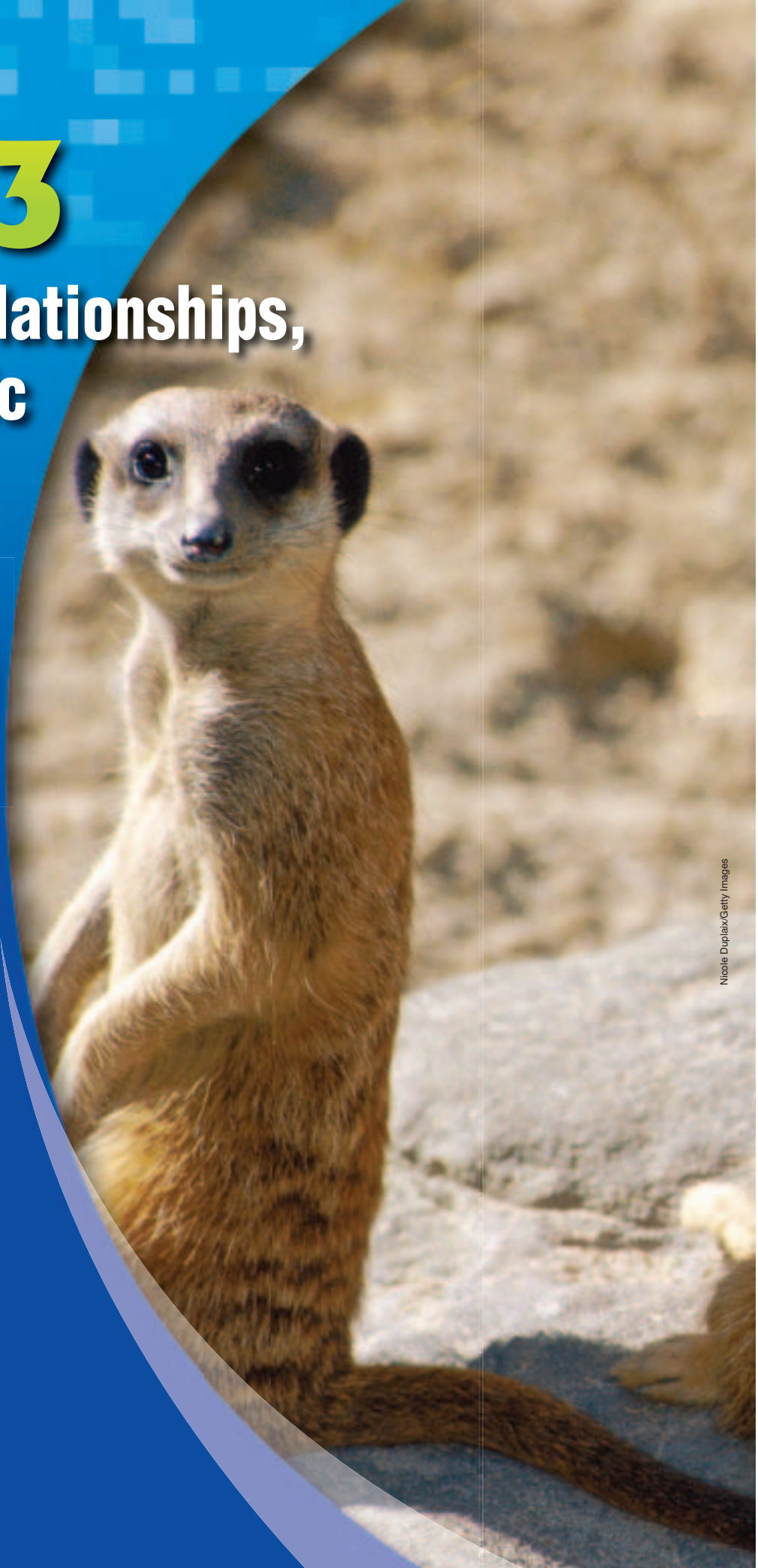
Unit 3

Patterns, Relationships, and Algebraic Thinking



Nicole Duplaix/Getty Images

Focal Point

Solve problems involving direct proportional relationships involving number, geometry, and measurement.

CHAPTER 7
Ratios and Proportions

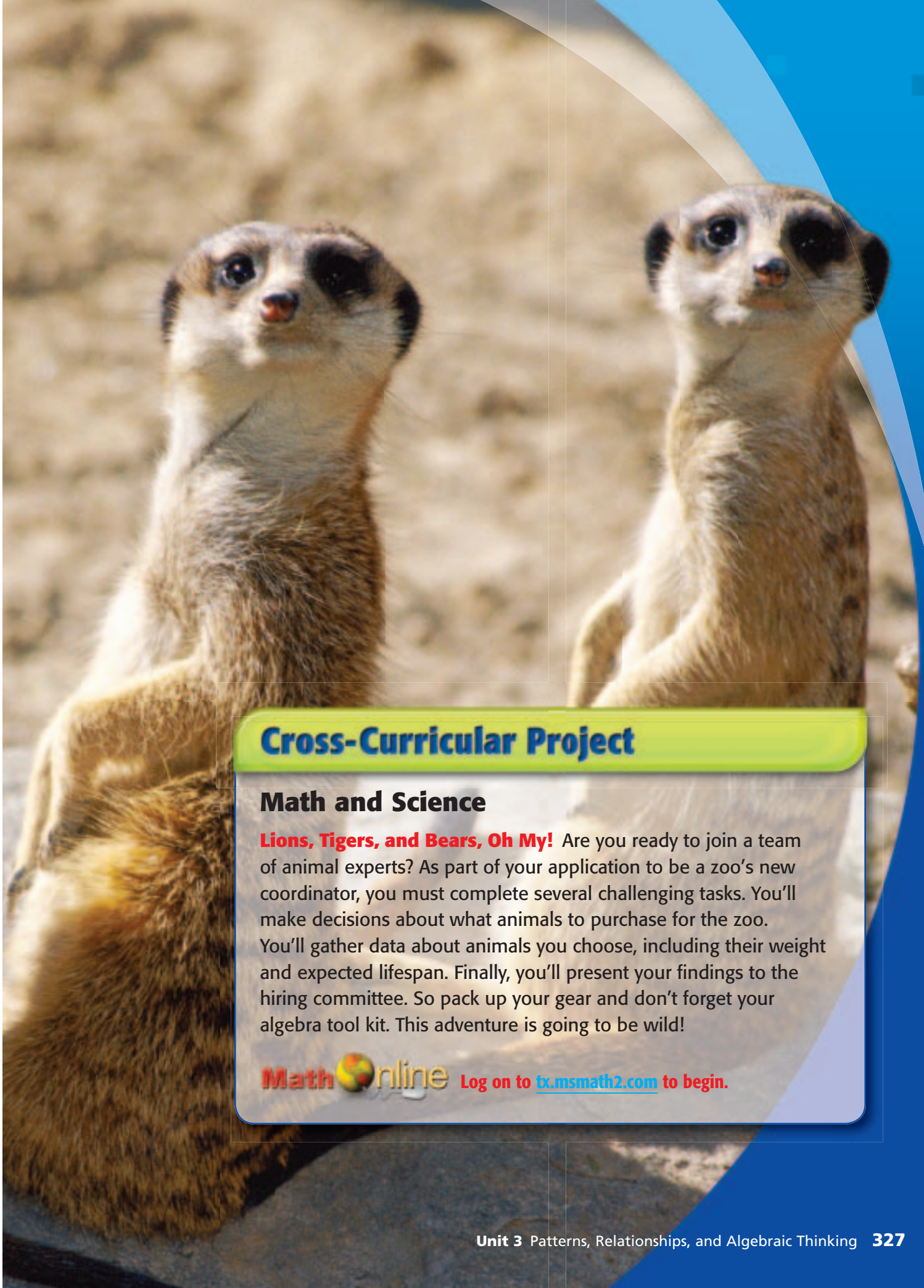
BIG Idea Solve problems involving direct proportional relationships.

BIG Idea Represent relationships in numerical, geometric, verbal, and symbolic form.

CHAPTER 8
Applying Percents

BIG Idea Solve problems involving direct proportional relationships.

BIG Idea Estimate and find solutions to application problems involving percent.



Cross-Curricular Project

Math and Science

Lions, Tigers, and Bears, Oh My! Are you ready to join a team of animal experts? As part of your application to be a zoo's new coordinator, you must complete several challenging tasks. You'll make decisions about what animals to purchase for the zoo. You'll gather data about animals you choose, including their weight and expected lifespan. Finally, you'll present your findings to the hiring committee. So pack up your gear and don't forget your algebra tool kit. This adventure is going to be wild!

MathOnline Log on to tx.msmath2.com to begin.

CHAPTER 7

Ratios and Proportions

Knowledge and Skills

- Solve problems involving direct proportional relationships. **TEKS 7.3**
- Represent relationships in numerical, geometric, verbal, and symbolic form. **TEKS 7.4**

Key Vocabulary

rate (p. 335)

ratio (p. 330)

proportional (p. 348)

Real-World Link

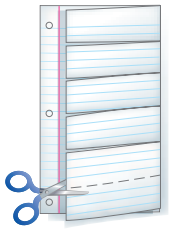
Statues A bronze replica of the Statue of Liberty can be found in Paris, France. The replica has the ratio 1 : 4 with the Statue of Liberty that stands in New York Harbor.

FOLDABLES Study Organizer

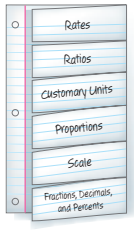
Ratios and Proportions Make this Foldable to help you organize your notes. Begin with a sheet of notebook paper.

1. **Fold** lengthwise to the holes.

2. **Cut** along the top line and then make equal cuts to form 6 tabs.



3. **Label** the major topics as shown.



Alan Schein/zefa/CORBIS

GET READY for Chapter 7

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 1

Take the Quick Quiz below. Refer to the Quick Review for help.

QUICK Quiz

Evaluate each expression. Round to the nearest tenth if necessary.
(Used in Lessons 7-2, 7-4, and 7-6)

1. $100 \times 25 \div 52$
2. $10 \div 4 \times 31$
3. $\frac{63 \times 4}{34}$
4. $\frac{2 \times 100}{68}$

Write each fraction in simplest form.
(Used in Lessons 7-1 through 7-4, 7-6, and 7-7)

5. $\frac{9}{45}$
6. $\frac{16}{24}$
7. $\frac{38}{46}$

8. **AGES** Mikhail is 14 years old. His father is 49 years old. What fraction, in simplest form, of his father's age is Mikhail?

Write each decimal as a fraction in simplest form. (Used in Lesson 7-7)

9. 0.78
10. 0.320
11. 0.06

12. **SAVINGS** Belinda has saved 0.92 of the cost of a new bicycle. What fraction, in simplest form, represents her savings?

Complete. (Used in Lesson 7-3)

13. 2 yd = ■ ft
14. 48 oz = ■ lb
15. $4\frac{1}{2}$ ft = ■ yd
16. $3\frac{1}{4}$ h = ■ min

QUICK Review

Example 1

Evaluate $15 \times 32 \div 40$.

$15 \times 32 \div 40 = 480 \div 40$ Multiply 15 by 32.
$= 12$ Divide.

Example 2

Write $\frac{16}{44}$ in simplest form.

$\frac{16}{44} = \frac{4}{11}$ (÷ 4 above and below)

Divide the numerator and denominator by their GCF, 4.

Example 3

Write 0.62 as a fraction in simplest form.

$0.62 = \frac{62}{100}$ 0.62 is sixty-two hundredths.
$= \frac{31}{50}$ Divide the numerator and denominator by their GCF, 2.

Example 4

Complete 22 lb = ■ oz.

