = $2 \times 10^{-3}$ g = $2\text{mg}$

(d)

30,000 seconds = $30 \times 10^3$ s = $30\text{ks}$

21 • Write each of the following without using prefixes: (a) 40 $\mu$W, (b) 4 ns, (c) 3 MW, (d) 25 km.
**Picture the Problem** We can use the definitions of the metric prefixes listed in Table 1-1 to express each of these quantities without prefixes.

\[(a)\] \(40 \mu W = 40 \times 10^{-6} W = \boxed{0.000040W}\)

\[(c)\] \(3 \text{ MW} = 3 \times 10^6 W = \boxed{3,000,000W}\)

\[(b)\] \(4 \text{ ns} = 4 \times 10^{-9} s = \boxed{0.000000004s}\)

\[(d)\] \(25 \text{ km} = 25 \times 10^3 m = \boxed{25,000m}\)

**22** • Write the following (which are not SI units) using prefixes (but not their abbreviations). For example, \(10^3\) meters = 1 kilometer: (a) \(10^{-12}\) boo, (b) \(10^9\) low, (c) \(10^{-6}\) phone, (d) \(10^{-18}\) boy, (e) \(10^6\) phone, (f) \(10^{-9}\) goat, (g) \(10^{12}\) bull.

**Picture the Problem** We can use the definitions of the metric prefixes listed in Table 1-1 to express each of these quantities without abbreviations.

\[(a)\] \(10^{-12}\) boo = \boxed{1\text{ picoboo}}

\[(c)\] \(10^6\) phone = \boxed{1\text{ megaphone}}

\[(b)\] \(10^9\) low = \boxed{1\text{ gigalow}}

\[(f)\] \(10^{-9}\) goat = \boxed{1\text{ nanogoat}}

\[(c)\] \(10^{-6}\) phone = \boxed{1\text{ microphone}}

\[(g)\] \(10^{12}\) bull = \boxed{1\text{ terabull}}

\[(d)\] \(10^{-18}\) boy = \boxed{1\text{ attoboy}}

**23** •• [SSM] In the following equations, the distance \(x\) is in meters, the time \(t\) is in seconds, and the velocity \(v\) is in meters per second. What are the SI units of the constants \(C_1\) and \(C_2\)? (a) \(x = C_1 + C_2t\), (b) \(x = \frac{1}{2}C_1t^2\), (c) \(v^2 = 2C_1x\), (d) \(x = C_1 \cos C_2t\), (e) \(v^2 = 2C_1v - (C_2x)^2\).

**Picture the Problem** We can determine the SI units of each term on the right-hand side of the equations from the units of the physical quantity on the left-hand side.

\[(a)\] Because \(x\) is in meters, \(C_1\) and \(C_2t\) must be in meters: \boxed{C_1 \text{ is in m}; C_2 \text{ is in m/s}}

\[(b)\] Because \(x\) is in meters, \(\frac{1}{2}C_1t^2\) must be in meters: \boxed{C_1 \text{ is in m}^2}$
(c) Because \( v^2 \) is in \( \text{m}^2/\text{s}^2 \), \( 2C_1x \) must be in \( \text{m}^2/\text{s}^2 \):

\[
C_1 \text{ is in m}^2
\]

(d) The argument of a trigonometric function must be dimensionless; i.e. without units. Therefore, because \( x \) is in meters:

\[
C_1 \text{ is in m}; C_2 \text{ is in s}^{-1}
\]

(e) All of the terms in the expression must have the same units. Therefore, because \( v \) is in m/s:

\[
C_1 \text{ is in m/s}; C_2 \text{ is in s}^{-1}
\]

**24** If \( x \) is in feet, \( t \) is in milliseconds, and \( v \) is in feet per second, what are the units of the constants \( C_1 \) and \( C_2 \) in each part of Problem 23?

**Picture the Problem** We can determine the US customary units of each term on the right-hand side of the equations from the units of the physical quantity on the left-hand side.

(a) Because \( x \) is in feet, \( C_1 \) and \( C_2t \) must be in feet:

\[
C_1 \text{ is in ft}; C_2 \text{ is in ft/ms}
\]

(b) Because \( x \) is in feet, \( \frac{1}{2}C_1t^2 \) must be in feet:

\[
C_1 \text{ is in ft}/(\text{ms})^2
\]

(c) Because \( v^2 \) is in \( \text{ft}^2/(\text{ms})^2 \), \( 2C_1x \) must be in \( \text{ft}^2/\text{s}^2 \):

\[
C_1 \text{ is in ft}/(\text{ms})^2
\]

(d) The argument of a trigonometric function must be dimensionless; that is, without units. Therefore, because \( x \) is in feet:

\[
C_1 \text{ is in ft}; C_2 \text{ is in (ms)}^{-1}
\]

(e) The argument of an exponential function must be dimensionless; that is, without units. Therefore, because \( v \) is in ft/s:

\[
C_1 \text{ is in ft/ms}; C_2 \text{ is in (ms)}^{-1}
\]

**Conversion of Units**

**25** From the original definition of the meter in terms of the distance along a meridian from the equator to the North Pole, find in meters (a) the
circumference of Earth and \((b)\) the radius of Earth. \((c)\) Convert your answers for \((a)\) and \((b)\) from meters into miles.

**Picture the Problem** We can use the formula for the circumference of a circle to find the radius of Earth and the conversion factor \(1 \text{ mi} = 1.609 \text{ km}\) to convert distances in meters into distances in miles.

\((a)\) The Pole-Equator distance is one-fourth of the circumference:

\[
C = 4D_{\text{pole-equator}} = 4 \times 10^7 \text{ m}
\]

\((b)\) The formula for the circumference of a circle is:

\[
C = \pi D = 2\pi R \Rightarrow R = \frac{C}{2\pi}
\]

Substitute numerical values and evaluate \(R\):

\[
R = \frac{4 \times 10^7 \text{ m}}{2\pi} = 6.37 \times 10^6 \text{ m} = 6 \times 10^6 \text{ m}
\]

\((c)\) Use the conversion factors \(1 \text{ km} = 1000 \text{ m}\) and \(1 \text{ mi} = 1.609 \text{ km}\) to express \(C\) in \(\text{mi}\):

\[
C = 4 \times 10^7 \text{ m} \times \frac{1 \text{ km}}{10^3 \text{ m}} \times \frac{1\text{ mi}}{1.609 \text{ km}} = 2 \times 10^4 \text{ mi}
\]

Use the conversion factors \(1 \text{ km} = 1000 \text{ m}\) and \(1 \text{ mi} = 1.61 \text{ km}\) to express \(R\) in \(\text{mi}\):

\[
R = 6.37 \times 10^6 \text{ m} \times \frac{1 \text{ km}}{10^3 \text{ m}} \times \frac{1\text{ mi}}{1.61 \text{ km}} = 4 \times 10^3 \text{ mi}
\]

26 • The speed of sound in air is 343 m/s at normal room temperature. What is the speed of a supersonic plane that travels at twice the speed of sound? Give your answer in kilometers per hour and miles per hour.

**Picture the Problem** We can use the conversion factor \(1 \text{ mi} = 1.61 \text{ km}\) to convert speeds in km/h into mi/h.

Find the speed of the plane in km/h:

\[
v = 2(343 \text{ m/s}) = 686 \text{ m/s}
\]

\[
= \left(\frac{686 \text{ m}}{s}\right) \left(\frac{1 \text{ km}}{10^3 \text{ m}}\right) \left(\frac{3600 \text{ s}}{\text{ h}}\right) = 2.47 \times 10^3 \text{ km/h}
\]
Convert \( v \) into mi/h:

\[
v = \left( 2.47 \times 10^3 \text{ km/h} \right) \left( \frac{1 \text{ mi}}{1.609 \text{ km}} \right) = 1.53 \times 10^3 \text{ mi/h}
\]

27 • A basketball player is 6 ft 10 \( \frac{1}{2} \) in tall. What is his height in centimeters?

**Picture the Problem** We’ll first express his height in inches and then use the conversion factor 1 in = 2.540 cm.

Express the player’s height in inches:

\[
h = 6 \text{ ft} \times \frac{12 \text{ in}}{\text{ ft}} + 10.5 \text{ in} = 82.5 \text{ in}
\]

Convert \( h \) into cm:

\[
h = 82.5 \text{ in} \times \frac{2.540 \text{ cm}}{\text{ in}} = 210 \text{ cm}
\]

28 • Complete the following: (a) 100 km/h = ____ mi/h, (b) 60 cm = ____ in, (c) 100 yd = ____ m.

**Picture the Problem** We can use the conversion factors 1 mi = 1.609 km, 1 in = 2.540 cm, and 1 m = 1.094 yd to perform these conversions.

(a) Convert 100 km/h to mi/h:

\[
100 \frac{\text{km}}{\text{h}} = 100 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ mi}}{1.609 \text{ km}} = 62.2 \text{ mi/h}
\]

(b) Convert 60 cm to in:

\[
60 \text{ cm} = 60 \text{ cm} \times \frac{1 \text{ in}}{2.540 \text{ cm}} = 23.6 \text{ in}
\]

(c) Convert 100 yd to m:

\[
100 \text{ yd} = 100 \text{ yd} \times \frac{1 \text{ m}}{1.094 \text{ yd}} = 91.4 \text{ m}
\]

29 • The main span of the Golden Gate Bridge is 4200 ft. Express this distance in kilometers.

**Picture the Problem** We can use the conversion factor 1.609 km = 5280 ft to convert the length of the main span of the Golden Gate Bridge into kilometers.
Convert 4200 ft into km by multiplying by 1 in the form
\[
\frac{1.609 \text{ km}}{5280 \text{ ft}}
\]

\[
4200 \text{ ft} = 4200 \text{ ft} \times \frac{1.609 \text{ km}}{5280 \text{ ft}} = 1.28 \text{ km}
\]

30. Find the conversion factor to convert from miles per hour into kilometers per hour.

**Picture the Problem** Let \( v \) be the speed of an object in mi/h. We can use the conversion factor 1 mi = 1.609 km to convert this speed to km/h.

Multiply \( v \) mi/h by 1.609 km/mi to convert \( v \) to km/h:

\[
\frac{\text{mi}}{\text{h}} = v \times \frac{1.609 \text{ km}}{\text{mi}} = 1.61v \text{ km/h}
\]

31. Complete the following: (a) \( 1.296 \times 10^5 \text{ km/h}^2 = \) \( \) km/(h·s), (b) \( 1.296 \times 10^5 \text{ km/h}^2 = \) \( \) m/s², (c) 60 mi/h = \( \) ft/s, (d) 60 mi/h = \( \) m/s.