There are 16 ounces in 1 pound. So, there are 16×22, or 352 ounces in 22 pounds.

Option 2

MathOnline Take the Online Readiness Quiz at tx.msmath2.com.

7-1 Ratios

Main IDEA

Write ratios as fractions in simplest form and determine whether two ratios are equivalent.

Targeted TEKS 7.2 The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. **(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio**. *Also addresses TEKS 7.14(B).*

NEW Vocabulary

ratio
equivalent ratios

GET READY for the Lesson

SCHOOL The student-teacher ratio of a school compares the total number of students to the total number of teachers.

Middle School	Students	Teachers
Prairie Lake	396	22
Green Brier	510	30

1. Write the student-teacher ratio of Prairie Lake Middle School as a fraction. Then write this fraction with a denominator of 1.
2. Can you determine which school has the lower student-teacher ratio by examining just the number of teachers at each school? just the number of students at each school? Explain.

KEY CONCEPT Ratios

Words A **ratio** is a comparison of two quantities by division.

Examples

Numbers	Algebra
3 to 4 $\quad$ 3:4 $\quad$ $\frac{3}{4}$	a to b $\quad$ $a:b$ $\quad$ $\frac{a}{b}$

Ratios can express part to part, part to whole, or whole to part relationships and are often written as fractions in simplest form.

EXAMPLE 1 Write Ratios in Simplest Form

GRILLING **Seasonings are often added to meat prior to grilling. Using the recipe, write a ratio comparing the amount of garlic powder to the amount of dried oregano as a fraction in simplest form.**

garlic powder / dried oregano $\quad \frac{4 \text{ tsp}}{6 \text{ tsp}} = \frac{\overset{2}{\cancel{4 \text{ tsp}}}}{\underset{3}{\cancel{6 \text{ tsp}}}}$ or $\frac{2}{3}$

The ratio of garlic powder to dried oregano is $\frac{2}{3}$, 2:3, or 2 to 3. That is, for every 2 units of garlic powder there are 3 units of dried oregano.

CHECK Your Progress

Use the recipe to write each ratio as a fraction in simplest form.

a. pepper : garlic powder

b. oregano : pepper

READING in the Content Area

For strategies in reading this lesson, visit tx.msmath2.com.

Ratios that express the same relationship between two quantities are called **equivalent ratios**. Equivalent ratios have the same value.

EXAMPLE 2 Identify Equivalent Ratios

Determine whether the ratios 250 miles in 4 hours and 500 miles in 8 hours are equivalent.

METHOD 1 **Compare the ratios written in simplest form.**

$250 \text{ miles} : 4 \text{ hours} = \frac{250 \div 2}{4 \div 2}$ or $\frac{125}{2}$ Divide the numerator and denominator by the GCF, 2

$500 \text{ miles} : 8 \text{ hours} = \frac{500 \div 4}{8 \div 4}$ or $\frac{125}{2}$ Divide the numerator and denominator by the GCF, 4

The ratios simplify to the same fraction. They are equivalent.

METHOD 2 **Look for a common multiplier relating the two ratios.**

$\frac{250}{4} \overset{\times 2}{=} \frac{500}{8}$

The numerator and denominator of the ratios are related by the same multiplier, 2. The ratios of miles to hours are equivalent

STUDY TIP

Writing Ratios
Ratios greater than 1 are expressed as improper fractions and not as mixed numbers.

Personal Tutor

Personal Tutor for MAC

CHOOSE Your Method

Determine whether the ratios are equivalent.

c. 20 nails for every 5 shingles, 12 nails for every 3 shingles

d. 2 cups flour to 8 cups sugar, 8 cups flour to 14 cups sugar

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Real-World Link
The recommended number of water lilies for a 9-square foot pond is 1.
Source: sustland.umn.edu

Real-World EXAMPLE

3 **PONDS** **For every 9 square feet of surface, a pond should have 2 fish. A pond that has a surface of 45 square feet contains 6 fish. Are these ratios equivalent? Justify your answer.**

Recommended Ratio: $9:2 = \frac{9}{2}$ square feet to fish

Actual Ratio: $45:6 = \frac{45}{6}$ or $\frac{15}{2}$ square feet to fish

Since $\frac{9}{2} \neq \frac{15}{2}$, the ratios are not equivalent. So, the number of fish is not correct for the pond.

CHECK Your Progress

e. **SWIMMING** A community pool requires there to be at least 3 lifeguards for every 20 swimmers. There are 60 swimmers and 9 lifeguards at the pool. Is this the correct number of lifeguards based on the above requirement? Justify your answer.

CHECK Your Understanding

Example 1 (p. 330)

FIELD TRIPS Use the information in the table to write each ratio as a fraction in simplest form.

Field Trip Statistics	
Students	180
Adults	24
Buses	4

1. adults : students
2. students : buses
3. buses : people
4. adults : people

Example 2 (p. 331)

Determine whether the ratios are equivalent. Explain.

5. 12 out of 20 doctors agree
   6 out of 10 doctors agree
6. 2 DVDs to 7 CDs
   10 DVDs to 15 CDs

Example 3 (p. 331)

7. **SHOPPING** A grocery store has a brand-name cereal on sale 2 boxes for $5. You buy 6 boxes and are charged $20. Based on the price ratio indicated, were you charged the correct amount? Justify your answer.

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–17	1
18–21	2
22–23	3

SOCCER Use the Madison Mavericks team statistics to write each ratio as a fraction in simplest form.

Madison Mavericks Team Statistics	
Wins	10
Losses	12
Ties	8

8. wins : losses
9. losses : ties
10. losses : games played
11. wins : games played

CARNIVALS Use the following information to write each ratio as a fraction in simplest form.

At its annual carnival, Brighton Middle School had 6 food booths and 15 games booths. A total of 66 adults and 165 children attended. The carnival raised a total of $1,600. Of this money, $550 came from ticket sales.

12. children : adults
13. food booths : games booths
14. children : games booths
15. booths : money raised
16. people : children
17. non-ticket sale money : total money

Determine whether the ratios are equivalent. Explain.

18. 20 female lions to 8 male lions,
    34 female lions to 10 male lions
19. $4 for every 16 ounces,
    $10 for every 40 ounces
20. 27 students to 6 microscopes,
    18 students to 4 microscopes
21. 8 roses to 6 babies breath,
    12 roses to 10 babies breath
22. **BAKING** It is recommended that a ham be baked 1 hour for every 2 pounds of meat. Latrell baked a 9-pound ham for 4.5 hours. Did he follow the above recommendation? Justify your answer.
23. **FISHING** Kamala catches two similar looking fish. The larger fish is 12 inches long and 3 inches wide. The smaller fish is 6 inches long and 1 inch wide. Do these fish have an equivalent length to width ratio? Justify your answer.

PHOTOGRAPHY **The *aspect ratio* of a photograph is a ratio comparing the length and width. A 35 mm negative has an aspect ratio of 1 : 1.5. Photo sizes with the same aspect ratio can be printed full frame without cropping. Determine which size photos can be printed full frame from a 35 mm negative. Justify your answers.**

24. 8″ × 10″ **25.** 5″ × 7.5″ **26.** 10″ × 15″

Real-World Link . . . . .
An orca whale, also called a killer whale, is not really a whale, but a dolphin. Its average birth weight is 300 pounds.
Source: coolantarctica.com

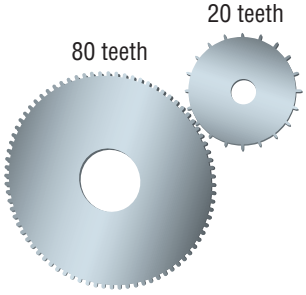
MAMMALS **For Exercises 27 and 28, use the information below.**

Mammal	Average Brain Weight (lb)	Average Body Weight (lb)
Adult Human	3	150
Adult Orca Whale	12	5,500

27. How much greater is the average weight of an adult orca whale's brain than the average weight of an adult human's brain?

28. Find the brain to body weight ratio for each mammal. Are these ratios equivalent? If not, which mammal has the greater brain to body weight ratio? Justify your answer and explain its meaning.

29. **GEARS** A *gear ratio* is a comparison of the number of teeth of a larger gear to the number of teeth of a smaller gear. What is the gear ratio of the two gears shown? How many times does the smaller gear turn for every turn of the larger gear?



30. **MONEY** A debt to income ratio compares the amount of money owed (debt) to the amount of money earned (income). What is the debt to income ratio for someone earning $2,000 a month who is making payments of $400 a month on their debts?

ANALYZE TABLES **For Exercises 31–33, use the table below that shows the logging statistics for three areas of forest.**

Area	Estimated Number of Trees Left to Grow	Estimated Number of Trees Removed for Timber
A	440	1,200
B	1,625	3,750
C	352	960

31. For which two areas was the growth-to-removal ratio the same? Explain.

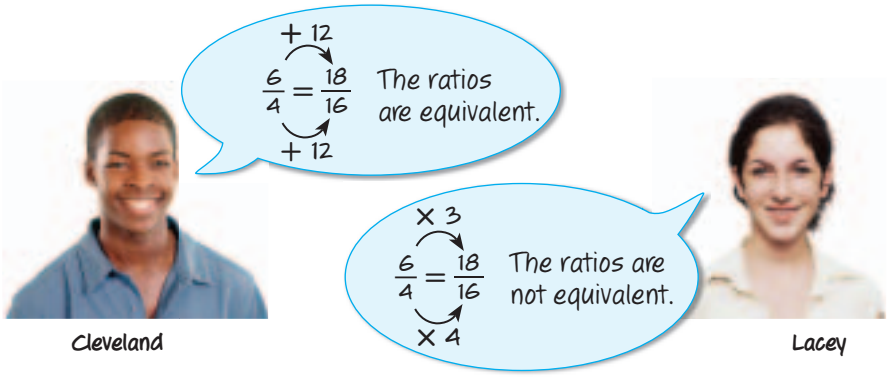
32. Which area had the greatest growth-to-removal ratio? Justify your answer.

33. Find the additional number of trees that should be planted and left to grow in area A so that its growth-to-removal ratio is the same as area B's. Justify your answer.

EXTRA PRACTICE
See pages 733, 761.
MathOnline
Self-Check Quiz at tx.msmath2.com

H.O.T. Problems

34. **FIND THE ERROR** Cleveland and Lacey are determining whether the ratios $\frac{6}{4}$ and $\frac{18}{16}$ are equivalent. Who is correct? Explain.



35. **CHALLENGE** Find the missing number in the following pattern. Explain your reasoning. (*Hint*: Look at the ratios of successive numbers.)

480, 240, 80, 20, ■.

36. **WRITING IN MATH** Refer to the application in Exercises 31–33. What would a growth-to-removal ratio greater than 1 indicate?

TEST PRACTICE

37. Which of the following ratios does NOT describe a relationship between the marbles in the jar?

A 8 white : 5 black

B 2 white : 5 black

C 5 black : 13 total

D 8 white : 13 total

38. A class of 24 students has 15 boys. What fraction represents the ratio of girls to boys in the class?

F $\frac{3}{5}$ **H** $\frac{3}{8}$

G $\frac{5}{8}$ **J** $\frac{5}{3}$

Spiral Review

39. Find $1\frac{4}{7} \div 1\frac{5}{6}$. Write in simplest form. (Lesson 6-7)

Solve each equation. Check your solution. (Lesson 6-6)

40. $\frac{y}{4} = 7$ 41. $\frac{1}{3}x = \frac{5}{9}$ 42. $4 = \frac{p}{2.7}$ 43. $2\frac{5}{6} = \frac{1}{2}a$

44. **MONEY** Grant and his brother put together their money to buy a present for their mom. If they had a total of $18 and Grant contributed $10, how much did his brother contribute? (Lesson 4-2)

GET READY for the Next Lesson

PREREQUISITE SKILL **Divide.** (Lesson 3-4)

45. 9.8 ÷ 2 46. $4.30 ÷ 5 47. $12.40 ÷ 40 48. 27.36 ÷ 3.2

7-2 Rates

Main IDEA

Determine unit rates.