**Picture the Problem** Use the conversion factors 1 h = 3600 s, 1.609 km = 1 mi, and 1 mi = 5280 ft to make these conversions.

\[
(a) 1.296 \times 10^5 \frac{\text{km}}{\text{h}^2} = \left(1.296 \times 10^5 \frac{\text{km}}{\text{h}^2}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 36.00 \text{ km/h·s}
\]

\[
(b) 1.296 \times 10^5 \frac{\text{km}}{\text{h}^2} = \left(1.296 \times 10^5 \frac{\text{km}}{\text{h}^2}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 \left(\frac{10^3 \text{ m}}{\text{km}}\right) = 10.00 \text{ m/s}^2
\]

\[
(c) 60 \frac{\text{mi}}{\text{h}} = \left(60 \frac{\text{mi}}{\text{h}}\right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 88 \text{ ft/s}
\]

\[
(d) 60 \frac{\text{mi}}{\text{h}} = \left(60 \frac{\text{mi}}{\text{h}}\right) \left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right) \left(\frac{10^3 \text{ m}}{\text{km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 26.8 \text{ m/s} = 27 \text{ m/s}
\]

32. There are 640 acres in a square mile. How many square meters are there in one acre?

**Picture the Problem** We can use the conversion factor given in the problem statement and the fact that 1 mi = 1.609 km to express the number of square meters in one acre. Note that, because there are exactly 640 acres in a square mile, 640 acres has as many significant figures as we may wish to associate with it.
Multiply by 1 twice, properly chosen, to convert one acre into square miles, and then into square meters:

\[
1 \text{acre} = \left(1 \text{acre} \times \frac{1 \text{mi}^2}{640 \text{ acres}} \times \frac{1609 \text{ m}^2}{\text{mi}} \right) = 4045 \text{ m}^2
\]

33. [SSM] You are a delivery person for the Fresh Aqua Spring Water Company. Your truck carries 4 pallets. Each pallet carries 60 cases of water. Each case of water has 24 one-liter bottles. You are to deliver 10 cases of water to each convenience store along your route. The dolly you use to carry the water into the stores has a weight limit of 250 lb. (a) If a milliliter of water has a mass of 1 g, and a kilogram has a weight of 2.2 lb, what is the weight, in pounds, of all the water in your truck? (b) How many full cases of water can you carry on the cart?

**Picture the Problem** The weight of the water in the truck is the product of the volume of the water and its weight density of 2.2 lb/L.

(a) Relate the weight \(w\) of the water on the truck to its volume \(V\) and weight density (weight per unit volume) \(D\):

\[w = DV\]

Find the volume \(V\) of the water:

\[V = (4 \text{ pallets}) \times \left(60 \frac{\text{ cases}}{\text{ pallet}} \times 24 \frac{\text{ L}}{\text{ case}}\right) = 5760 \text{ L}\]

Substitute numerical values for \(D\) and \(L\) and evaluate \(w\):

\[w = \left(2.2 \frac{\text{ lb}}{\text{ L}}\right)(5760 \text{ L}) = 12.67 \times 10^4 \text{ lb} = 1.3 \times 10^4 \text{ lb}\]

(b) Express the number of cases of water in terms of the weight limit of the cart and the weight of each case of water:

\[N = \frac{\text{ weight limit of the cart}}{\text{ weight of each case of water}}\]

Substitute numerical values and evaluate \(N\):

\[N = \frac{250 \text{ lb}}{\left(2.2 \frac{\text{ lb}}{\text{ L}}\right) \times \left(24 \frac{\text{ L}}{\text{ case}}\right)} = 4.7 \text{ cases}\]

You can carry 4 cases.
A right circular cylinder has a diameter of 6.8 in and a height of 2.0 ft. What is the volume of the cylinder in (a) cubic feet, (b) cubic meters, (c) liters?

**Picture the Problem** The volume of a right circular cylinder is the area of its base multiplied by its height. Let \( d \) represent the diameter and \( h \) the height of the right circular cylinder; use conversion factors to express the volume \( V \) in the given units.

(a) Express the volume of the cylinder:

\[
V = \frac{1}{4} \pi d^2 h
\]

Substitute numerical values and evaluate \( V \):

\[
V = \frac{1}{4} \pi (6.8\text{ in})^2 (2.0\text{ ft}) = \frac{1}{4} \pi (6.8\text{ in})^2 (2.0\text{ ft}) \left(\frac{1\text{ ft}}{12\text{ in}}\right)^2 = 0.504\text{ ft}^3 = 0.50\text{ ft}^3
\]

(b) Use the fact that 1 m = 3.281 ft to convert the volume in cubic feet into cubic meters:

\[
V = (0.504\text{ ft}^3) \left(\frac{1\text{ m}}{3.281\text{ ft}}\right)^3 = 0.0143\text{ m}^3 = 0.014\text{ m}^3
\]

(c) Because 1 L = 10\(^{-3}\) m\(^3\):

\[
V = (0.0143\text{ m}^3) \left(\frac{1\text{ L}}{10^{-3}\text{ m}^3}\right) = 14\text{ L}
\]

In the following, \( x \) is in meters, \( t \) is in seconds, \( v \) is in meters per second, and the acceleration \( a \) is in meters per second squared. Find the SI units of each combination: (a) \( \frac{v^2}{x} \), (b) \( \sqrt{\frac{x}{a}} \), (c) \( \frac{1}{2} at^2 \).

**Picture the Problem** We can treat the SI units as though they are algebraic quantities to simplify each of these combinations of physical quantities and constants.

(a) Express and simplify the units of \( \frac{v^2}{x} \):

\[
\frac{(m/s)^2}{m} = \frac{m^2}{m \cdot s^2} = \frac{m}{s^2}
\]

(b) Express and simplify the units of \( \sqrt{\frac{x}{a}} \):

\[
\sqrt{\frac{m}{m/s^2}} = \sqrt{s^2} = s
\]
(c) Noting that the constant factor \( \frac{1}{2} \) has no units, express and simplify
the units of \( \frac{1}{2}at^2 \):

**Dimensions of Physical Quantities**

36 • What are the dimensions of the constants in each part of Problem 23?

**Picture the Problem** We can use the facts that each term in an equation must have the same dimensions and that the arguments of a trigonometric or exponential function must be dimensionless to determine the dimensions of the constants.

\[
\begin{align*}
(a) & \quad x = C_1 + C_2 \quad t \\
& \quad \text{L} \quad \text{L} \quad \text{L} \quad \text{T} \\
& \\
(b) & \quad x = \frac{1}{2} \quad C_1 \quad t^2 \\
& \quad \text{L} \quad \text{L} \quad \text{T} \quad \text{T} \\
& \\
(c) & \quad v^2 = 2 \quad C_1 \quad x \\
& \quad \text{L}^2 \quad \text{L} \quad \text{T} \quad \text{L} \\
& \\
(d) & \quad x = C_1 \cos C_2 \quad t \\
& \quad \text{L} \quad \text{L} \quad \frac{1}{\text{T}}\quad \text{T} \\
& \\
(e) & \quad v^2 = 2 \quad C_1 \quad v - (C_2)^2 \quad x^2 \\
& \quad \left(\frac{\text{L}}{\text{T}}\right)^2 \quad \text{L} \quad \text{T} \quad \text{T} \quad \frac{1}{\text{T}} \quad \text{L}^2 \\
&
\end{align*}
\]

37 • The law of radioactive decay is \( N(t) = N_0 e^{-\lambda t} \), where \( N_0 \) is the number of radioactive nuclei at \( t = 0 \), \( N(t) \) is the number remaining at time \( t \), and \( \lambda \) is a quantity known as the decay constant. What is the dimension of \( \lambda \)?

**Picture the Problem** Because the exponent of the exponential function must be dimensionless, the dimension of \( \lambda \) must be \( \text{T}^{-1} \).

38 •• The SI unit of force, the kilogram-meter per second squared (kg·m/s²) is called the newton (N). Find the dimensions and the SI units of the constant \( G \) in Newton’s law of gravitation \( F = Gm_1m_2/r^2 \).
**Picture the Problem** We can solve Newton’s law of gravitation for \( G \) and substitute the dimensions of the variables. Treating them as algebraic quantities will allow us to express the dimensions in their simplest form. Finally, we can substitute the SI units for the dimensions to find the units of \( G \).

Solve Newton’s law of gravitation for \( G \) to obtain:

\[
G = \frac{Fr^2}{m_1m_2}
\]

Substitute the dimensions of the variables:

\[
[G] = \left( \frac{ML}{T^2} \right) \times L^2 = \frac{L^3}{MT^2}
\]

Use the SI units for \( L, M, \) and \( T \) to obtain:

\[
\frac{m^3}{kg \cdot s^2}
\]

**39**   ••  The magnitude of the force \( (F) \) that a spring exerts when it is stretched a distance \( x \) from its unstressed length is governed by Hooke’s law, \( F = kx \).

(a) What are the dimensions of the force constant, \( k \)? (b) What are the dimensions and SI units of the quantity \( kx^2 \)?

**Picture the Problem** The dimensions of mass and velocity are \( M \) and \( LT^{-1} \), respectively. We note from Table 1-2 that the dimensions of force are \( MLT^{-2} \).

(a) We know, from Hooke’s law, that:

\[
k = \frac{F}{x}
\]

Write the corresponding dimensional equation:

\[
[k] = \frac{[F]}{[x]}
\]

Substitute the dimensions of \( F \) and \( x \) and simplify to obtain:

\[
[k] = \frac{M \cdot L}{T^2} = \frac{M}{T^2}
\]

(b) Substitute the dimensions of \( k \) and \( x^2 \) and simplify to obtain:

\[
[kx^2] = \frac{M}{T^2} \cdot L^2 = \frac{ML^2}{T^2}
\]

Substitute the units of \( kx^2 \) to obtain:

\[
\frac{kg \cdot m^2}{s^2}
\]
40  **  Show that the product of mass, acceleration, and speed has the dimensions of power.

**Picture the Problem** We note from Table 1-2 that the dimensions of power are $ML^2/T^3$. The dimensions of mass, acceleration, and speed are $M$, $L/T^2$, and $L/T$ respectively.

Express the dimensions of $mav$:

$$[mav] = M \times \frac{L}{T^2} \times \frac{L}{T} = \frac{ML^2}{T^3}$$

From Table 1-2:

$$[p] = \frac{ML^2}{T^3}$$

Comparing these results, we see that the product of mass, acceleration, and speed has the dimensions of power.

41  **  [SSM]  The momentum of an object is the product of its velocity and mass. Show that momentum has the dimensions of force multiplied by time.

**Picture the Problem** The dimensions of mass and velocity are $M$ and $L/T$, respectively. We note from Table 1-2 that the dimensions of force are $ML/T^2$.

Express the dimensions of momentum:

$$[mv] = M \times \frac{L}{T} = \frac{ML}{T}$$

From Table 1-2:

$$[F] = \frac{ML}{T^2}$$

Express the dimensions of force multiplied by time:

$$[Ft] = \frac{ML}{T^2} \times T = \frac{ML}{T}$$

Comparing these results, we see that momentum has the dimensions of force multiplied by time.

42  **  What combination of force and one other physical quantity has the dimensions of power?

**Picture the Problem** Let $X$ represent the physical quantity of interest. Then we can express the dimensional relationship between $F$, $X$, and $P$ and solve this relationship for the dimensions of $X$. 
Express the relationship of $X$ to force and power dimensionally:

$[F][X] = [P]$

Solve for $[X]$:

$[X] = \frac{[P]}{[F]}$

Substitute the dimensions of force and power and simplify to obtain:

$[X] = \frac{ML}{T^2} = \frac{L}{T}$

Because the dimensions of velocity are $L/T$, we can conclude that:

$[P] = \left[\frac{[F]}{[v]}\right]$  

Remarks: While it is true that $P = Fv$, dimensional analysis does not reveal the presence of dimensionless constants. For example, if $P = \pi Fv$, the analysis shown above would fail to establish the factor of $\pi$.

43 [SSM] When an object falls through air, there is a drag force that depends on the product of the cross sectional area of the object and the square of its velocity, that is, $F_{\text{air}} = CAv^2$, where $C$ is a constant. Determine the dimensions of $C$.

**Picture the Problem** We can find the dimensions of $C$ by solving the drag force equation for $C$ and substituting the dimensions of force, area, and velocity.

Solve the drag force equation for the constant $C$:

$C = \frac{F_{\text{air}}}{Av^2}$

Express this equation dimensionally:

$[C] = \frac{[F_{\text{air}}]}{[A][v]^2}$

Substitute the dimensions of force, area, and velocity and simplify to obtain:

$[C] = \frac{ML}{T^2} = \left(\frac{L}{T}\right)^2 = \frac{M}{L^3}$

44 [SSM] Kepler’s third law relates the period of a planet to its orbital radius $r$, the constant $G$ in Newton’s law of gravitation ($F = \frac{Gm_1m_2}{r^2}$), and the mass of the sun $M_s$. What combination of these factors gives the correct dimensions for the period of a planet?
**Picture the Problem** We can express the period of a planet as the product of the factors \( r, G, \) and \( M_S \) (each raised to a power) and then perform dimensional analysis to determine the values of the exponents.

Express the period \( T \) of a planet as the product of \( r^a, G^b, \) and \( M_S^c \):

\[
T = Cr^a G^b M_S^c \tag{1}
\]

where \( C \) is a dimensionless constant.

Solve the law of gravitation for the constant \( G \):

\[
G = \frac{F r^2}{m_1 m_2}
\]

Express this equation dimensionally:

\[
[G] = \frac{[F][r]^2}{[m_1][m_2]}
\]

Substitute the dimensions of \( F, r, \) and \( m \):

\[
[G] = \frac{ML}{T^2} \times \frac{(L)^2}{M \times M} = \frac{L^3}{MT^2}
\]

Noting that the dimension of time is represented by the same letter as is the period of a planet, substitute the dimensions in equation (1) to obtain:

\[
T = (L)^a \left( \frac{L^3}{MT^2} \right)^b (M)^c
\]

Introduce the product of \( M^0 \) and \( L^0 \) in the left hand side of the equation and simplify to obtain:

\[
M^0 L^0 T^1 = M^{c-b} L^{a+3b} T^{-2b}
\]

Equating the exponents on the two sides of the equation yields:

\[
0 = c - b, \quad 0 = a + 3b, \quad \text{and} \quad 1 = -2b
\]

Solve these equations simultaneously to obtain:

\[
a = \frac{3}{2}, \quad b = -\frac{1}{2}, \quad \text{and} \quad c = -\frac{1}{2}
\]

Substitute for \( a, b, \) and \( c \) in equation (1) and simplify to obtain:

\[
T = Cr^{3/2} G^{-1/2} M_S^{-1/2} = \frac{C}{\sqrt{GM_S}} r^{3/2}
\]

**Scientific Notation and Significant Figures**

45 • [SSM] Express as a decimal number without using powers of 10 notation: (a) \( 3 \times 10^4 \), (b) \( 6.2 \times 10^{-3} \), (c) \( 4 \times 10^{-6} \), (d) \( 2.17 \times 10^5 \).
22  Chapter 1

**Picture the Problem** We can use the rules governing scientific notation to express each of these numbers as a decimal number.

\[(a) \ 3 \times 10^4 = \boxed{30,000}\]

\[(c) \ 4 \times 10^{-6} = \boxed{0.000004}\]

\[(b) \ 6.2 \times 10^{-3} = \boxed{0.0062}\]

\[(d) \ 2.17 \times 10^5 = \boxed{217,000}\]

46  •  Write the following in scientific notation: (a) 1345100 m = ____ km, (b) 12340. kW = ____ MW, (c) 54.32 ps = ____ s, (d) 3.0 m = ____ mm

**Picture the Problem** We can use the rules governing scientific notation to express each of these numbers in scientific notation.

\[(a) \ \text{km} \ 10^{3451.1} m \ \text{km} 10^{3451.1} m 1345100 \times=\times=\]

\[(c) \ \text{s} \ 10^{-5.432} s \ \text{s} 10^{-54.32}\ \text{ps} 32.54 \times=-\times=-\]

\[(b) \ \text{MW} \ 10^{-1.2340} W \ \text{MW} 10^{-1.2340}\ \text{kW} .12340 \times=-\times=-\]

\[(d) \ \text{mm} \ 10^{-0.3}\ m \ \text{mm} 10\ m 3.0\ m 0.3 \times=\times=\times=\]

47  •  [SSM] Calculate the following, round off to the correct number of significant figures, and express your result in scientific notation: (a) (1.14)(9.99 \times 10^4), (b) (2.78 \times 10^{-8}) – (5.31 \times 10^{-9}), (c) 12\pi /(4.56 \times 10^{-3}), (d) 27.6 + (5.99 \times 10^2).

**Picture the Problem** Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

\[(a) \ \text{The number of significant figures in each factor is three; therefore the result has three significant figures:} \ \boxed{1.14 \times 10^5}\]
(b) Express both terms with the same power of 10. Because the first measurement has only two digits after the decimal point, the result can have only two digits after the decimal point:

\[
\begin{align*}
(2.78 \times 10^{-8}) - (5.31 \times 10^{-9}) &= (2.78 - 0.531) \times 10^{-8} \\
&= 2.25 \times 10^{-8}
\end{align*}
\]

\[
\begin{align*}
8898 &- 1025.2 \\
&= 78.2
\end{align*}
\]

\[
\begin{align*}
10531.078.2 &- 1031.5 \\
&= 78.2
\end{align*}
\]

(c) We’ll assume that 12 is exact. Hence, the answer will have three significant figures:

\[
\begin{align*}
\frac{12\pi}{4.56 \times 10^{-3}} &= \frac{8.27}{10^3}
\end{align*}
\]

\[
\begin{align*}
31027.8 &- 1056.4 \\
&= 31027.8
\end{align*}
\]

(d) Proceed as in (b):

\[
\begin{align*}
27.6 + (5.99 \times 10^2) &= 27.6 + 599 \\
&= 627 \\
&= 6.27 \times 10^2
\end{align*}
\]

48 • Calculate the following, round off to the correct number of significant figures, and express your result in scientific notation: (a) $(200.9)(569.3)$, (b) $(0.000000513)(62.3 \times 10^7)$, (c) $28401 + (5.78 \times 10^4)$, (d) $63.25/(4.17 \times 10^{-3})$.

**Picture the Problem** Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(a) Note that both factors have four significant figures.

\[
(200.9)(569.3) = 1.144 \times 10^5
\]

(b) Express the first factor in scientific notation and note that both factors have three significant figures.

\[
(0.000000513)(62.3 \times 10^7) = (5.13 \times 10^{-7})(62.3 \times 10^7) = 3.20 \times 10^2
\]

(c) Express both terms in scientific notation and note that the second has only three significant figures. Hence the result will have only three significant figures.

\[
28401 + (5.78 \times 10^4) = (2.841 \times 10^4) + (5.78 \times 10^4) \\
= (2.841 + 5.78) \times 10^4 \\
= 8.62 \times 10^4
\]
(d) Because the divisor has three significant figures, the result will have three significant figures.

\[
\frac{63.25}{4.17 \times 10^{-3}} = 1.52 \times 10^4
\]

49 • [SSM] A cell membrane has a thickness of 7.0 nm. How many cell membranes would it take to make a stack 1.0 in high?

**Picture the Problem** Let \( N \) represent the required number of membranes and express \( N \) in terms of the thickness of each cell membrane.

Express \( N \) in terms of the thickness of a single membrane:

\[
N = \frac{1.0 \text{ in}}{7.0 \text{ nm}}
\]

Convert the units into SI units and simplify to obtain:

\[
N = \frac{1.0 \text{ in} \times 2.540 \text{ cm}}{7.0 \text{ nm} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1 \text{ nm}}{10^{-9} \text{ m}}} = 3.63 \times 10^6 = \boxed{3.6 \times 10^6}
\]

50 •• A circular hole of radius \( 8.470 \times 10^{-1} \text{ cm} \) must be cut into the front panel of a display unit. The *tolerance* is \( 1.0 \times 10^{-3} \text{ cm} \), which means the actual hole cannot differ by more than this much from the desired radius. If the actual hole is larger than the desired radius by the allowed tolerance, what is the difference between the actual area and the desired area of the hole?

**Picture the Problem** Let \( r_0 \) represent the larger radius and \( r \) the desired radius of the hole. We can find the difference between the actual and the desired area of the hole by subtracting the smaller area from the larger area.

Express the difference between the two areas in terms of \( r \) and \( r_0 \):

\[
\Delta A = \pi r_0^2 - \pi r^2 = \pi (r_0^2 - r^2)
\]

Factoring \( r_0^2 - r^2 \) to obtain:

\[
\Delta A = \pi (r_0 - r)(r_0 + r)
\]

Substitute numerical values and evaluate \( \Delta A \):

\[
\Delta A = \pi \left[ 1.0 \times 10^{-3} \text{ cm} \right] \left[ 8.470 \times 10^{-1} \text{ cm} + \left( 8.470 \times 10^{-1} \text{ cm} + 1.0 \times 10^{-3} \text{ cm} \right) \right]
\]

\[
= \boxed{5.3 \times 10^{-3} \text{ cm}^2}
\]

51 •• [SSM] A square peg must be made to fit through a square hole. If you have a square peg that has an edge length of 42.9 mm, and the square hole has an edge length of 43.2 mm, (a) what is the area of the space available when
the peg is in the hole? \((b)\) If the peg is made rectangular by removing 0.10 mm of material from one side, what is the area available now?