Targeted TEKS 7.2 The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. **(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio.** *Also addresses TEKS 7.3(B).*

NEW Vocabulary

rate
unit rate

MINI Lab

Choose a page in a textbook and take turns reading as much as possible in 2 minutes.

1. Count the number of words that each of you read.
2. Write the ratio *number of words to number of minutes* as a fraction.
3. Simplify the fractions by dividing the numerator and the denominator by 2.

A ratio that compares two quantities with different kinds of units is called a **rate**.

$\frac{100 \text{ words}}{2 \text{ minutes}}$ ← The units *words* and *minutes* are different.

When a rate is simplified so that it has a denominator of 1 unit, it is called a **unit rate**.

$\frac{50 \text{ words}}{1 \text{ minute}}$ ← The denominator is 1 unit.

The unit rate $\frac{50 \text{ words}}{1 \text{ minute}}$ can be read as *50 words per minute*.

The table below shows some common unit rates.

Rate	Unit Rate	Abbreviation	Name
$\frac{\text{number of miles}}{1 \text{ hour}}$	miles per hour	mi/h or mph	average speed
$\frac{\text{number of miles}}{1 \text{ gallon}}$	miles per gallon	mi/gal or mpg	gas mileage
$\frac{\text{number of dollars}}{1 \text{ pound}}$	price per pound	dollars/lb	unit price
$\frac{\text{number of dollars}}{1 \text{ hour}}$	dollars per hour	dollars/h	hourly wage

EXAMPLES Find Unit Rates

1 **RUNNING** **Alethia ran 24 miles in 3 hours. What was her average speed in miles per hour?**

Write the rate as a fraction. Then find an equivalent rate with a denominator of 1.

$$24 \text{ miles in 3 hours} = \frac{24 \text{ mi}}{3 \text{ h}}$$ Write the rate as a fraction.

$$= \frac{24 \text{ mi} \div 3}{3 \text{ h} \div 3}$$ Divide the numerator and the denominator by 3.

$$= \frac{8 \text{ mi}}{1 \text{ h}}$$ Simplify.

Alethia's average speed, or unit rate, was 8 miles per hour.

STUDY TIP

Mental Math
To find a unit rate mentally, divide the first quantity in the rate by the second quantity.
24 miles in 3 hours
$= \frac{24}{3}$ mi/h
$= 8$ mi/h

2 **GROCERIES** **Find the unit price if it costs $2 for six oranges. Round to the nearest cent if necessary.**

$$\$2 \text{ for six oranges} = \frac{\$2}{6 \text{ oranges}}$$ Write the rate as a fraction.

$$= \frac{\$2 \div 6}{6 \text{ oranges} \div 6}$$ Divide the numerator and the denominator by 6.

$$\approx \frac{\$0.33}{1 \text{ orange}}$$ Simplify.

The unit price is about $0.33 per orange.

CHECK Your Progress

Find each unit rate. Round to the nearest hundredth if necessary.

a. $300 for 6 hours

b. 220 miles on 8 gallons

c. **ESTIMATION** Estimate the unit rate if a 4-pack of mixed fruit sells for $2.12.

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Unit rates are useful when you want to make comparisons.

TEST EXAMPLE **Compare Using Unit Rates**

3 The prices of 3 different bags of dog food are given in the table. Which size bag has the lowest price per pound?

Dog Food Prices	
Bag Size (pounds)	**Price**
40	$49.00
20	$23.44
8	$9.88

A The 40-lb bag only

B The 20-lb bag only

C The 8-lb bag only

D All three bag sizes have the same price per pound.

Test-Taking Tip

Alternative Method
One 40-lb bag is equivalent to two 20-lb bags or five 8-lb bags. The cost for one 40-lb bag is $49, the cost for two 20-lb bags is about 2 × $23 or $46, and the cost for five 8-lb bags is about 5 × $10 or $50. So the 20-lb bag has the lowest price per pound.

Read the Test Item

To determine the lowest price per pound, find and compare the unit price for each size bag.

Solve the Test Item

40-pound bag $49.00 ÷ 40 pounds = $1.225 per pound

20-pound bag $23.44 ÷ 20 pounds = $1.172 per pound

8-pound bag $9.88 ÷ 8 pounds = $1.235 per pound

At $1.172 per pound, the 20-pound bag sells for the lowest price per pound. The answer is B.

CHECK Your Progress

d. Tito wants to buy some peanut butter to donate to the local food pantry. If Tito wants to save as much money as possible, which brand should he buy?

Peanut Butter Sales	
Brand	**Sale Price**
Nutty	12 ounces for $2.19
Grandma's	18 ounces for $2.79
Bee's	28 ounces for $4.69
Save-A-Lot	40 ounces for $6.60

F Nutty, because the quality of the peanut butter is better

G Grandma's, because the price per ounce is about $0.16

H Bee's, because the price per ounce is about $0.14

J Save-A-Lot, because he wants to buy 40 ounces

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Real-World Link
Face paint can be made from 1 teaspoon cornstarch and $\frac{1}{2}$ teaspoon each of water and cold cream.
Source: painting.about.com

Real-World EXAMPLE Use a Unit Rate

4 **FACE PAINTING** Lexi painted 3 faces in 12 minutes at the Arts and Crafts fair. At this rate, how many faces can she paint in 40 minutes?

Find the unit rate. Then multiply this unit rate by 40 to find the number of faces she can paint in 40 minutes.

$$3 \text{ faces in } 12 \text{ minutes} = \frac{3 \text{ faces} \div 12}{12 \text{ min} \div 12} = \frac{0.25 \text{ faces}}{1 \text{ min}}$$ Find the unit rate.

$$\frac{0.25 \text{ faces}}{1 \cancel{\text{ min}}} \cdot 40 \cancel{\text{ min}} = 10 \text{ faces}$$ Divide out the common units.

Lexi can paint 10 faces in 40 minutes or $\frac{4 \text{ minutes}}{1 \text{ face}}$.

CHECK Your Progress

e. **SCHOOL SUPPLIES** Kimbel bought 4 notebooks for $6.32. At this same unit price, how much would he pay for 5 notebooks?

CHECK Your Understanding

Examples 1, 2 (pp. 335–336)

Find each unit rate. Round to the nearest hundredth if necessary.

1. 90 miles on 15 gallons
2. 1,680 kilobytes in 4 minutes
3. 5 pounds for $2.49
4. 152 feet in 16 seconds

Example 3 (pp. 336–337)

5. **TEST PRACTICE** Four stores offer customers bulk CD rates. Which store offers the best buy?

Bulk CD Offers	
Store	**Offer**
CD Express	4 CDs for $60
Music Place	6 CDs for $75
CD Rack	5 CDs for $70
Music Shop	3 CDs for $40

A CD Express **C** Music Place

B CD Rack **D** Music Shop

Example 4 (p. 337)

6. **TRAVEL** After 3.5 hours, Pasha had traveled 217 miles. At this same speed, how far will she have traveled after 4 hours?

Exercises

HOMEWORK HELP

For Exercises	See Examples
7–16	1, 2
17–20	3
21–24	4

Find each unit rate. Round to the nearest hundredth if necessary.

7. 360 miles in 6 hours
8. 6,840 customers in 45 days
9. 152 people for 5 classes
10. 815 Calories in 4 servings
11. 45.5 meters in 13 seconds
12. $7.40 for 5 pounds
13. $1.12 for 8.2 ounces
14. 144 miles in 4.5 gallons

15. **ESTIMATION** Estimate the unit rate if 12 pairs of socks sell for $5.79.

16. **ESTIMATION** Estimate the unit rate if a 26 mile marathon was completed in 5 hours.

17. **SPORTS** The results of a swim meet are shown. Who swam the fastest? Explain your reasoning.

Name	Event	Time (s)
Tawni	50-m Freestyle	40.8
Pepita	100-m Butterfly	60.2
Susana	200-m Medley	112.4

18. **MONEY** A grocery store sells three different packages of bottled water. Which package costs the least per bottle? Explain your reasoning.

6-pack for $3.79

9-pack for $4.50

12-pack for $6.89

NUTRITION For Exercises 19 and 20, use the table at the right.

Soft Drink Nutritional Information			
Soft Drink	Serving Size (oz)	Sodium (mg)	Sugar (g)
A	12	40	22
B	8	24	15
C	7	42	30

19. Which soft drink has about twice the amount of sodium per ounce than the other two? Explain.
20. Which soft drink has the least amount of sugar per ounce? Explain.

Real-World Link
North Carolina has approximately 8.2 million people in approximately 48,718 square miles.
Source: quickfacts.census.gov

21. **WORD PROCESSING** Ben can type 153 words in 3 minutes. At this rate, how many words can he type in 10 minutes?

22. **FABRIC** Marcus buys 3 yards of fabric for $7.47. Later he realizes that he needs 2 more yards. How much will he pay for the extra fabric?

23. **ESTIMATION** A player scores 87 points in 6 games. At this rate, about how many points would she score in the next 4 games?

24. **JOBS** Dalila earns $94.20 for working 15 hours as a holiday helper wrapping gifts. If she works 18 hours the next week, how much money will she earn?

25. **POPULATION** Use the information at the left. What is the *population density* or number of people per square mile in North Carolina?

ESTIMATION Estimate the unit price for each item. Justify your answers.

26. 

$2.49

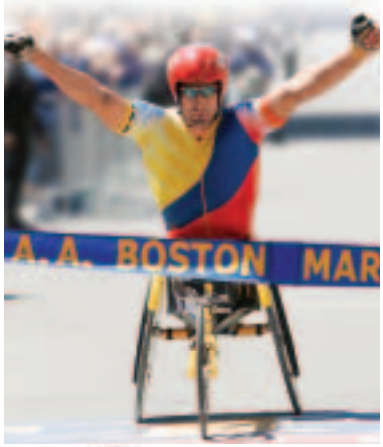
27. 

$1.89

28. 

$1.13

29. **RECIPES** A recipe that makes 10 mini-loaves of banana bread calls for $1\frac{1}{4}$ cups flour. How much flour is needed to make 2 dozen mini-loaves using this recipe?



Real-World Link . . . . .
The winning time for the men's wheelchair division of the 2005 Boston Marathon in Massachusetts was 1 hour, 24 minutes, and 11 seconds.
Source: boston.com

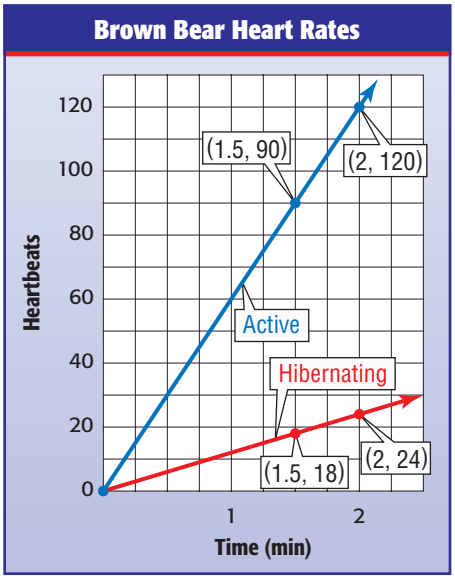
SPORTS **For Exercises 30 and 31, use the information at the left.**

30. The wheelchair division for the Boston Marathon is 26.2 miles long. What was the average speed of the winner of the wheelchair division in 2005? Round to the nearest hundredth.