**Picture the Problem** Let \(s_h\) represent the side of the square hole and \(s_p\) the side of the square peg. We can find the area of the space available when the peg is in the hole by subtracting the area of the peg from the area of the hole.

\[(a)\] Express the difference between the two areas in terms of \(s_h\) and \(s_p\):

\[
\Delta A = s_h^2 - s_p^2
\]

Substitute numerical values and evaluate \(\Delta A\):

\[
\Delta A = (43.2 \text{ mm})^2 - (42.9 \text{ mm})^2 \\
\approx 26 \text{ mm}^2
\]

\[(b)\] Express the difference between the area of the square hole and the rectangular peg in terms of \(s_h\), \(s_p\) and the new length of the peg \(l_p\):

\[
\Delta A' = s_h^2 - s_p l_p
\]

Substitute numerical values and evaluate \(\Delta A'\):

\[
\Delta A' = (43.2 \text{ mm})^2 - (42.9 \text{ mm})(42.9 \text{ mm} - 0.10 \text{ mm}) \\
\approx 30 \text{ mm}^2
\]

**Vectors and Their Properties**

\(52\) • A vector 7.00 units long and a vector 5.50 units long are added. Their sum is a vector 10.0 units long. \((a)\) Show graphically at least one way that this can be accomplished. \((b)\) Using your sketch in Part \((a)\), determine the angle between the original two vectors.

**Picture the Problem** Let \(\vec{A}\) be the vector whose length is 7.00 units and let \(\vec{B}\) be the vector whose length is 5.50 units. \(\vec{C} = \vec{A} + \vec{B}\) is the sum of \(\vec{A}\) and \(\vec{B}\). The fact that their sum is 10.0 long tells us that the vectors are not collinear. We can use the of the vectors to find the angle between \(\vec{A}\) and \(\vec{B}\).

\[(a)\] A graphical representation of vectors \(\vec{A}\), \(\vec{B}\) and \(\vec{C}\) is shown below. \(\theta\) is the angle between \(\vec{A}\) and \(\vec{B}\).
(b) The components of the vectors are related as follows:

\[ A_x + B_x = C_x \]
\[ A_y + B_y = C_y \]

Substituting for the components gives:

\[ 7.00 + (5.50) \cos \theta = (10.0) \cos \alpha \]
\[ (5.50) \sin \theta = (10.0) \sin \alpha \]

Squaring and adding these equations yields:

\[ (100) \sin^2 \alpha + (100) \cos^2 \alpha = (5.50) \sin^2 \theta + \left[ 7.00 + (5.50) \cos \theta \right]^2 \]

or

\[ (100)(\sin^2 \alpha + \cos^2 \alpha) = (5.50) \sin^2 \theta + \left[ 7.00 + (5.50) \cos \theta \right]^2 \]

Because \( \sin^2 \alpha + \cos^2 \alpha = 1 \):

\[ 100 = (5.50)^2 \sin^2 \theta + \left[ 7.00 + (5.50) \cos \theta \right]^2 \]

Solve this equation for \( \theta \) (you can (1) use the same trigonometric identity used in the previous step to eliminate either \( \sin^2 \theta \) or \( \cos^2 \theta \) in favor of the other and then solve the resulting equation or (2) use your graphing calculator’s SOLVER program) to obtain:

\[ \theta = 74.4^\circ \]

Remarks: You could also solve Part (b) of this problem by using the law of cosines.

53 • [SSM] Determine the \( x \) and \( y \) components of the following three vectors in the \( xy \) plane. (a) A 10-m displacement vector that makes an angle of \( 30^\circ \) clockwise from the \(+y\) direction. (b) A 25-m/s velocity vector that makes an
angle of $-40^\circ$ counterclockwise from the $-x$ direction. (c) A 40-lb force vector that makes an angle of $120^\circ$ counterclockwise from the $-y$ direction.

**Picture the Problem** The $x$ and $y$ components of these vectors are their projections onto the $x$ and $y$ axes. Note that the components are calculated using the angle each vector makes with the $+x$ axis.

(a) Sketch the displacement vector (call it $\vec{A}$) and note that it makes an angle of $60^\circ$ with the $+x$ axis:  

\[
\begin{align*}
A_x &= (10 \text{ m}) \cos 60^\circ = 5.0 \text{ m} \\
A_y &= (10 \text{ m}) \sin 60^\circ = 8.7 \text{ m}
\end{align*}
\]

(b) Sketch the velocity vector (call it $\vec{v}$) and note that it makes an angle of $220^\circ$ with the $+x$ axis:  

\[
\begin{align*}
v_x &= (25 \text{ m/s}) \cos 220^\circ = -19 \text{ m/s} \\
v_y &= (25 \text{ m/s}) \sin 220^\circ = -16 \text{ m/s}
\end{align*}
\]
(c) Sketch the force vector (call it $\vec{F}$) and note that it makes an angle of $30^\circ$ with the $+x$ axis:

\[ F_x = (40 \text{ lb}) \cos 30^\circ = 35 \text{ lb} \]

and

\[ F_y = (40 \text{ lb}) \sin 30^\circ = 20 \text{ lb} \]

54 • Rewrite the following vectors in terms of their magnitude and angle (counterclockwise from the $+x$ direction). (a) A displacement vector with an $x$ component of $+8.5$ m and a $y$ component of $-5.5$ m. (b) A velocity vector with an $x$ component of $-75$ m/s and a $y$ component of $+35$ m/s. (c) A force vector with a magnitude of $50$ lb that is in the third quadrant with an $x$ component whose magnitude is $40$ lb.

**Picture the Problem** We can use the Pythagorean Theorem to find magnitudes of these vectors from their $x$ and $y$ components and trigonometry to find their direction angles.

(a) Sketch the components of the displacement vector (call it $\vec{A}$) and show their resultant:

Use the Pythagorean Theorem to find the magnitude of $\vec{A}$:

\[ A = \sqrt{A_x^2 + A_y^2} = \sqrt{(8.5 \text{ m})^2 + (-5.5 \text{ m})^2} \]

\[ = 10 \text{ m} \]

Find the direction angle of $\vec{A}$:

\[ \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-5.5 \text{ m}}{8.5 \text{ m}}\right) \]

\[ = -33^\circ = 327^\circ \]
(b) Sketch the components of the velocity vector (call it \( \vec{v} \)) and show their resultant:

Use the Pythagorean Theorem to find the magnitude of \( \vec{v} \) and trigonometry to find its direction angle:

\[
v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-75 \text{ m/s})^2 + (35 \text{ m/s})^2} = 83 \text{ m/s}
\]

\[
\alpha = \tan^{-1} \left( \frac{v_y}{v_x} \right) = \tan^{-1} \left( \frac{35 \text{ m/s}}{75 \text{ m/s}} \right) = 25^\circ
\]

and

\[
\theta = 180^\circ - \alpha = 180^\circ - 25^\circ = 155^\circ
\]

(c) Sketch the force vector (call it \( \vec{F} \)) and its \( x \)-component and show \( F_y \):

Use trigonometry to find \( \alpha \) and, hence, \( \theta \):

\[
\alpha = \cos^{-1} \left( \frac{F_x}{F} \right) = \cos^{-1} \left( \frac{40 \text{ lb}}{50 \text{ lb}} \right) = 37^\circ
\]

\[
\theta = 180^\circ + \alpha = 180^\circ + 37^\circ = 217^\circ
\]

Use the relationship between a vector and its vertical component to find \( F_y \):

\[
F_y = F \sin \theta = (50 \text{ lb}) \sin 217^\circ = -30 \text{ lb}
\]

Remarks: In Part (c) we could have used the Pythagorean Theorem to find the magnitude of \( F_y \).

55 • You walk 100 m in a straight line on a horizontal plane. If this walk took you 50 m east, what are your possible north or south movements? What are the possible angles that your walk made with respect to due east?
Picture the Problem There are two directions that you could have walked that are consistent with your having walked 50 m to the east. One possibility is walking in the north-east direction and the other is walking in the south-east direction. We can use the Pythagorean Theorem to find the distance you walked either north or south and then use trigonometry to find the two angles that correspond to your having walked in either of these directions.

Letting $A$ represent the magnitude of your displacement on the walk, use the Pythagorean Theorem to relate $A$ to its horizontal ($A_x$) and vertical ($A_y$) components:

$$A_y = \sqrt{A^2 - A_x^2}$$

Substitute numerical values and evaluate $A_y$:

$$A_y = \sqrt{(100 \text{ m})^2 - (50 \text{ m})^2} = \pm 87 \text{ m}$$

The plus-and-minus-signs mean that you could have gone 87 m north or 87 m south.

Use trigonometry to relate the directions you could have walked to the distance you walked and its easterly component:

$$\theta = \cos^{-1}\left(\frac{50 \text{ m}}{100 \text{ m}}\right) = \pm 60^\circ$$

The plus-and-minus-signs mean that you could have walked 60° north of east or 60° south of east.

56 • The final destination of your journey is 300 m due east of your starting point. The first leg of this journey is the walk described in Problem 55, and the second leg is also a walk along a single straight-line path. Estimate graphically the length and heading for the second leg of your journey.

Picture the Problem Let \(\vec{A}\) be the vector whose length is 100 m and whose direction is 60° N of E. Let \(\vec{B}\) be the vector whose direction and magnitude we are to determine and assume that you initially walked in a direction north of east. The graphical representation that we can use to estimate these quantities is shown below. Knowing that the magnitude of \(\vec{A}\) is 100 m, use any convenient scale to determine the length of \(\vec{B}\) and a protractor to determine the value of \(\theta\).
The magnitude of the second leg of your journey is about 260 m at an angle of approximately 20° S of E. If you had initially walked into the southeast, the magnitude of the second leg of your journey would still be about 260 m but its direction would be approximately 20° N of E.

Remarks: If you use the Pythagorean Theorem and right-triangle trigonometry, you’ll find that the length of the second leg of your journey is 265 m and that $\theta = 19°$ S of E.

57  **  Given the following vectors: $\vec{A} = 3.4\hat{i} + 4.7\hat{j}$, $\vec{B} = (-7.7)\hat{i} + 3.2\hat{j}$, and $\vec{C} = 5.4\hat{i} + (-9.1)\hat{j}$. (a) Find the vector $\vec{D}$, in unit vector notation, such that $\vec{D} + 2\vec{A} - 3\vec{C} + 4\vec{B} = 0$. (b) Express your answer in Part (a) in terms of magnitude and angle with the +x direction.

**Picture the Problem** We can find the vector $\vec{D}$ by solving the equation $\vec{D} + 2\vec{A} - 3\vec{C} + 4\vec{B} = 0$ for $\vec{D}$ and then substituting for the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. In (b) we can use the components of $\vec{D}$ to find its magnitude and direction.

(a) Solve the vector equation that gives the condition that must satisfied for $\vec{D}$:

$$\vec{D} = -2\vec{A} + 3\vec{C} - 4\vec{B}$$

Substitute for $\vec{A}$, $\vec{C}$ and $\vec{B}$ and simplify to obtain:

$$\vec{D} = -2(3.4\hat{i} + 4.7\hat{j}) + 3(5.4\hat{i} - 9.1\hat{j}) - 4(-7.7\hat{i} + 3.2\hat{j})$$

$$= (-6.8 + 16.2 + 30.8)\hat{i} + (-27.3 - 12.8)\hat{j}$$

$$= 40.2\hat{i} - 49.5\hat{j} = 40\hat{i} - 50\hat{j}$$

(b) Use the Pythagorean Theorem to relate the magnitude of $\vec{D}$ to its components $D_x$ and $D_y$:

$$D = \sqrt{D_x^2 + D_y^2}$$
Substitute numerical values and evaluate \( D \):

\[
D = \sqrt{(40.2)^2 + (-49.5)^2} = 63.8
\]

Use trigonometry to express and evaluate the angle \( \theta \):

\[
\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{-49.5}{40.2}\right)
\]

\[
= -51^\circ
\]

where the minus sign means that \( \vec{D} \) is in the 4th quadrant.

**58**

Given the following force vectors: \( \vec{A} \) is 25 lb at an angle of 30° clockwise from the +x axis, and \( \vec{B} \) is 42 lb at an angle of 50° clockwise from the +y axis. \((a)\) Make a sketch and visually estimate the magnitude and angle of the vector \( \vec{C} \) such that \( 2\vec{A} + \vec{C} - \vec{B} \) results in a vector with a magnitude of 35 lb pointing in the +x direction. \((b)\) Repeat the calculation in Part \((a)\) using the method of components and compare your result to the estimate in \((a)\).

**Picture the Problem** A diagram showing the condition that \( 2\vec{A} + \vec{C} - \vec{B} = 35\hat{i} \) and from which one can scale the values of \( \vec{C} \) and \( \theta \) is shown below. In \((b)\) we can use the two scalar equations corresponding to this vector equation to check our graphical results.

\((a)\) A diagram showing the conditions imposed by \( 2\vec{A} + \vec{C} - \vec{B} = 35\hat{i} \) approximately to scale is shown to the right.

The magnitude of \( \vec{C} \) is approximately \( 57 \) lb and the angle \( \theta \) is approximately \( 68^\circ \).

\((b)\) Express the condition relating the vectors \( \vec{A}, \vec{B}, \) and \( \vec{C} \):

\[
2\vec{A} + \vec{C} - \vec{B} = 35\hat{i}
\]

The corresponding scalar equations are:

\[
2A_x + C_x - B_x = 35
\]

and

\[
2A_y + C_y - B_y = 0
\]
Solve these equations for $C_x$ and $C_y$ to obtain:

$$C_x = 35 - 2A_x + B_x$$
and
$$C_y = -2A_y + B_y$$

Substitute for the $x$ and $y$ components of $\vec{A}$ and $\vec{B}$ to obtain:

$$C_x = 35 \text{ lb} - 2[(25 \text{ lb})\cos 330^\circ] + (42 \text{ lb})\cos 40^\circ = 23.9 \text{ lb}$$
and
$$C_y = -2[(25 \text{ lb})\sin 330^\circ] + (42 \text{ lb})\sin 40^\circ = 52 \text{ lb}$$

Use the Pythagorean Theorem to relate the magnitude of $\vec{C}$ to its components:

$$C = \sqrt{C_x^2 + C_y^2}$$

Substitute numerical values and evaluate $C$:

$$C = \sqrt{(23.9 \text{ lb})^2 + (52.0 \text{ lb})^2} = 57 \text{ lb}$$

Use trigonometry to find the direction of $\vec{C}$:

$$\theta = \tan^{-1} \left( \frac{C_y}{C_x} \right) = \tan^{-1} \left( \frac{52.0 \text{ lb}}{23.9 \text{ lb}} \right) = 65^\circ$$

Remarks: The analytical results for $C$ and $\theta$ are in excellent agreement with the values determined graphically.

59 [SSM] Calculate the unit vector (in terms of $\hat{i}$ and $\hat{j}$) in the direction opposite to the direction of each of the vectors in Problem 57.

Picture the Problem

The unit vector in the direction opposite to the direction of a vector is found by taking the negative of the unit vector in the direction of the vector. The unit vector in the direction of a given vector is found by dividing the given vector by its magnitude.

The unit vector in the direction of $\vec{A}$ is given by:

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{\sqrt{A_x^2 + A_y^2}}$$

Substitute for $\vec{A}$, $A_x$, and $A_y$ and evaluate $\hat{A}$:

$$\hat{A} = \frac{3.4\hat{i} + 4.7\hat{j}}{\sqrt{(3.4)^2 + (4.7)^2}} = 0.59\hat{i} + 0.81\hat{j}$$
The unit vector in the direction opposite to that of $\vec{A}$ is given by:

$$\hat{A} = -0.59\hat{i} - 0.81\hat{j}$$

The unit vector in the direction of $\vec{B}$ is given by:

$$\hat{B} = \frac{\vec{B}}{\|\vec{B}\|} = \frac{\vec{B}}{\sqrt{B_x^2 + B_y^2}}$$

Substitute for $\vec{B}$, $B_x$, and $B_y$ and evaluate $\hat{B}$:

$$\hat{B} = \frac{-7.7\hat{i} + 3.2\hat{j}}{\sqrt{(-7.7)^2 + (3.2)^2}} = -0.92\hat{i} + 0.38\hat{j}$$

The unit vector in the direction opposite to that of $\vec{B}$ is given by:

$$-\hat{B} = 0.92\hat{i} - 0.38\hat{j}$$

The unit vector in the direction of $\vec{C}$ is given by:

$$\hat{C} = \frac{\vec{C}}{\|\vec{C}\|} = \frac{\vec{C}}{\sqrt{C_x^2 + C_y^2}}$$

Substitute for $\vec{C}$, $C_x$, and $C_y$ and evaluate $\hat{C}$:

$$\hat{C} = \frac{5.4\hat{i} - 9.1\hat{j}}{\sqrt{(5.4)^2 + (-9.1)^2}} = 0.51\hat{i} - 0.86\hat{j}$$

The unit vector in the direction opposite that of $\vec{C}$ is given by:

$$-\hat{C} = -0.51\hat{i} + 0.86\hat{j}$$

Unit vectors $\hat{i}$ and $\hat{j}$ are directed east and north, respectively. Calculate the unit vector (in terms of $\hat{i}$ and $\hat{j}$) in the following directions. (a) northeast, (b) 70° clockwise from the $-y$ axis, (c) southwest.

**Picture the Problem** The unit vector in a given direction is a vector pointing in that direction whose magnitude is 1.

(a) The unit vector in the northeast direction is given by:

$$\hat{u}_{\text{NE}} = (1)\cos 45^\circ \hat{i} + (1)\sin 45^\circ \hat{j} = 0.707\hat{i} + 0.707\hat{j}$$

(b) The unit vector 70° clockwise from the $-y$ axis is given by:

$$\hat{u} = (1)\cos 200^\circ \hat{i} + (1)\sin 200^\circ \hat{j} = -0.940\hat{i} - 0.342\hat{j}$$
(c) The unit vector in the southwest direction is given by:

$$\mathbf{u}_{\text{SW}} = (1)\cos 225^\circ \mathbf{i} + (1)\sin 225^\circ \mathbf{j}$$

$$= -0.707\mathbf{i} - 0.707\mathbf{j}$$

Remarks: One can confirm that a given vector is, in fact, a unit vector by checking its magnitude.

**General Problems**

61 • [SSM]  The Apollo trips to the moon in the 1960's and 1970's typically took 3 days to travel the Earth-moon distance once they left Earth orbit. Estimate the spacecraft's average speed in kilometers per hour, miles per hour, and meters per second.

**Picture the Problem** Average speed is defined to be the distance traveled divided by the elapsed time. The Earth-moon distance and the distance and time conversion factors can be found on the inside-front cover of the text. We’ll assume that 3 days means exactly three days.

Express the average speed of Apollo as it travels to the moon:

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{elapsed time}}$$

Substitute numerical values to obtain:

$$v_{\text{av}} = \frac{2.39 \times 10^5 \text{ mi}}{3 \text{ d}}$$

Use the fact that there are 24 h in 1 d to convert 3 d into hours:

$$v_{\text{av}} = \frac{2.39 \times 10^5 \text{ mi}}{3 \text{ d} \times \frac{24 \text{ h}}{\text{d}}}$$

$$= 3.319 \times 10^3 \text{ mi/h}$$

$$= 3.32 \times 10^3 \text{ mi/h}$$

Use the fact that 1 mi is equal to 1.609 km to convert the spacecraft’s average speed to km/h:

$$v_{\text{av}} = 3.319 \times 10^3 \frac{\text{mi}}{\text{h}} \times 1.609 \frac{\text{km}}{\text{mi}} = 5.340 \times 10^3 \frac{\text{km}}{\text{h}} = 5.34 \times 10^3 \text{ km/h}$$
Use the facts that there are 3600 s in 1 h and 1000 m in 1 km to convert the spacecraft’s average speed to m/s:

\[ v_{av} = 5.340 \times 10^6 \text{ km/h} \times \frac{10^3 \text{ m}}{\text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 1.485 \times 10^3 \text{ m/s} = 1.49 \times 10^3 \text{ m/s} \]

**Remarks:** An alternative to multiplying by \(10^3 \text{ m/km}\) in the last step is to replace the metric prefix "\(k\)" in "\(\text{km}\)" by \(10^3\).

**62** • On many of the roads in Canada the speed limit is 100 km/h. What is this speed limit in miles per hour?

**Picture the Problem** We can use the conversion factor \(1 \text{ mi} = 1.609 \text{ km}\) to convert 100 km/h into mi/h.

Multiply 100 km/h by \(1 \text{ mi}/1.609 \text{ km}\) to obtain:

\[ 100 \frac{\text{km}}{\text{h}} = 100 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ mi}}{1.609 \text{ km}} = 62.2 \text{ mi/h} \]

**63** • If you could count $1.00 per second, how many years would it take to count 1.00 billion dollars?

**Picture the Problem** We can use a series of conversion factors to convert 1 billion seconds into years.

Multiply 1 billion seconds by the appropriate conversion factors to convert into years:

\[ 10^9 \text{ s} = \frac{10^9 \text{ s}}{3600 \text{ s}} \times \frac{1 \text{ h}}{24 \text{ h}} \times \frac{1 \text{ day}}{365.24 \text{ days}} = 31.7 \text{ y} \]

**64** • (a) The speed of light in vacuum is 186,000 mi/s = \(3.00 \times 10^8 \text{ m/s}\). Use this fact to find the number of kilometers in a mile. (b) The weight of 1.00 ft³ of water is 62.4 lb, and 1.00 ft = 30.5 cm. Use this and the fact that 1.00 cm³ of water has a mass of 1.00 g to find the weight in pounds of a 1.00-kg mass.