31. At this rate, about how long would it take this competitor to complete a 30 mile race?

32. **GROCERIES** Salami is on sale for $4.48 per pound. If its regular price is $5.28 per pound, how much do you save on a 2.5-pound package of this salami? Justify your answer using two different methods.

ANIMALS **For Exercises 33–37, use the graph that shows the average number of heartbeats for an active adult brown bear and a hibernating brown bear.**



33. What does the point (2, 120) represent on the graph?

34. What does the point (1.5, 18) represent on the graph?

35. What does the ratio of the y-coordinate to the x-coordinate for each pair of points on the graph represent?

36. Use the graph to find the bear's average heart rate when it is active and when it is hibernating.

37. When is the bear's heart rate greater, when it is active or when it is hibernating? How can you tell this from the graph?

38. **TIRES** At Tire Depot, a set of 2 brand new tires sells for $216. The manager's special advertises the same tires selling at a rate of $380 for 4 tires. How much do you save per tire if you purchase the manager's special?

39. **FIND THE DATA** Refer to the Texas Data File on pages 16–19. Choose some data and write a real-world problem in which you would compare unit rates or ratios.

EXTRA PRACTICE
See pages 733, 761.
MathOnline
Self-Check Quiz at tx.msmath2.com

H.O.T. Problems

CHALLENGE **Determine whether each statement is *sometimes*, *always*, or *never* true. Give an example or a counterexample.**

40. A ratio is a rate.

41. A rate is a ratio.

42. **OPEN ENDED** Create a rate and then convert it to a unit rate.

43. **NUMBER SENSE** In which situation will the rate $\frac{x \text{ feet}}{y \text{ minutes}}$ increase? Give an example to explain your reasoning.

a. x increases, y is unchanged

b. x is unchanged, y increases

44. **WRITING IN MATH** Explain how you can use multiplication to check your answer when finding the unit rate for 5.2 miles in 4 hours.

TEST PRACTICE

45. Mrs. Ross needs to buy dish soap. There are four different size containers at a store.

Dish Soap Prices	
Brand	**Price**
Lots of Suds	$0.98 for 8 ounces
Bright Wash	$1.29 for 12 ounces
Spotless Soap	$3.14 for 30 ounces
Lemon Bright	$3.45 for 32 ounces

Mrs. Ross wants to buy the one that costs the least per ounce. Which brand should she buy?

A Lots of Suds **C** Spotless Soap

B Bright Wash **D** Lemon Bright

46. The table shows the total distance traveled by a car driving at a constant rate of speed.

Time (h)	Distance (mi)
2	130
3.5	227.5
4	260
7	455

Based on this information, how far will the car have traveled after 10 hours?

F 520 miles **H** 650 miles

G 585 miles **J** 715 miles

Spiral Review

FLOWERS **For Exercises 47–50, use the information in the table to write each ratio as a fraction in simplest form.** (Lesson 7-1)

Flower Arrangement	
Lilies	4
Roses	18
Snapdragons	6

47. lilies : roses

48. snapdragons : lilies

49. roses : flowers

50. flowers : snapdragons

51. **SANDWICHES** Lawanda is making subs. She puts $1\frac{1}{2}$ slices of cheese on each sub. If she has 12 slices of cheese, how many subs can she make? (Lesson 6-6)

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve.** (Lessons 3-2 and 3-4)

52. 2.5×20

53. 3.5×4

54. $104 \div 16$

55. $4{,}200 \div 2{,}000$

Extend 7-2

Math Lab
Rate of Change

Main IDEA

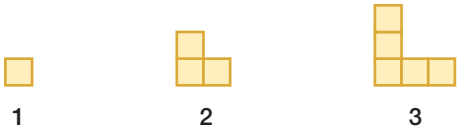
Investigate rate of change.

Targeted TEKS
7.2 The student adds, subtracts, multiplies or divides to solve problems and justify solutions. **(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio.** *Also addresses TEKS 7.14(A).*

A *rate of change* is a ratio that shows a change in one quantity with respect to a change in another quantity. In this lab, you will use tables and graphs to investigate constant rates of change.

ACTIVITY

STEP 1 Use tiles to build the figures shown below. Then continue the pattern to build the fourth and fifth figures.



1 2 3

STEP 2 For each figure, record the number of tiles and the perimeter of the figure in a table like the one shown at the right.

Figures	Number of Tiles (x)	Perimeter (y)
1	1	
2	3	
3	5	
4		
5		

STEP 3 Draw a coordinate plane on grid paper and graph the ordered pairs (x, y).

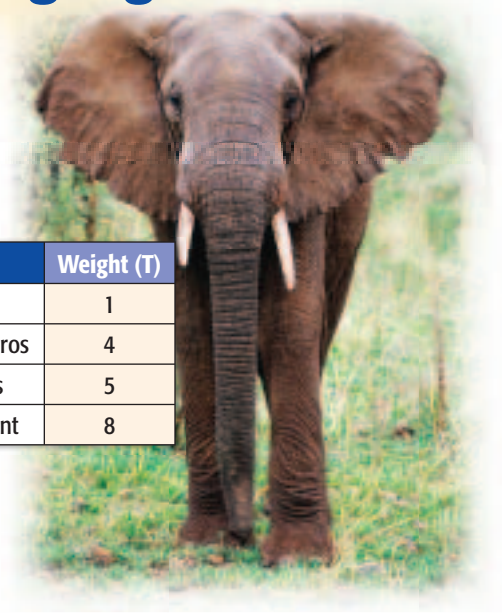
Analyze the Results

1. What do you notice about the points on the graph?
2. Find the ratio $\frac{\text{change in perimeter}}{\text{change in tiles}}$ between the second and third points, the third and fourth points, and the fourth and fifth points. Each ratio is a rate of change. Describe what you observe.
3. Complete: As the number of tiles increases by 2 units, the perimeter of the models increases by ▪ units.
4. **MAKE A PREDICTION** Refer to the table at the right. Find the ratio $\frac{\text{change in earnings}}{\text{change in hours worked}}$ for Greg and Monica. Which person's earnings when graphed will form the steeper line? Explain your reasoning.

Hours Worked	Earnings ($)	
	Greg	Monica
1	4	5
2	8	10
3	12	15
4	16	20

5. Graph the ordered pairs (hours worked, earnings) for each person and connect to form a line. The graph of which relationship has the steeper line?

7-3 Measurement: Changing Customary Units



Main IDEA

Change units in the customary system.

Reinforcement of TEKS 6.8 The student solves application problems involving estimation and measurement of length, area, time, temperature, volume, weight, and angles. **(D) convert measures within the same measurement system (customary** and metric**) based on relationships between units**. *Also addresses TEKS 7.3(B), 7.14(A).*

NEW Vocabulary

unit ratio

GET READY for the Lesson

ANIMALS The table shows the approximate weights in tons of several large land animals. One ton is equivalent to 2,000 pounds.

Animal	Weight (T)
Grizzly Bear	1
White Rhinoceros	4
Hippopotamus	5
African Elephant	8

You can use a *ratio table*, whose columns are filled with ratios that have the same value, to convert each weight from tons to pounds.

1. Copy and complete the ratio table. The first two ratios are done for you.

×4

Tons	1	4	5	8
Pounds	2,000	8,000	■	■

×4

To produce equivalent ratios, multiply the quantities in each row by the same number.

2. Then graph the ordered pairs (tons, pounds) from the table. Label the horizontal axis *Weight in Tons* and the vertical axis *Weight in Pounds*. Connect the points. What do you notice about the graph of these data?

The relationships among the most commonly used customary units of length, weight, and capacity are shown in the table below.

KEY CONCEPT Equality Relationships for Customary Units

Type of Measure	Larger Unit	→	Smaller Unit
Length	1 foot (ft)	=	12 inches (in.)
	1 yard (yd)	=	3 feet
	1 mile (mi)	=	5,280 feet
Weight	1 pound (lb)	=	16 ounces (oz)
	1 ton (T)	=	2,000 pounds
Capacity	1 cup (c)	=	8 fluid ounces (fl oz)
	1 pint (pt)	=	2 cups
	1 quart (qt)	=	2 pints
	1 gallon (gal)	=	4 quarts

Each of the relationships above can be written as a unit ratio. Like a unit rate, a **unit ratio** is one in which the denominator is 1 unit.

$$\frac{3 \text{ ft}}{1 \text{ yd}} \qquad \frac{2{,}000 \text{ lb}}{1 \text{ T}} \qquad \frac{4 \text{ qt}}{1 \text{ gal}}$$

Notice that the numerator and denominator of each fraction above are equivalent, so the value of each ratio is 1. You can multiply by a unit ratio of this type to *convert* or change from larger units to smaller units.

EXAMPLES Convert Larger Units to Smaller Units

STUDY TIP

Multiplying by 1
Although the number and units changed in Example 1, because the measure is multiplied by 1, the *value* of the converted measure is the same as the original.

1 **Convert 20 feet into inches.**

Since 1 foot = 12 inches, the unit ratio is $\frac{12 \text{ in.}}{1 \text{ ft}}$.

$20 \text{ ft} = 20 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}}$ Multiply by $\frac{12 \text{ in.}}{1 \text{ ft}}$.

$= 20 \cancel{\text{ft}} \cdot \frac{12 \text{ in.}}{1 \cancel{\text{ft}}}$ Divide out common units, leaving the desired unit, inches.

$= 20 \cdot 12 \text{ in. or } 240 \text{ in.}$ Multiply.

So, 20 feet = 240 inches.

2 **GARDENING** **Clarence mixes $\frac{1}{4}$ cup of fertilizer with soil before planting each bulb. How many ounces of fertilizer does he use per bulb?**

$\frac{1}{4} \text{ c} = \frac{1}{4} \cancel{\text{c}} \cdot \frac{8 \text{ fl oz}}{1 \cancel{\text{c}}}$ Since 1 cup = 8 fluid ounces, multiply by $\frac{8 \text{ fl oz}}{1 \text{ c}}$. Then, divide out common units.

$= \frac{1}{4} \cdot 8 \text{ fl oz or } 2 \text{ fl oz}$ Multiply.

So, 2 fluid ounces of fertilizer are used per bulb.

CHECK Your Progress

Complete.

a. 36 yd = ■ ft b. $\frac{3}{4}$ T = ■ lb c. $1\frac{1}{2}$ qt = ■ pt

REVIEW Vocabulary

reciprocal The product of a number and its reciprocal is 1; *Example*: The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. (Lesson 6-7)

To convert from smaller units to larger units, multiply by the reciprocal of the appropriate unit ratio.

EXAMPLES Convert Smaller Units to Larger Units

3 **Convert 15 quarts into gallons.**

Since 1 gallon = 4 quarts, the unit ratio is $\frac{4 \text{ qt}}{1 \text{ gal}}$, and its reciprocal is $\frac{1 \text{ gal}}{4 \text{ qt}}$.

$15 \text{ qt} = 15 \text{ qt} \cdot \frac{1 \text{ gal}}{4 \text{ qt}}$ Multiply by $\frac{1 \text{ gal}}{4 \text{ qt}}$.

$= 15 \cancel{\text{qt}} \cdot \frac{1 \text{ gal}}{4 \cancel{\text{qt}}}$ Divide out common units, leaving the desired unit, gallons

$= 15 \cdot \frac{1}{4} \text{ gal or } 3.75 \text{ gal}$ Multiplying 15 by $\frac{1}{4}$ is the same as dividing 15 by 4.

4 **COSTUMES** **Umeka needs $4\frac{1}{2}$ feet of fabric to make a costume for a play. How many yards of fabric does she need?**

$4\frac{1}{2}$ ft $= 4\frac{1}{2}$ ft $\cdot \frac{1 \text{ yd}}{3 \text{ ft}}$ — Since 1 yard = 3 feet, multiply by $\frac{1 \text{ yd}}{3 \text{ ft}}$. Then, divide out common units.