**Picture the Problem** In both the examples cited we can equate expressions for the physical quantities, expressed in different units, and then divide both sides of the equation by one of the expressions to obtain the desired conversion factor.
(a) Divide both sides of the equation expressing the speed of light in the two systems of measurement by 186,000 mi/s to obtain:

\[
1 = \frac{3.00 \times 10^8 \text{ m/s}}{1.86 \times 10^5 \text{ mi/h}} = 1.61 \times 10^3 \text{ m/mi}
\]

\[
= \left(1.61 \times 10^3 \frac{\text{m}}{\text{mi}}\right) \left(\frac{1 \text{ km}}{10^3 \text{ m}}\right) = 1.61 \text{ km/mi}
\]

(b) Find the volume of 1.00 kg of water:

Volume of 1.00 kg = \(10^3 \text{ g}\) is \(10^3 \text{ cm}^3\)

Express \(10^3 \text{ cm}^3\) in \(\text{ft}^3\):

\[
(10\text{ cm})^3 \left(\frac{1.00\text{ ft}}{30.5\text{ cm}}\right)^3 = 0.03525 \text{ ft}^3
\]

Relate the weight of 1.00 ft\(^3\) of water to the volume occupied by 1.00 kg of water:

\[
\frac{1.00\text{ kg}}{0.03525\text{ ft}^3} = 62.4 \frac{\text{ lb}}{\text{ ft}^3}
\]

Divide both sides of the equation by the left-hand side to obtain:

\[
1 = \frac{62.4 \frac{\text{ lb}}{\text{ ft}^3}}{1.00\text{ kg}} = \frac{2.20\text{ lb/kg}}{0.03525\text{ ft}^3}
\]

65 • The mass of one uranium atom is \(4.0 \times 10^{-26} \text{ kg}\). How many uranium atoms are there in 8.0 g of pure uranium?

**Picture the Problem** We can use the given information to equate the ratios of the number of uranium atoms in 8 g of pure uranium and of 1 atom to its mass.

Express the proportion relating the number of uranium atoms \(N_U\) in 8.0 g of pure uranium to the mass of 1 atom:

\[
\frac{N_U}{8.0\text{ g}} = \frac{1\text{ atom}}{4.0 \times 10^{-26} \text{ kg}}
\]

Solve for and evaluate \(N_U\):

\[
N_U = \left(8.0\text{ g} \times \frac{1\text{ kg}}{10^3\text{ g}}\right) \times \frac{1\text{ atom}}{4.0 \times 10^{-26} \text{ kg}} = 2.0 \times 10^{23}
\]
During a thunderstorm, a total of 1.4 in of rain falls. How much water falls on one acre of land? (1 mi² = 640 acres.) Express your answer in (a) cubic inches, (b) cubic feet, (c) cubic meters, and (d) kilograms. Note that the density of water is 1000 kg/m³.

**Picture the Problem** Assuming that the water is distributed uniformly over the one acre of land, its volume is the product of the area over which it is distributed and its depth. The mass of the water is the product of its density and volume. The required conversion factors can be found in the front material of the text.

(a) Express the volume \( V \) of water in terms of its depth \( h \) and the area \( A \) over which it falls:

Substitute numerical values and evaluate \( V \):

\[
V = (1 \text{ acre}) \left( \frac{1 \text{ mi}^2}{640 \text{ acres}} \right) \left( \frac{5280 \text{ ft}}{\text{mi}} \right)^2 \left( \frac{12 \text{ in}}{\text{ft}} \right)^2 \left( 1.4 \text{ in} \right) = 8.782 \times 10^6 \text{ in}^3
\]

\[
= 8.8 \times 10^6 \text{ in}^3
\]

(b) Convert in³ into ft³:

\[
V = 8.782 \times 10^6 \text{ in}^3 \times \left( \frac{1 \text{ ft}}{12 \text{ in}} \right)^3 = 5.082 \times 10^3 \text{ ft}^3 = 5.1 \times 10^3 \text{ ft}^3
\]

(c) Convert ft³ into m³:

\[
V = 5.082 \times 10^3 \text{ ft}^3 \times \left( \frac{12 \text{ in}}{\text{ft}} \right)^3 \times \left( \frac{2.540 \text{ cm}}{\text{in}} \right)^3 \times \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^3 = 1.439 \times 10^2 \text{ m}^3
\]

\[
= 1.4 \times 10^2 \text{ m}^3
\]

(d) The mass of the water is the product of its density \( \rho \) and volume \( V \):

Substitute numerical values and evaluate \( m \):

\[
m = \rho V = \left( 10^3 \frac{\text{kg}}{\text{m}^3} \right) \left( 1.439 \times 10^2 \text{ m}^3 \right) = 1.4 \times 10^5 \text{ kg}
\]
An iron nucleus has a radius of $5.4 \times 10^{-15}$ m and a mass of $9.3 \times 10^{-26}$ kg. (a) What is its mass per unit volume in kg/m$^3$? (b) If Earth had the same mass per unit volume, what would be its radius? (The mass of Earth is $5.98 \times 10^{24}$ kg.)

**Picture the Problem** The mass per unit volume of an object is its density.

(a) The density $\rho$ of an object is its mass $m$ per unit volume $V$:

$$\rho = \frac{m}{V}$$

Assuming the iron nucleus to be spherical, its volume as a function of its radius $r$ is given by:

$$V = \frac{4}{3} \pi r^3$$

Substitute for $V$ and simplify to obtain:

$$\rho = \frac{m}{\frac{4}{3} \pi r^3} = \frac{3m}{4 \pi r^3}$$

(1)

Substitute numerical values and evaluate $\rho$:

$$\rho = \frac{3(9.3 \times 10^{-26} \text{ kg})}{4 \pi (5.4 \times 10^{-15} \text{ m})^3}$$

$$= 1.410 \times 10^{17} \text{ kg/m}^3$$

$$= \boxed{1.4 \times 10^{17} \text{ kg/m}^3}$$

(b) Solve equation (1) for $r$ to obtain:

$$r = \sqrt[3]{\frac{3m}{4 \pi \rho}}$$

Substitute numerical values and evaluate $r$:

$$r = \sqrt[3]{\frac{3(5.98 \times 10^{24} \text{ kg})}{4 \pi (1.410 \times 10^{17} \text{ kg/m}^3)}}$$

$$= \boxed{2.2 \times 10^2 \text{ m}}$$

or about 200 m!

The Canadian Norman Wells Oil Pipeline extends from Norman Wells, Northwest Territories, to Zama, Alberta. The $8.68 \times 10^5$-m-long pipeline has an inside diameter of 12 in and can be supplied with oil at 35 L/s. (a) What is the volume of oil in the pipeline if it is full at some instant in time? (b) How long would it take to fill the pipeline with oil if it is initially empty?
Picture the Problem The volume of a cylinder is the product of its cross-sectional area and its length. The time required to fill the pipeline with oil is the ratio of its volume to the flow rate \( R \) of the oil. We’ll assume that the pipe has a diameter of exactly 12 in.

(a) Express the volume \( V \) of the cylindrical pipe in terms of its radius \( r \) and its length \( L \):

\[
V = \pi r^2 L
\]

Substitute numerical values and evaluate \( V \):

\[
V = \pi \left( 6 \text{ in} \times \frac{2.540 \times 10^{-2} \text{ m}}{\text{in}} \right)^2 \left( 8.68 \times 10^5 \text{ m} \right) = 6.3 \times 10^4 \text{ m}^3
\]

(b) Express the time \( \Delta t \) to fill the pipe in terms of its volume \( V \) and the flow rate \( R \) of the oil:

\[
\Delta t = \frac{V}{R}
\]

Substitute numerical values and evaluate \( \Delta t \):

\[
\Delta t = \frac{6.3 \times 10^4 \text{ m}^3}{\left( 35 \frac{\text{L}}{\text{s}} \right) \left( 10^{-3} \text{ m}^3 / \text{L} \right)} = 1.8 \times 10^6 \text{ s}
\]

or about 21 days!

69 ** The astronomical unit (AU) is defined as the mean center-to-center distance from Earth to the sun, namely \( 1.496 \times 10^{11} \text{ m} \). The parsec is the radius of a circle for which a central angle of 1 s intercepts an arc of length 1 AU. The light-year is the distance that light travels in 1 y. (a) How many parsecs are there in one astronomical unit? (b) How many meters are in a parsec? (c) How many meters in a light-year? (d) How many astronomical units in a light-year? (e) How many light-years in a parsec?

Picture the Problem We can use the relationship between an angle \( \theta \), measured in radians, subtended at the center of a circle, the radius \( R \) of the circle, and the length \( L \) of the arc to answer these questions concerning the astronomical units of measure. We’ll take the speed of light to be \( 2.998 \times 10^8 \text{ m/s} \).

(a) Relate the angle \( \theta \) subtended by an arc of length \( S \) to the distance \( R \):

\[
\theta = \frac{S}{R} \Rightarrow S = R \theta \quad (1)
\]
Substitute numerical values and evaluate \( S \):

\[
S = (1 \text{ parsec})(1 \text{s}) \left( \frac{1 \text{ min}}{60 \text{s}} \right) \left( \frac{1^\circ}{60 \text{ min}} \right) \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = 4.848 \times 10^{-6} \text{ parsec}
\]

\( (b) \) Solving equation (1) for \( R \) yields:

\[
R = \frac{S}{\theta}
\]

Substitute numerical values and evaluate \( R \):

\[
R = \frac{1.496 \times 10^{11} \text{ m}}{(1 \text{s}) \left( \frac{1 \text{ min}}{60 \text{s}} \right) \left( \frac{1^\circ}{60 \text{ min}} \right) \left( \frac{2\pi \text{ rad}}{360^\circ} \right)} = 3.086 \times 10^{16} \text{ m}
\]

\( (c) \) The distance \( D \) light travels in a given interval of time \( \Delta t \) is given by:

\[
D = c\Delta t
\]

Substitute numerical values and evaluate \( D \):

\[
D = \left( 2.998 \times 10^8 \frac{\text{m}}{\text{s}} \right) \left( 365.24 \frac{\text{d}}{\text{y}} \right) \left( 24 \frac{\text{h}}{\text{d}} \right) \left( 60 \frac{\text{min}}{\text{h}} \right) \left( 60 \frac{\text{s}}{\text{min}} \right) = 9.461 \times 10^{15} \text{ m}
\]

\( (d) \) Use the definition of 1 AU and the result from Part (c) to obtain:

\[
1 c \cdot y = \left( 9.461 \times 10^{15} \text{ m} \right) \left( \frac{1 \text{AU}}{1.496 \times 10^{11} \text{ m}} \right) = 6.324 \times 10^4 \text{ AU}
\]

\( (e) \) Combine the results of Parts (b) and (c) to obtain:

\[
1 \text{ parsec} = \left( 3.086 \times 10^{16} \text{ m} \right) \left( \frac{1 c \cdot y}{9.461 \times 10^{15} \text{ m}} \right) = 3.262 c \cdot y
\]

70 ** If the average density of the universe is at least \( 6 \times 10^{-27} \text{ kg/m}^3 \), then the universe will eventually stop expanding and begin contracting. \( (a) \) How many electrons are needed in each cubic meter to produce the critical density? \( (b) \) How many protons per cubic meter would produce the critical density?

\( (m_e = 9.11 \times 10^{-31} \text{ kg}; m_p = 1.67 \times 10^{-27} \text{ kg}) \)
Picture the Problem Let \( N_e \) and \( N_p \) represent the number of electrons and the number of protons, respectively and \( \rho \) the critical average density of the universe. We can relate these quantities to the masses of the electron and proton using the definition of density.

(a) Using its definition, relate the required density \( \rho \) to the electron density \( N_e/V \):

\[
\rho = \frac{m_e}{V} = \frac{N_e m_e}{V} = \frac{N_e}{V} \frac{m_e}{m_e} = \frac{\rho}{m_e} \quad (1)
\]

Substitute numerical values and evaluate \( N_e/V \):

\[
\frac{N_e}{V} = \frac{6 \times 10^{-27} \text{ kg/m}^3}{9.11 \times 10^{-31} \text{ kg/electron}} = 6.586 \times 10^3 \text{ electrons/m}^3 \\
\approx 7 \times 10^3 \text{ electrons/m}^3
\]

(b) Express and evaluate the ratio of the masses of an electron and a proton:

\[
\frac{m_e}{m_p} = \frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = 5.455 \times 10^{-4}
\]

Rewrite equation (1) in terms of protons:

\[
\frac{N_p}{V} = \frac{\rho}{m_p} \quad (2)
\]

Divide equation (2) by equation (1) to obtain:

\[
\frac{N_p}{V} = \frac{m_e}{m_p} \text{ or } \frac{N_p}{N_e} = \frac{m_e}{m_p} \left( \frac{N_e}{V} \right)
\]

Substitute numerical values and use the result from Part (a) to evaluate \( N_p/V \):

\[
\frac{N_p}{V} = (5.455 \times 10^{-4})(6.586 \times 10^3 \text{ electrons/m}^3) \approx 4 \text{ protons/m}^3
\]

You are an astronaut doing physics experiments on the moon. You are interested in the experimental relationship between distance fallen, \( y \), and time elapsed, \( t \), of falling objects dropped from rest. You have taken some data for a falling penny, which is represented in the table below. You expect that a general relationship between distance \( y \) and time \( t \) is \( y = Bt^C \), where \( B \) and \( C \) are constants to be determined experimentally. To accomplish this, create a log-log plot of the data: (a) graph \( \log(y) \) vs. \( \log(t) \), with \( \log(y) \) the ordinate variable and \( \log(t) \) the abscissa variable. (b) Show that if you take the log of each side of your equation, you get \( \log(y) = \log(B) + C\log(t) \). (c) By comparing this linear relationship to the graph of the data, estimate the values of \( B \) and \( C \). (d) If you
drop a penny, how long should it take to fall 1.0 m? (e) In the next chapter, we will show that the expected relationship between $y$ and $t$ is $y = \frac{1}{2}at^2$, where $a$ is the acceleration of the object. What is the acceleration of objects dropped on the moon?

<table>
<thead>
<tr>
<th>$y$ (m)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$ (s)</td>
<td>3.5</td>
<td>5.2</td>
<td>6.0</td>
<td>7.3</td>
<td>7.9</td>
</tr>
</tbody>
</table>

**Picture the Problem** We can plot log $y$ versus log $t$ and find the slope of the best-fit line to determine the exponent $C$. The value of $B$ can be determined from the intercept of this graph. Once we know $C$ and $B$, we can solve $y = Bt^C$ for $t$ as a function of $y$ and use this result to determine the time required for an object to fall a given distance on the surface of the moon.

(a) The following graph of log $y$ versus log $t$ was created using a spreadsheet program. The equation shown on the graph was obtained using Excel’s "Add Trendline" function. (Excel’s "Add Trendline" function uses regression analysis to generate the trendline.)

(b) Taking the logarithm of both sides of the equation $y = Bt^C$ yields:

$$\log y = \log(Bt^C) = \log B + \log t^C = \log B + C \log t$$

(c) Note that this result is of the form:

$$Y = b + mX$$

where

$Y = \log y$, $b = \log B$, $m = C$, and $X = \log t$
From the regression analysis (trendline) we have:

\[ \log B = -0.076 \]

Solving for \( B \) yields:

\[ B = 10^{-0.076} = 0.84 \text{ m/s}^2 \]

where we have inferred the units from the given for \( y = Bt^C \).

Also, from the regression analysis we have:

\[ C = 1.96 \approx 2.0 \]

(d) Solve \( y = Bt^C \) for \( t \) to obtain:

\[ t = \left( \frac{y}{B} \right)^{\frac{1}{C}} \]

Substitute numerical values and evaluate \( t \) to determine how long it would take a penny to fall 1.0 m:

\[ t = \left( \frac{1.0 \text{ m}}{0.84 \text{ m/s}^2} \right)^{\frac{1}{2}} \approx 1.1 \text{ s} \]

(e) Substituting for \( B \) and \( C \) in \( y = Bt^C \) yields:

\[ y = \left( 0.84 \frac{\text{m}}{\text{s}^2} \right) t^2 \]

Compare this equation to \( y = \frac{1}{2} at^2 \) to obtain:

\[ \frac{1}{2} a = 0.84 \frac{\text{m}}{\text{s}^2} \]

and

\[ a = 2 \left( 0.84 \frac{\text{m}}{\text{s}^2} \right) = 1.7 \frac{\text{m}}{\text{s}^2} \]

Remarks: One could use a graphing calculator to obtain the results in Parts (a) and (c).

72 A particular company’s stock prices vary with the market and with the company’s type of business, and can be very unpredictable, but people often try to look for mathematical patterns where they may not belong. Corning is a materials-engineering company located in upstate New York. Below is a table of the price of Corning stock on August 3, for every 5 years from 1981 to 2001. Assume that the price follows a power law: price (in $) = Bt^C \) where \( t \) is expressed in years. (a) Evaluate the constants \( B \) and \( C \). (b) According to the power law, what should the price of Corning stock have been on August 3, 2000? (It was actually $82.83!)
### Picture the Problem

We can plot $\log P$ versus $\log t$ and find the slope of the best-fit line to determine the exponent $C$. The value of $B$ can be determined from the intercept of this graph. Once we know $C$ and $B$, we can use $P = B t^C$ to predict the price of Corning stock as a function of time.

(a) The following graph of $\log P$ versus $\log t$ was created using a spreadsheet program. The equation shown on the graph was obtained using Excel’s “Add Trendline” function. (Excel’s “Add Trendline” function uses regression analysis to generate the trendline.)

\[
\log P = 0.658 \log t + 0.2614
\]

Taking the logarithm of both sides of the equation $P = B t^C$ yields:

\[
\log P = \log(B t^C) = \log B + \log(t^C) = \log B + C \log t
\]

Note that this result is of the form: $Y = b + mX$

where

$Y = \log P$, $b = \log B$, $m = C$, and $X = \log t$

From the regression analysis (trendline) we have:

\[
\log B = 0.2614 \Rightarrow B = 10^{0.2614} = \boxed{1.83}
\]

Also, from the regression analysis we have:

\[
C = \boxed{0.658}
\]

(b) Substituting for $B$ and $C$ in $P = B t^C$ yields:

\[
P = (1.83) t^{0.658}
\]
Evaluate $P(20\ y)$ to obtain:

\[
P(20\ y) = (1.83)(20)^{0.658} = \$13.14
\]

**Remarks:** One could use a graphing calculator to obtain these results.

73  **[SSM]** The Super-Kamiokande neutrino detector in Japan is a large transparent cylinder filled with ultra pure water. The height of the cylinder is 41.4 m and the diameter is 39.3 m. Calculate the mass of the water in the cylinder. Does this match the claim posted on the official Super-K Web site that the detector uses 50000 tons of water?

**Picture the Problem** We can use the definition of density to relate the mass of the water in the cylinder to its volume and the formula for the volume of a cylinder to express the volume of water used in the detector’s cylinder. To convert our answer in kg to lb, we can use the fact that 1 kg weighs about 2.205 lb.

Relate the mass of water contained in the cylinder to its density and volume:

\[
m = \rho V
\]

Express the volume of a cylinder in terms of its diameter $d$ and height $h$:

\[
V = A_{\text{base}} h = \frac{\pi}{4} d^2 h
\]

Substitute in the expression for $m$ to obtain:

\[
m = \rho \frac{\pi}{4} d^2 h
\]

Substitute numerical values and evaluate $m$:

\[
m = (10^3 \text{ kg/m}^3) \left(\frac{\pi}{4}\right) (39.3 \text{ m})^2 (41.4 \text{ m})
\]

\[
= 5.022 \times 10^7 \text{ kg}
\]

Convert $5.02 \times 10^7$ kg to tons:

\[
m = 5.022 \times 10^7 \text{ kg} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} \times \frac{1 \text{ ton}}{2000 \text{ lb}}
\]

\[
= 55.4 \times 10^3 \text{ ton}
\]

The 50,000-ton claim is conservative. The actual weight is closer to 55,000 tons.

74  **[SSM]** You and a friend are out hiking across a large flat plain and decide to determine the height of a distant mountain peak, and also the horizontal distance from you to the peak (Figure 1-19). In order to do this, you stand in one spot and determine that the sightline to the top of the peak is inclined at 7.5° above the horizontal. You also make note of the heading to the peak at that point: 13° east of north. You stand at the original position, and your friend hikes due west for 1.5 km. He then sights the peak and determines that its sightline has a heading of 15° east of north. How far is the mountain from your position, and how high is its summit above your position?
Picture the Problem  Vector $\vec{A}$ lies in the plane of the plain and locates the base or the peak relative to you. Vector $\vec{B}$ also lies in the plane of the plain and locates the base of the peak relative to your friend when he/she has walked 1.5 km to the west. We can use the geometry of the diagram and the E-W components of the vectors $\vec{d}$, $\vec{B}$ and $\vec{A}$ to find the distance to the mountain from your position. Once we know the distance $A$, we can use a trigonometric relationship to find the height of the peak above your position.