$= \frac{9}{2} \cdot \frac{1}{3}$ yd — Write $4\frac{1}{2}$ as an improper fraction. Then divide out common factors.

$= \frac{3}{2}$ yd or $1\frac{1}{2}$ yd — Multiply.

So Umeka needs $1\frac{1}{2}$ yards of fabric.

CHECK Your Progress

Complete.

d. 2,640 ft = ▢ mi

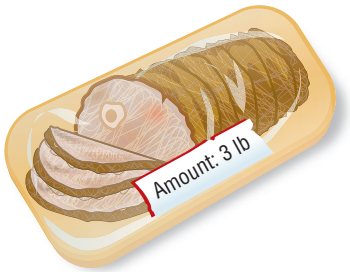
e. 5 pt = ▢ qt

f. 100 oz = ▢ lb

g. 76c = ▢ gal

Real-World EXAMPLE

5 **FOOD** **The pork loin roast shown is cut into 10 smaller pork chops of equal weight. How many ounces does each pork chop weigh? Justify your answer.**



Begin by converting 3 pounds to ounces.

3 lb $= 3$ lb $\cdot \frac{16 \text{ oz}}{1 \text{ lb}}$ — Since 1 pound = 16 ounces, multiply by $\frac{16 \text{ oz}}{1 \text{ lb}}$. Then, divide out common units.

$= 3 \cdot 16$ oz or 48 oz — Multiply.

Find the unit rate which gives the number of ounces per pork chop.

$\frac{\text{ounces}}{\text{pork chops}} = \frac{48 \text{ oz}}{10 \text{ pork chops}}$ or 4.8 ounces per pork chop

So, each pork chop weighs 4.8 ounces.

CHECK Your Progress

h. **RECIPES** A recipe calls for 5 cups of strawberries. Are 2 pints of strawberries enough? Justify your answer.

i. **TRUCKS** The height of a semi-truck is $4\frac{1}{2}$ yards. Will the truck fit under an overpass that is $14\frac{1}{2}$ feet tall? Justify your answer.

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STUDY TIP

Alternative Method
For Example 5, you could also begin by finding the number of pounds per pork chop.

$\frac{3 \text{ lb}}{10 \text{ pork chops}}$ or $\frac{3}{10}$ lb per pork chop

Then convert the number of pounds per pork chop to ounces.

$\frac{\frac{3}{10} \text{ lb}}{1 \text{ pork chop}} \cdot \frac{16 \text{ oz}}{1 \text{ lb}}$

$= \frac{4.8 \text{ oz}}{1 \text{ pork chop}}$

CHECK Your Understanding

Examples 1, 2 (p. 343)

Complete.

1. 3 lb = ▢ oz
2. $5\frac{1}{3}$ yd = ▢ ft
3. 6.5 c = ▢ fl oz

4. **FISH** Grouper are members of the sea bass family. A large grouper can weigh $\frac{1}{3}$ ton. About how much does a large grouper weigh to the nearest pound?

Examples 3, 4 (pp. 343–344)

Complete.

5. 12 qt = ▢ gal
6. 28 in. = ▢ ft
7. 15 pt = ▢ qt

8. **VEHICLES** The world's narrowest electric vehicle is about 35 inches wide and is designed to move down narrow aisles in warehouses. About how wide is this vehicle to the nearest foot?

Example 5 (p. 344)

9. **BIRDS** How many times greater is the weight of an ostrich egg that weighs about 4 pounds than a hummingbird egg that weighs about 0.05 ounce? Justify your answer.

Exercises

HOMEWORK HELP

For Exercises	See Examples
10–21	1–4
22–23	2
24–25	4
26–29	5

Complete.

10. 18 ft = ▢ yd
11. 72 oz = ▢ lb
12. 2 lb = ▢ oz
13. 4 gal = ▢ qt
14. $4\frac{1}{2}$ pt = ▢ c
15. 3 c = ▢ fl oz
16. 2 mi = ▢ ft
17. $1\frac{1}{4}$ mi = ▢ ft
18. 5,000 lb = ▢ T
19. 13 c = ▢ pt
20. $2\frac{3}{4}$ qt = ▢ pt
21. $3\frac{3}{8}$ T = ▢ lb

22. **PUMPKINS** One of the largest pumpkins ever grown weighed about $\frac{1}{2}$ ton. How many pounds did the pumpkin weigh?

23. **SKIING** Speed skiing takes place on a course that is $\frac{2}{3}$ mile long. How many feet long is the course?

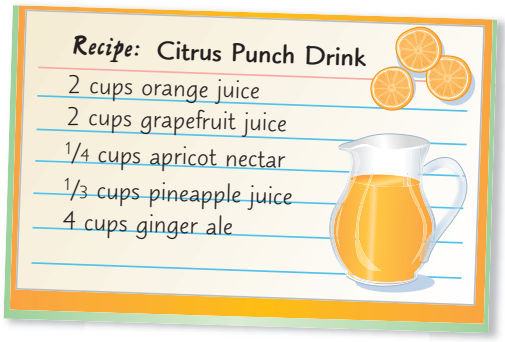
24. **BOATING** A 40-foot power boat is for sale by owner. About how long is this boat to the nearest yard?

25. **BLOOD** A total of 35 pints of blood were collected at a local blood drive. How many quarts of blood is this?

26. **CAR REPAIR** A car repair shop changes the oil of 50 cars. They recover $3\frac{1}{2}$ quarts of oil from each car. How many gallons of oil did they recover? Justify your answer.

27. **ESTIMATION** One bushel of apples weighs about 40 pounds. About how many bushels of apples would weigh 1 ton?

28. **PUNCH** Will a 2-quart pitcher hold the entire recipe of citrus punch given at the right? Explain your reasoning.



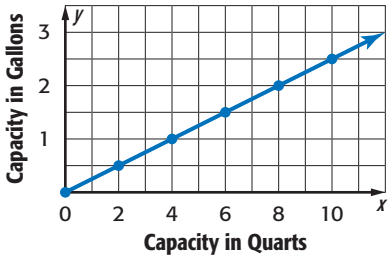
29. **WEATHER** On Monday, it snowed a total of 15 inches. On Tuesday and Wednesday, it snowed an additional $4\frac{1}{2}$ inches and $6\frac{3}{4}$ inches, respectively. A weather forecaster says that over the last three days, it snowed about $2\frac{1}{2}$ feet. Is this a valid claim? Justify your answer.

Complete.

30. $1\frac{1}{4}$ gal = ■ c
31. 880 yd = ■ mi
32. 24 fl oz = ■ qt
33. 9 c = ■ qt
34. 2.3 yd = ■ in.
35. $3\frac{1}{2}$ T = ■ oz

36. **ESTIMATION** Cristos is a member of the swim team and trains by swimming an average of 3,000 yards a day. About how many miles would he swim by training at this rate for 5 days, to the nearest half-mile?

ANALYZE GRAPHS For Exercises 37–40, use the graph at the right.



37. What does an ordered pair from this graph represent?
38. Write two sentences that describe the graph.
39. Use the graph to find the capacity in quarts of a 2.5 gallon container. Explain your reasoning.
40. Use the graph to predict the capacity in gallons of a 12 quart container. Explain your reasoning.

EXTRA PRACTICE
See pages 733, 761.
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H.O.T. Problems

41. **OPEN ENDED** Write a problem about a real-world situation in which you would need to convert pints to cups.

REASONING Replace each ● with <, >, or = to make a true sentence. Justify your answers.

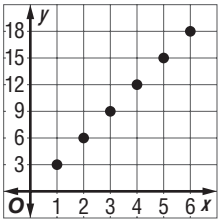
42. 16 in. ● $1\frac{1}{2}$ ft
43. $8\frac{3}{4}$ gal ● 32 qt
44. 2.7 T ● 86,400 oz

45. **CHALLENGE** To whiten fabrics, a certain Web site recommends that you soak them in a mixture of $\frac{3}{4}$ cup vinegar, 2 quarts water, and some salt. Does a mixture that contains 1.5 ounces vinegar and 16 ounces water have the same vinegar to water ratio as the recommended mixture? Explain your reasoning.

46. **WRITING IN MATH** Use multiplication by unit ratios of equivalent measures to convert 5 square feet to square inches. Justify your answer.

TEST PRACTICE

47. Which situation is represented by the graph?



A Conversion of inches to yards

B Conversion of feet to inches

C Conversion of miles to feet

D Conversion of yards to feet

48. How many cups of milk are shown below?

F $\frac{3}{4}$ c

G $1\frac{1}{4}$ c

H $2\frac{1}{2}$ c

J 10 c

Spiral Review

49. **GROCERIES** Three pounds of pears cost $3.57. At this rate, how much would 10 pounds cost? (Lesson 7-2)

Write each ratio as a fraction in simplest form. (Lesson 7-1)

50. 9 feet in 21 minutes

51. 36 calls in 2 hours

52. 14 SUVs out of 56 vehicles

53. $85 for 5 hours

54. 16 cats out of 44 animals

55. 3,048 people per 32 square miles

56. **ICE SKATING** By doubling just the length of the rectangular ice skating rink in Will's backyard from 16 to 32 feet, he increased its area from 128 square feet to 256 square feet. Find the width of both rinks. (Lesson 4-6)

EARNINGS **For Exercises 57–59, use the pay stub at the right.** (Lesson 4-3)

57. Write and solve an equation to find the regular hourly wage.

58. Write and solve an equation to find the overtime hourly wage.

59. Write and solve an equation to find how many times greater Grace's overtime hourly wage is than her regular hourly wage.

Martin, Grace		Employee #: 4211
Description:	**Hours:**	**Earnings ($):**
Regular hours:	40	300.00
Overtime hours:	2	22.50

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** (Lesson 4-3)

60. $5 \cdot 4 = x \cdot 2$ **61.** $9 \cdot 24 = 27 \cdot x$ **62.** $x \cdot 15 = 12 \cdot 4$ **63.** $8\frac{1}{2} \cdot x = 11 \cdot 17$

7-4 Algebra: Solving Proportions

Main IDEA

Solve proportions.

Targeted TEKS 7.3 The student solves problems involving direct proportional relationships. **(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.** *Also addresses TEKS 7.2(D), 7.5(B), 7.13(D).*

NEW Vocabulary

proportional
cross product

GET READY for the Lesson

NUTRITION The table shows the amount of vitamin C in different serving sizes of a certain cereal.

1. Write the rate $\frac{\text{vitamin C}}{\text{serving size}}$ for each serving size of cereal.
2. Find the number of milligrams per cup for each serving size. What do you notice?

Vitamin C (mg)	Serving Size (c)
15	0.5
60	2

Two quantities are **proportional** if they have a constant rate or ratio. In the example above, notice that the serving size and amount of vitamin C change or *vary* in the same way.

$$\frac{15 \text{ mg}}{0.5 \text{ c}} \overset{\times 4}{=} \frac{60 \text{ mg}}{2 \text{ c}}$$

When the serving size quadruples, the number of milligrams of vitamin C also quadruples.

The unit rates for these different-sized servings are the same, a constant 30 milligrams per cup. So, the amount of vitamin C is proportional to the serving size. Another way of expressing this relationship is by writing a proportion.

$$\frac{15 \text{ mg}}{0.5 \text{ c}} \overset{\times 4}{=} \frac{60 \text{ mg}}{2 \text{ c}} \overset{\div 2}{=} \frac{30 \text{ mg}}{1 \text{ c}}$$

KEY CONCEPT Proportion

Words A **proportion** is an equation stating that two ratios or rates are equivalent.

Symbols

Numbers	Algebra
$\frac{1}{2} = \frac{3}{6}$, $\frac{8 \text{ ft}}{10 \text{ s}} = \frac{4 \text{ ft}}{5 \text{ s}}$	$\frac{a}{b} = \frac{c}{d}$, where $b, d \neq 0$

Consider the following proportion.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} \cdot bd = \frac{c}{d} \cdot bd$$ Multiply each side by bd.

$$ad = bc$$ Simplify.

The products ad and bc are called the **cross products** of this proportion. The cross products of any proportion are equal. You can compare unit rates or cross products to identify proportional relationships.

EXAMPLE 1 Identify Proportional Relationships

RECREATION A carousel makes 4 complete turns after 64 seconds and 5 complete turns after 76 seconds. Based on this information, is the number of turns made by this carousel proportional to the time in seconds? Explain.

METHOD 1 Compare unit rates.

$$\frac{\text{seconds}}{\text{complete turns}} \rightarrow \quad \frac{64 \text{ s}}{4 \text{ turns}} = \frac{16 \text{ s}}{1 \text{ turn}} \qquad \frac{76 \text{ s}}{5 \text{ turns}} = \frac{15.2 \text{ s}}{1 \text{ turn}}$$

Since the unit rates are not equal, the number of turns is not proportional to the time in seconds.

METHOD 2 Compare ratios by comparing cross products.

$$\begin{matrix} \text{seconds} \rightarrow \\ \text{complete turns} \rightarrow \end{matrix} \quad \frac{64}{4} \stackrel{?}{=} \frac{76}{5} \quad \begin{matrix} \leftarrow \text{seconds} \\ \leftarrow \text{complete turns} \end{matrix}$$

$64 \cdot 5 \stackrel{?}{=} 4 \cdot 76$ Find the cross products.

$320 \neq 304$ Multiply.

Since the cross products are not equal, the number of turns is not proportional to the time in seconds.

CHOOSE Your Method

Determine if the quantities in each pair of ratios are proportional. Explain.

a. 60 voted out of 100 registered and 84 voted out of 140 registered

b. $12 for 16 yards of fabric and $9 for 24 yards fabric

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You can also use cross products to find a missing value in a proportion. This is known as *solving the proportion*.

STUDY TIP

Mental Math
Some proportions can be solved using mental math.

$\frac{2.5}{10} = \frac{x}{30}$

$\times 3$

$\frac{2.5}{10} = \frac{7.5}{30}$

$\times 3$

EXAMPLES 2 Solve a Proportion

Solve $\frac{21}{5} = \frac{c}{7}$.

$\frac{21}{5} = \frac{c}{7}$ Write the proportion.

$21 \cdot 7 = 5 \cdot c$ Find the cross products.

$147 = 5c$ Multiply.

$\frac{147}{5} = \frac{5c}{5}$ Divide each side by 5.

$29.4 = c$ Simplify.

Check for Reasonableness Since $\frac{21}{5} \approx \frac{20}{5}$ or $\frac{4}{1}$ and $\frac{29.4}{7} \approx \frac{28}{7}$ or $\frac{4}{1}$, the answer is reasonable. ✔

Concepts in Motion
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3 Solve $\frac{2.6}{13} = \frac{8}{n}$.

$\frac{2.6}{13} = \frac{8}{n}$ Write the proportion.

$2.6 \cdot n = 13 \cdot 8$ Find the cross products.

$2.6n = 104$ Multiply.

$\frac{2.6n}{2.6} = \frac{104}{2.6}$ Divide each side by 2.6.

$n = 40$ Simplify.

CHECK Your Progress **Solve each proportion.**

c. $\frac{16}{k} = \frac{2}{3}$ d. $\frac{2}{6} = \frac{5}{h}$ e. $\frac{10}{k} = \frac{2.5}{4}$

Real-World Career . . .
How Does a Paleontologist use Math?
In 2001, paleontologists discovered a 6-foot skull of a prehistoric crocodile called *SuperCroc*. By using proportions, they estimated its total length as 40 feet.

MathOnline
For more information, go to tx.msmath2.com.

Real-World EXAMPLE

4 **SCIENCE** **In one species, a 6-foot crocodile has a 2-foot skull. If skull length is proportional to body length, what is the length of a crocodile of that same species with a 3.5-foot skull?**

METHOD 1 **Write and solve a proportion.**

Let b represent the length of the crocodile with a 3.5-foot skull.

body length → $\frac{6 \text{ ft}}{2 \text{ ft}} = \frac{b \text{ ft}}{3.5 \text{ ft}}$ Write a proportion.
skull length →

$6 \cdot 3.5 = 2 \cdot b$ Find the cross products.

$21 = 2b$ Multiply.

$10.5 = b$ Divide each side by 2.

METHOD 2 **Find and use a unit rate or ratio.**

body length → $\frac{6 \text{ ft} \div 2}{2 \text{ ft} \div 2} = \frac{3}{1}$ There ratio of body length to skull length is 3 : 1.
skull length →

Words	The body length is 3 times the skull length.
Variable	Let b represent the length of the crocodile with a 3.5 foot skull.
Equation	$b = 3 \cdot 3.5$

$b = 3 \cdot 3.5$ or 10.5 Multiply.

So, a crocodile with a 3.5-foot skull is about 10.5 feet long.

CHOOSE Your Method

f. **RUNNING** How long will it take Salvador to run a 300-meter race if he can run 120 meters in 24 seconds?

CHECK Your Understanding

Example 1 (p. 349)

Determine if the quantities in each pair of ratios are proportional. Explain.

1. 2 adults for 10 children and 3 adults for 12 children
2. 12 inches by 8 inches and 18 inches by 12 inches
3. 8 feet in 21 seconds and 12 feet in 31.5 seconds
4. $5.60 for 5 pairs of socks and $7.12 for 8 pairs of socks

Examples 2, 3 (pp. 349–350)

Solve each proportion.

5. $\frac{5}{6} = \frac{t}{18}$
6. $\frac{6}{k} = \frac{24}{28}$
7. $\frac{21}{5} = \frac{c}{7}$
8. $\frac{15}{w} = \frac{2}{5}$
9. $\frac{3}{n} = \frac{2.7}{18}$
10. $\frac{0.2}{3} = \frac{3}{d}$

Example 4 (p. 350)

11. **GROCERIES** Orange juice is on sale 3 half-gallons for $5. At this rate, find the cost of 5 half-gallons of orange juice to the nearest cent.
12. **TRAVEL** Franco drove 203 miles in 3.5 hours. At this rate, how long will it take him to drive another 29 miles to the next town?

Exercises

HOMEWORK HELP

For Exercises	See Examples
13–20	1
21–32	2, 3
33–40	4

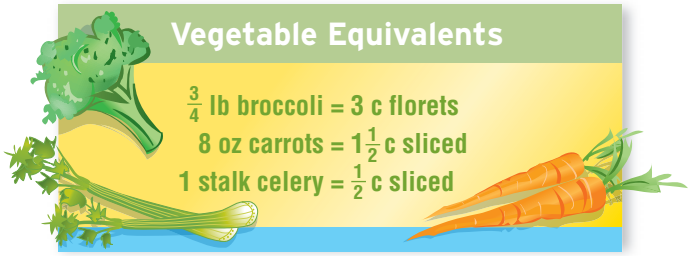
Determine if the quantities in each pair of ratios are proportional. Explain.

13. 20 children from 6 families to 16 children from 5 families
14. 5 pounds of dry ice melts in 30 hours and 4 pounds melts in 24 hours
15. 16 winners out of 200 entries and 28 winners out of 350 entries
16. 5 meters in 7 minutes and 25 meters in 49 minutes
17. 1.4 tons produced every 18 days and 10.5 tons every 60 days
18. 3 inches for every 4 miles and 7.5 inches for every 10 miles
19. **READING** Leslie reads 25 pages in 45 minutes. After 60 minutes, she has read a total of 30 pages. Is her time proportional to the number of pages she reads? Explain.
20. **PETS** A store sells 2 hamsters for $11 and 6 hamsters for $33. Is the cost proportional to the number of hamsters sold? Explain.

Solve each proportion.

21. $\frac{3}{8} = \frac{b}{40}$
22. $\frac{x}{12} = \frac{12}{4}$
23. $\frac{c}{7} = \frac{18}{42}$
24. $\frac{5}{k} = \frac{10}{22}$
25. $\frac{3}{8} = \frac{n}{4}$
26. $\frac{15}{4} = \frac{3}{g}$
27. $\frac{45}{5} = \frac{d}{7}$
28. $\frac{30}{a} = \frac{8}{20}$
29. $\frac{1.6}{m} = \frac{2}{3}$
30. $\frac{4.5}{5} = \frac{t}{7}$
31. $\frac{2.5}{4.5} = \frac{7.5}{x}$
32. $\frac{3.8}{5.2} = \frac{7.6}{z}$
33. **SCHOOL** If 4 notebooks weigh 2.8 pounds, how much do 6 of the same notebooks weigh?
34. **COOKING** There are 6 teaspoons in 2 tablespoons. How many teaspoons are in 1.5 tablespoons?

ANALYZE TABLES For Exercises 35–38, use the table.



35. How many stalks of celery yield $3\frac{1}{2}$ cups of celery slices?

36. How many cups of broccoli florets would 1 pound of broccoli produce?

37. About how many ounces of carrots yields 2 cups sliced?

38. How many cups of sliced celery would $7\frac{1}{2}$ stalks of celery yield?

39. **SCIENCE** The ratio of salt to water in a certain solution is 4 to 15. If the solution contains 6 ounces of water, how many ounces of salt does it contain?

40. **CONCERTS** Alethia purchased 7 advanced tickets for herself and her friends to a concert and paid $164.50. If the total cost of tickets to the concert is proportional to the number purchased, how many tickets to the same concert did Serefina purchase if she paid a total of $94?

41. **SAVINGS** Pao spent $140 of his paycheck and put the remaining $20 in his savings account. If the number of dollars he spends is proportional to the number he saves, how much of a $156-paycheck will he put into savings?

42. **MOVIES** After 30 seconds, 720 frames of film have passed through a movie projector. At this rate, what is the approximate running time in minutes of a movie made up of 57,000 frames of film?

43. **SCHOOL** There are 325 students and 13 teachers at a school. Next school year, the enrollment is expected to increase by 100 students. Write and solve a proportion to find the number of teachers that must be hired so the student-teacher ratio remains the same.

44. **FIND THE DATA** Refer to the Texas Data File on pages 16–19. Choose some data and write a real-world problem in which you would solve a proportion.

H.O.T. Problems

45. **Which One Doesn't Belong?** Identify the rate that is not proportional to the other three. Explain your reasoning.

$4.50 for 5 lb	$2.88 for 3.2 lb	$5.70 for 6 lb	$4.86 for 5.4 lb

46. **CHALLENGE** A box contains juice boxes, bottles of water, and cans of soft drinks. If the ratio of water to juice is 1:6 and the ratio of juice to soft drinks is 3:5, what is the ratio of water to soft drinks? Explain your reasoning.

Real-World Link
Film for an IMAX projection system passes through the projector at the rate of 330 feet per minute or 5.5 feet per second.
Source: hfmgv.org

EXTRA PRACTICE
See pages 734, 761.
MathOnline
Self-Check Quiz at tx.msmath2.com

47. **SELECT A TECHNIQUE** Sweet corn is on sale at $2.50 for a dozen at a farmer's market. Select one or more of the following technique(s) to determine how many ears you can buy for $10. Then use this technique to solve the problem.

mental math | estimation | number sense

48. **WRITING IN MATH** Marshall walks 3 large dogs and 2 small dogs for a dog-walking service. If he is hired to walk 2 more large dogs and 2 more small dogs, will the ratio of large dogs to small dogs stay the same? Explain your reasoning.