Referring to the diagram, note that:

$$B \sin \beta = h$$

and

$$A \sin \alpha = h$$

Equating these expressions for $h$ gives:

$$B \sin \beta = A \sin \alpha \Rightarrow B = \frac{\sin \alpha}{\sin \beta} A$$

Adding the E-W components of the vectors $\vec{d}$, $\vec{B}$ and $\vec{A}$ yields:

$$1.5 \text{ km} = B \cos \beta - A \cos \alpha$$

Substitute for $B$ and simplify to obtain:

$$1.5 \text{ km} = A \left( \frac{\sin \alpha}{\sin \beta} \cos \beta - A \cos \alpha \right)$$

$$= A \left( \frac{\sin \alpha}{\tan \beta} - \cos \alpha \right)$$

Solving for $A$ yields:

$$A = \frac{1.5 \text{ km}}{\frac{\sin \alpha}{\tan \beta} - \cos \alpha}$$
Substitute numerical values and evaluate $A$:

$$A = \frac{1.5 \text{ km}}{\sin 77^\circ \tan 75^\circ - \cos 77^\circ} = 41.52 \text{ km}$$

$$= 42 \text{ km}$$

Referring to the following diagram, we note that:

$$h = A \tan 7.5^\circ$$

$$= (41.52 \text{ km}) \tan 7.5^\circ$$

$$= 5.5 \text{ km}$$

**Remarks:** One can also solve this problem using the law of sines.

75  •••  The table below gives the periods $T$ and orbit radii $r$ for the motions of four satellites orbiting a dense, heavy asteroid. (a) These data can be fitted by the formula $T = Cr^n$. Find the values of the constants $C$ and $n$. (b) A fifth satellite is discovered to have a period of 6.20 y. Find the radius for the orbit of this satellite, which fits the same formula.

<table>
<thead>
<tr>
<th>Period $T$, y</th>
<th>0.44</th>
<th>1.61</th>
<th>3.88</th>
<th>7.89</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius $r$, Gm</td>
<td>0.088</td>
<td>0.208</td>
<td>0.374</td>
<td>0.600</td>
</tr>
</tbody>
</table>

**Picture the Problem** We can plot log $T$ versus log $r$ and find the slope of the best-fit line to determine the exponent $n$. We can then use any of the ordered pairs to evaluate $C$. Once we know $n$ and $C$, we can solve $T = Cr^n$ for $r$ as a function of $T$.

(a) Take the logarithm (we’ll arbitrarily use base 10) of both sides of $T = Cr^n$ and simplify to obtain:

$$\log(T) = \log(Cr^n) = \log C + \log r^n$$

$$= n \log r + \log C$$

Note that this equation is of the form $y = mx + b$. Hence a graph of log $T$ vs. log $r$ should be linear with a slope of $n$ and a log $T$ -intercept log $C$. 


The following graph of log $T$ versus log $r$ was created using a spreadsheet program. The equation shown on the graph was obtained using Excel’s "Add Trendline" function. (Excel’s "Add Trendline" function uses regression analysis to generate the trendline.)

From the regression analysis we note that:

\[ n = 1.50, \]

\[ C = 10^{1.2311} = 17.0 \text{ y/(Gm)}^{3/2}, \]

and

\[ T = \left( \frac{17.0 \text{ y/(Gm)}^{3/2}}{T} \right)^{1.50} \tag{1} \]

(b) Solve equation (1) for the radius of the planet’s orbit:

\[ r = \left( \frac{T}{17.0 \text{ y/(Gm)}^{3/2}} \right)^{2/3} \]

Substitute numerical values and evaluate $r$:

\[ r = \left( \frac{6.20 \text{ y}}{17.0 \text{ y/(Gm)}^{3/2}} \right)^{2/3} = 0.510 \text{ Gm} \]

76. The period $T$ of a simple pendulum depends on the length $L$ of the pendulum and the acceleration of gravity $g$ (dimensions $L/T^2$). (a) Find a simple combination of $L$ and $g$ that has the dimensions of time. (b) Check the dependence of the period $T$ on the length $L$ by measuring the period (time for a complete swing back and forth) of a pendulum for two different values of $L$. (c) The correct formula relating $T$ to $L$ and $g$ involves a constant that is a multiple of $\pi$, and cannot be obtained by the dimensional analysis of Part (a). It can be found by
experiment as in Part (b) if \( g \) is known. Using the value \( g = 9.81 \, \text{m/s}^2 \) and your experimental results from Part (b), find the formula relating \( T \) to \( L \) and \( g \).

**Picture the Problem** We can express the relationship between the period \( T \) of the pendulum, its length \( L \), and the acceleration of gravity \( g \) as \( T = CL^a g^b \) and perform dimensional analysis to find the values of \( a \) and \( b \) and, hence, the function relating these variables. Once we’ve performed the experiment called for in Part (b), we can determine an experimental value for \( C \).

(a) Express \( T \) as the product of \( L \) and \( g \) raised to powers \( a \) and \( b \):

\[
T = CL^a g^b \tag{1}
\]

where \( C \) is a dimensionless constant.

Write this equation in dimensional form:

\[
[T] = [L]^a [g]^b
\]

Substituting the dimensions of the physical quantities yields:

\[
T = L^a \left( \frac{L}{T^2} \right)^b
\]

Because \( L \) does not appear on the left-hand side of the equation, we can write this equation as:

\[
L^0 T^1 = L^{a+b} T^{-2b}
\]

Equate the exponents to obtain:

\[ a + b = 0 \quad \text{and} \quad -2b = 1 \]

Solve these equations simultaneously to find \( a \) and \( b \):

\[ a = \frac{1}{2} \quad \text{and} \quad b = -\frac{1}{2} \]

Substitute in equation (1) to obtain:

\[
T = CL^{\frac{1}{2}} g^{-\frac{1}{2}} = C \sqrt{\frac{L}{g}} \tag{2}
\]

(b) If you use pendulums of lengths 1.0 m and 0.50 m; the periods should be about:

\[
T(1.0\, \text{m}) = 2.0\, \text{s}
\]

and

\[
T(0.50\, \text{m}) = 1.4\, \text{s}
\]

(c) Solving equation (2) for \( C \) yields:

\[
C = T \sqrt{\frac{g}{L}}
\]
Evaluate \( C \) with \( L = 1.0 \) m and \( T = 2.0 \) s:

\[
C = (2.0 \text{s}) \sqrt{\frac{9.81 \text{m/s}^2}{1.0 \text{m}}} = 6.26 \approx 2\pi
\]

Substitute in equation (2) to obtain:

\[
T = 2\pi \sqrt{\frac{L}{g}}
\]

---

A sled at rest is suddenly pulled in three horizontal directions at the same time but it goes nowhere. Paul pulls to the northeast with a force of 50 lb. Johnny pulls at an angle of 35° south of due west with a force of 65 lb. Connie pulls with a force to be determined. (a) Express the boys' two forces in terms of the usual unit vectors (b) Determine the third force (from Connie), expressing it first in component form and then as a magnitude and angle (direction).

**Picture the Problem** A diagram showing the forces exerted by Paul, Johnny, and Connie is shown below. Once we've expressed the forces exerted by Paul and Johnny in vector form we can use them to find the force exerted by Connie.

(a) The force that Paul exerts is:

\[
\vec{F}_{\text{Paul}} = [(50 \text{lb})\cos 45°] \hat{i} + [(50 \text{lb})\sin 45°] \hat{j} = (35.4 \text{ lb}) \hat{i} + (35.4 \text{ lb}) \hat{j}
\]

The force that Johnny exerts is:

\[
\vec{F}_{\text{Johnny}} = [(65 \text{ lb})\cos 215°] \hat{i} + [(65 \text{ lb})\sin 215°] \hat{j} = (-53.2 \text{ lb}) \hat{i} - (37.3 \text{ lb}) \hat{j}
\]
The sum of the forces exerted by Paul and Johnny is:

\[
\vec{F}_{\text{Paul}} + \vec{F}_{\text{Johnny}} = (35.4 \text{ lb})\hat{i} + (35.4 \text{ lb})\hat{j} - (53.2 \text{ lb})\hat{i} - (37.3 \text{ lb})\hat{j} \\
= (-17.8 \text{ lb})\hat{i} - (1.9 \text{ lb})\hat{j}
\]

(b) The condition that the three forces must satisfy is:

\[
\vec{F}_{\text{Paul}} + \vec{F}_{\text{Johnny}} + \vec{F}_{\text{Connie}} = 0
\]

Solving for \( \vec{F}_{\text{Connie}} \) yields:

\[
\vec{F}_{\text{Connie}} = -\left(\vec{F}_{\text{Paul}} + \vec{F}_{\text{Johnny}}\right)
\]

Substitute for \( \vec{F}_{\text{Paul}} + \vec{F}_{\text{Johnny}} \) to obtain:

\[
\vec{F}_{\text{Connie}} = -\left[(-17.8 \text{ lb})\hat{i} - (1.9 \text{ lb})\hat{j}\right] = (18 \text{ lb})\hat{i} + (1.9 \text{ lb})\hat{j}
\]

The magnitude of \( \vec{F}_{\text{Connie}} \) is:

\[
F_{\text{Connie}} = \sqrt{(17.8 \text{ lb})^2 + (1.9 \text{ lb})^2} = 18 \text{ lb}
\]

The direction that the force exerted by Connie acts is given by:

\[
\theta = \tan^{-1}\left(\frac{1.9 \text{ lb}}{17.8 \text{ lb}}\right) = 6.1^\circ \text{ N of E}
\]

You spot a plane that is 1.50 km North, 2.5 km East and at an altitude of 5.0 km above your position. (a) How far from you is the plane? (b) At what angle from due north (in the horizontal plane) are you looking? (c) Determine the plane's position vector (from your location) in terms of the unit vectors, letting \( \hat{i} \) be toward the east direction, \( \hat{j} \) be toward the north direction, and \( \hat{k} \) be vertically upward. (d) At what elevation angle (above the horizontal plane of Earth) is the airplane?

**Picture the Problem** A diagram showing the given information is shown below. We can use the Pythagorean Theorem, trigonometry, and vector algebra to find the distance, angles, and expression called for in the problem statement.
(a) Use the Pythagorean Theorem to express $d$ in terms of $h$ and $\ell$:

$$d = \sqrt{\ell^2 + h^2}$$

Substitute numerical values and evaluate $d$:

$$d = \sqrt{(2.5 \text{ km})^2 + (1.5 \text{ km})^2 + (5.0 \text{ km})^2} = 5.8 \text{ km}$$

(b) Use trigonometry to evaluate the angle from due north at which you are looking at the plane:

$$\theta = \tan^{-1}\left(\frac{2.5 \text{ km}}{1.5 \text{ km}}\right) = 59^\circ \text{ E of N}$$

(c) We can use the coordinates of the plane relative to your position to express the vector $\vec{d}$:

$$\vec{d} = (2.5 \text{ km})\hat{i} + (1.5 \text{ km})\hat{j} + (5.0 \text{ km})\hat{k}$$

(d) Express the elevation angle $\phi$ in terms of $h$ and $\ell$:

$$\phi = \tan^{-1}\left(\frac{h}{\ell}\right)$$

Substitute numerical values and evaluate $\phi$:

$$\phi = \tan^{-1}\left(\frac{5.0 \text{ km}}{\sqrt{(2.5 \text{ km})^2 + (1.5 \text{ km})^2}}\right) = 60^\circ \text{ above the horizon}$$