TEST PRACTICE

49. Mirma gives away 84 flyers over a 3-hour period. If the number of flyers she is able to give away per hour remains the same, which proportion can be used to find x, the number of flyers that she would give away over a 5-hour period?

A $\frac{3}{84} = \frac{x}{5}$ C $\frac{5}{3} = \frac{84}{x}$

B $\frac{84}{3} = \frac{x}{5}$ D $\frac{3}{84} = \frac{x}{8}$

50. A recipe that makes 16 muffins calls for $\frac{1}{2}$ cup of flour. How much flour is needed to make 3 dozen muffins using this recipe?

F $1\frac{1}{8}$ c

G 1 c

H $1\frac{1}{4}$ c

J $1\frac{1}{2}$ c

Spiral Review

MEASUREMENT Complete. (Lesson 7-3)

51. 5 qt = ■ pt

52. $3\frac{1}{2}$ lb = ■ oz

53. 28 c = ■ qt

54. **GROCERIES** Which is a better buy: 5 pounds of onions for $2.99 or 3 pounds of onions for $1.59? (Lesson 7-2)

Multiply. Write in simplest form. (Lesson 6-5)

55. $3\frac{1}{2} \times 5\frac{7}{8}$

56. $1\frac{2}{3} \times 5\frac{4}{5}$

57. $2\frac{1}{4} \times 7\frac{5}{8}$

GET READY for the Next Lesson

58. **PREREQUISITE SKILL** Mr. Andres is filling up his car with gas that costs $2.50 per gallon. His car's gas gauge before filling up is shown at the right. If his car's gas tank holds 16 gallons, about how much will Mr. Andres pay to fill up his tank? Use the *eliminate possibilities* strategy. (Lesson 6-4)

A $15.00

B $25.00

C $27.00

D $35.00

Extend 7-4

Math Lab
Wildlife Sampling

Main IDEA

Use proportions to estimate populations.

Targeted TEKS 7.3 The student solves problems involving direct proportional relationships. **(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.**

Naturalists can estimate the population in a wildlife preserve by using the capture-recapture technique. In this lab, you will model this technique using dried beans in a bowl to represent deer in a forest.

ACTIVITY

STEP 1 Fill a small bowl with dried beans.

STEP 2 Use the paper cup to scoop out some of the beans. These represent the original *captured* deer. Record the number in a table like the one shown at the right. Mark each bean with an × on both sides. Then return these beans to the bowl and mix well.

Original Number Captured			
Trial	Sample	Recaptured	*P*
A			
B			
C			
⋮			
J			
Total			

STEP 3 Scoop another cup of beans from the bowl and count them. This is the *sample* for Trial A. Count the beans with the ×'s. These are the *recaptured* deer. Record both numbers.

STEP 4 Use the proportion below to estimate the total number of beans in the bowl. This represents the total population *P*. Record the value of *P* in the table.

$$\frac{\text{captured}}{\text{total population } (P)} = \frac{\text{recaptured}}{\text{sample}}$$

STEP 5 Return all of the beans to the bowl.

STEP 6 Repeat Steps 3–5 nine times.

Analyze the Results

1. **ESTIMATION** Find the average of the estimates in column *P*. Is this a good estimate of the number of beans in the bowl? Explain your reasoning.
2. Count the actual number of beans in the bowl. How does this number compare to your estimate?

CHAPTER 7

Mid-Chapter Quiz

Lessons 7-1 through 7-4

RECIPE **For Exercises 1–3, use the information in the table to write each ratio as a fraction in simplest form.** (Lesson 7-1)

Cherry Punch Recipe	
Cherry Juice	4 cups
Apple Juice	2 cups
Ginger Ale	16 cups

1. cherry juice : apple juice
2. apple juice : ginger ale
3. cherry juice : ginger ale

Determine whether the ratios are equivalent. Explain. (Lesson 7-1)

4. 6 out of 9 words spelled correctly
   2 out of 3 words spelled correctly
5. 150 athletes to 15 coaches
   3 athletes to 1 coach
6. 24 points in 4 games
   72 points in 8 games

Find each unit rate. Round to the nearest hundredth if necessary. (Lesson 7-2)

7. 200 miles in 4 hours
8. $6.20 for 5 pounds
9. 98 carbohydrates in 8 servings

10. **TEST PRACTICE** Celeste's favorite type of chocolate comes in four different amounts. Which amount of chocolate shown in the table has the best unit price? (Lesson 7-2)

Weight (oz)	Cost ($)
12	2.50
18	3.69
24	4.95
30	6.25

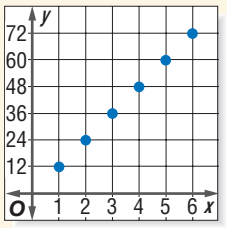
A 12 oz **C** 24 oz

B 18 oz **D** 36 oz

Complete. (Lesson 7-3)

11. 42 ft = ■ yd
12. 9 pt = ■ qt
13. 7,600 lb = ■ T
14. $7\frac{1}{2}$ gal = ■ qt

15. **TEST PRACTICE** Which situation is best represented by the graph? (Lesson 7-3)



F Conversion of inches to yards

G Conversion of feet to inches

H Conversion of inches to miles

J Conversion of yards to feet

Determine if the quantities in each pair of ratios are proportional. Explain. (Lesson 7-4)

16. 8 pages in 5 minutes and 40 pages in 25 minutes
17. 40 blank CDs for $9.60 and 24 blank CDs for $4.80
18. 45 pounds in 30 bags and 72 pounds in 48 bags

Solve each proportion. (Lesson 7-4)

19. $\frac{3}{d} = \frac{12}{20}$
20. $\frac{7}{8} = \frac{m}{48}$
21. $\frac{w}{8} = \frac{1}{3}$
22. $\frac{1}{4} = \frac{3}{t}$

23. **TEST PRACTICE** It took 45 minutes to fill a circular pool of uniform depth to a level of 17 inches. At this rate, which of the following represents the approximate length of time will it take to fill the pool to a level of 35 inches? (Lesson 7-4)

A 138 min **C** 93 min

B 63 min **D** 22 min

7-5 Problem-Solving Investigation

MAIN IDEA: Solve problems by drawing a diagram.

Targeted TEKS 7.13 The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. **(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture,… to solve a problem.** *Also addresses TEKS 7.13(B).*

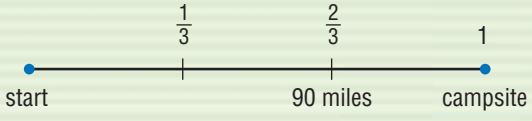
P.S.I. TEAM +

I-M: DRAW A DIAGRAM

YOUR MISSION: Draw a diagram to solve the problem.

THE PROBLEM: How much farther do we have to drive?

Frieda: We've gone about 90 miles, which is about $\frac{2}{3}$ of the way to the campsite. So, how much farther do we have to go?

EXPLORE	You know that 90 miles is about $\frac{2}{3}$ of the total distance.
PLAN	Draw a diagram showing the fractional part of the distance.
SOLVE	 If $\frac{2}{3}$ of the distance is 90 miles, then $\frac{1}{3}$ of the distance would be 45 miles. So, the missing third must be another 45 miles.  The total distance is 90 + 45 or 135 miles.
CHECK	Since $\frac{2}{3}$ of the total distance is 90 miles, and $\frac{2}{3}(135) = 90$, the solution checks.

Analyze The Strategy

1. Determine how far the trip would have been if the 90 miles were only $\frac{1}{3}$ of the total distance. Draw a new diagram for this situation.
2. WRITING IN MATH Write a problem that could be solved by drawing a diagram. Exchange your problem with a classmate and solve.

Mixed Problem Solving

Solve Exercises 3–5. Use the *draw a diagram* strategy.

3. **PHYSICAL SCIENCE** A ball is dropped from a height of 10 feet. It hits the ground and bounces up half as high as it fell. This is true for each successive bounce. What height does the ball reach on the fourth bounce?

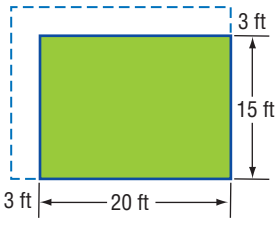
4. **FAMILY** At Nelia's family reunion, 80% of the people are 18 years of age or older. Half of the remaining people are under 12 years old. If 20 children are under 12 years old, how many people are at the reunion?

5. **VOLUME** A swimming pool is being filled with water. After 25 minutes, $\frac{1}{6}$ of the swimming pool is filled. How much longer will it take to completely fill the pool, assuming the water rate is constant?

Use any strategy to solve Exercises 6–8. Some strategies are shown below.

PROBLEM-SOLVING STRATEGIES
- Use the four-step plan.
- Work backward.
- Eliminate possibilities.
- Draw a diagram.

6. **GEOMETRY** Jamila is adding 3 feet to the length and width of her rectangular garden, as shown.



Which expression represents the area of land to be added to the original garden?

A $(15 + 3)(20 + 3)$

B $(15 + 3)(20 + 3) - (15)(20)$

C $(15 - 3)(20 - 3)$

D $(15 - 3)(20 - 3) - (15)(20)$

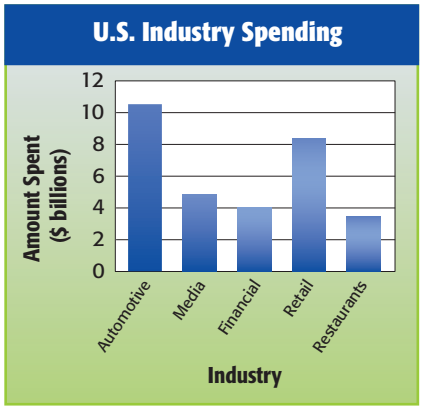
7. **GAMES** Six members of a video game club are having a tournament. In the first round, every player will play a video game against every other player. How many games will be in the first round of the tournament?

8. **MONEY** Mr. Li has $240 in his checking account after writing checks for $15.70, $43.20, and $18. What was his balance before he wrote the three checks?

Select the Operation

For Exercises 9 and 10, select the appropriate operation(s) to solve the problem. Justify your selection(s) and solve the problem.

9. **BUSINESS** The graph shows the annual spending by five industries in the United States.



Source: *The Top Ten of Everything*

Estimate how much more the automotive industry spends than the retail industry.

10. **TESTS** The scores on a test are found by adding or subtracting points as shown below. If Salazar's score on a 15-question test was 86 points, how many of his answers were correct, incorrect, and blank?

Answer	Points
Correct	+8
Incorrect	−4
No answer	−2

7-6 Scale Drawings

Main IDEA

Solve problems involving scale drawings.

Targeted TEKS 7.3 The student solves problems involving direct proportional relationships. **(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.** *Also addresses TEKS 7.5(B).*

NEW Vocabulary

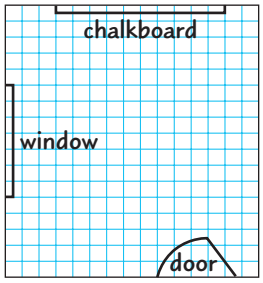
scale drawing
scale model
scale
scale factor

MINI Lab

Concepts in Motion In Motion
Interactive Lab tx.msmath2.com

- Measure the length of each wall, door, window, and chalkboard in your classroom.
- Record each length to the nearest $\frac{1}{2}$ foot.

1. Let 1 unit on the grid paper represent 1 foot. So, 6 units = 6 feet. Convert all of your measurements to units.
2. On grid paper, make a drawing of your classroom like the one shown at the right.



A map is an example of a scale drawing. **Scale drawings** and **scale models** are used to represent objects that are too large or too small to be drawn or built at actual size. The **scale** gives the ratio that compares the measurements on the drawing or model to the measurements of the real object. The measurements on a drawing or model are proportional to measurements of the actual object.

EXAMPLE 1 Use a Map Scale

MAPS What is the actual distance between Corpus Christi and Austwell?

Step 1 Use a centimeter ruler to find the map distance between the two cities. The map distance is about 4.4 centimeters.

Step 2 Write and solve a proportion using the scale. Let d represent the actual distance between the cities.

	Scale	Corpus Christi to Austwell	
map →	$\frac{1 \text{ centimeter}}{20 \text{ kilometers}}$	$= \frac{4.4 \text{ centimeters}}{d \text{ kilometers}}$	← map
actual →			← actual

$1 \times d = 20 \times 4.4$ Cross products

$d = 88$ Simplify.

The distance between the cities is about 88 kilometers.

STUDY TIP

Scale A map scale can be written in different ways, including the following:

1 cm = 20 km

1 cm : 20 km

$\frac{1 \text{ cm}}{20 \text{ km}}$

CHECK Your Progress

a. **MAPS** On the map of Arkansas shown, find the distance between Clarksville and Little Rock. Use a ruler to measure.

A blueprint is another example of a scale drawing.

STUDY TIP

Alternate Method
You could also set up and solve the proportion below.

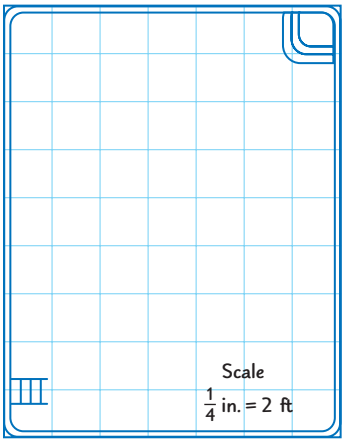
$$\frac{\frac{1}{4}\text{ in.}}{2\text{ ft}} \overset{\times 7}{=} \frac{\frac{7}{4}\text{ in.}}{w\text{ ft}}$$

So, $w = 14$ ft.

EXAMPLE Use a Blueprint Scale

2 **POOLS** **On the blueprint of the pool, each square has a side length of $\frac{1}{4}$ inch. What is the actual width of the pool?**

The pool on the blueprint is $1\frac{3}{4}$ inches wide. Write and solve a proportion using the scale. Let w represent the actual width of the pool.

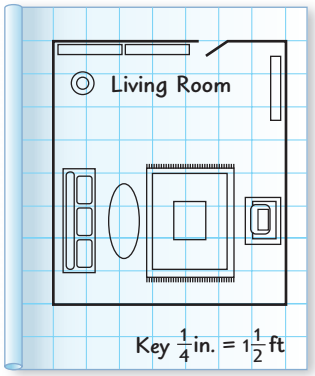


	Scale	Width of Pool	
blueprint →	$\frac{1}{4}$ inch	$1\frac{3}{4}$ inches	← blueprint
actual →	2 feet	w feet	← actual
	$\frac{1}{4} \cdot w = 2 \cdot 1\frac{3}{4}$		Cross products
	$\frac{1}{4}w = \frac{14}{4}$		Multiply.
	$w = 14$		Simplify. Multiply each side by 4.

The actual width of the pool is 14 feet.

CHECK Your Progress

b. **INTERIOR DESIGN** On the blueprint of the living room, each square has a side length of $\frac{1}{4}$ inch. What are the actual dimensions of the living room?

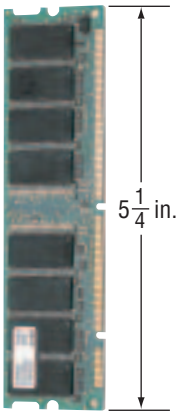


STUDY TIP

Scale The scale is the ratio of the drawing/model measure to the actual measure. It is not always the ratio of a smaller measure to a larger measure.

EXAMPLE Use a Scale Model

3 **COMPUTERS** **Designers are creating a larger model of the computer memory board. If they use a scale of 20 inches = 1 inch, what is the length of the model?**



Write a proportion using the scale.
Let m represent the length of the model.

	Scale	Length	
model →	$\frac{20 \text{ inches}}{1 \text{ inch}}$	$= \frac{m \text{ inches}}{5\frac{1}{4} \text{ inches}}$	← model
actual →			← actual

$20 \cdot 5\frac{1}{4} = 1 \cdot m$ Cross products

$105 = m$ Multiply.

The scale model is 105 inches long.

CHECK Your Progress

c. **SCOOTERS** A scooter is $3\frac{1}{2}$ feet long. Find the length of a scale model of the scooter if the scale is 1 inch $= \frac{3}{4}$ feet.

Online **Personal Tutor at** tx.msmath2.com

In Lesson 7–3, you used ratios to convert units. You can use a similar method to simplify a scale. A scale written as a ratio without units in simplest form is called the **scale factor**.

scale → $\frac{\frac{1}{4} \text{ inch}}{2 \text{ feet}} = \frac{\frac{1}{4} \text{ inch}}{24 \text{ inches}}$ Convert 2 feet to inches.

$= \frac{4}{4} \cdot \frac{\frac{1}{4} \text{ inch}}{24 \text{ inches}}$ Mutiply by $\frac{4}{4}$ to eliminate the fraction in the numerator.

$= \frac{1}{96}$ ← scale factor

EXAMPLE Find a Scale Factor

STUDY TIP

Equivalent Scales The scales below are equivalent because their scale factors are equal, $\frac{1}{72}$.

- 1 inch = 6 feet
- $\frac{1}{2}$ inch = 3 feet

4 **SAILBOATS** **Find the scale factor of a model sailboat if the scale is 1 inch = 6 feet.**

$\frac{1 \text{ inch}}{6 \text{ feet}} = \frac{1 \text{ inch}}{72 \text{ inches}}$ Convert 6 feet to inches.

$= \frac{1}{72}$ Divide out the common units.

The scale factor is $\frac{1}{72}$. That is, each measure on the model is $\frac{1}{72}$ of the actual measure.

CHECK Your Progress

d. **CARS** What is the scale factor of a model car if the scale is 1 inch = 2 feet?

Matt Meadows

CHECK Your Understanding

Example 1 (pp. 358–359)

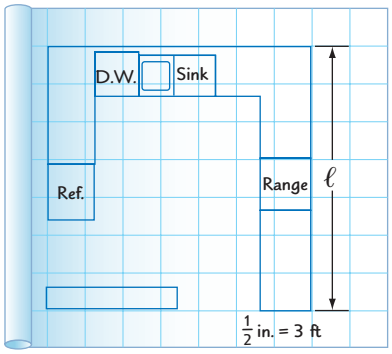
GEOGRAPHY **Find the actual distance between each pair of cities in New Mexico. Use a ruler to measure.**

1. Carlsbad and Artesia
2. Hobbs and Eunice
3. Artesia and Eunice
4. Lovington and Carlsbad

Example 2 (p. 359)

BLUEPRINTS **For Exercises 5 and 6, use the blueprint. Each square has a side length of $\frac{1}{2}$ inch.**

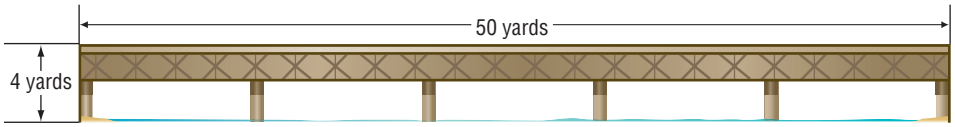
5. Find the actual length ℓ of the kitchen.
6. Estimate the walking distance from the range to the refrigerator.



Example 3 (p. 360)

BRIDGES **For Exercises 7 and 8, use the following information.**

A engineer makes a model of the bridge using a scale of 1 inch = 3 yards.

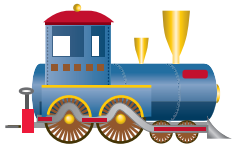


7. What is the length of the model?
8. What is the height of the model?

Example 4 (p. 360)

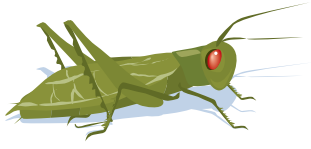
Find the scale factor of each scale drawing or model.

9.



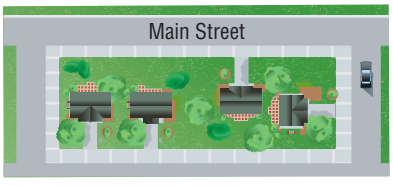
1 inch = 4 feet

10.



1 centimeter = 15 millimeters

11. **CITY PLANNING** In the aerial view of a city block at the right, the length of Main Street is 2 inches. If Main Street's actual length is 2 miles, find the scale factor of the drawing.



Exercises

HOMEWORK HELP

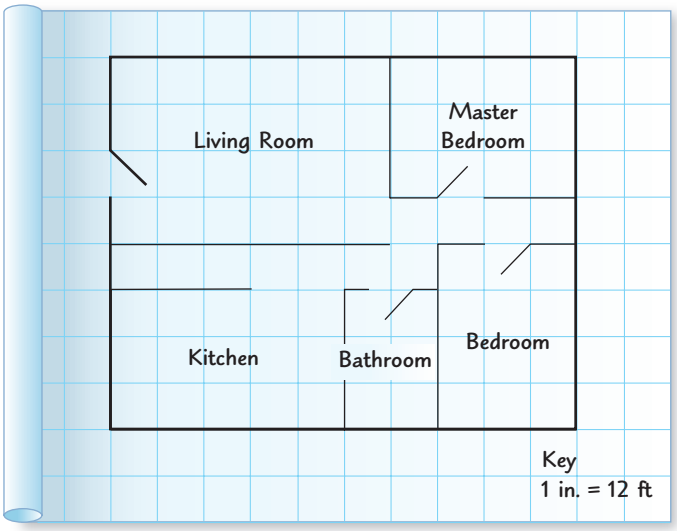
For Exercises	See Examples
12–15	1
16–17	2
18–24	3, 4

GEOGRAPHY **Find the actual distance between each pair of locations in South Carolina. Use a ruler to measure.**

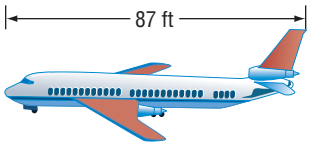
12. Columbia and Charleston
13. Hollywood and Sumter
14. Congaree Swamp and Charleston
15. Sumter and Columbia

For Exercises 16–18, use the blueprint of an apartment at the right. Each square has a side length of $\frac{1}{4}$ inch.

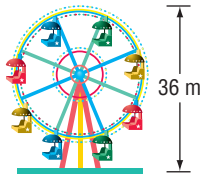
16. What is the actual length of the living room?
17. Find the actual dimensions of the master bedroom.
18. Find the scale factor for this blueprint.



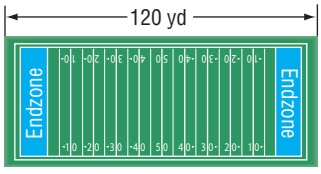
Find the length of each model. Then find the scale factor.

19. 

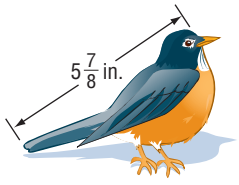
2 in. = 15 ft

20. 

0.5 cm = 1.5 m

21. 

1 in. = 20 yd

22. 

1 in. = 0.5 in.

EXTRA PRACTICE

See pages 734, 761.

Math Online
Self-Check Quiz at tx.msmath2.com

Real-World Link . . . . .
Mount Rushmore is a sculpture that was carved using a model with a scale of 1 inch : 1 foot.

23. **GEOGRAPHY** A map of Bakersfield, California, has a scale of 1 inch to 5 miles. If the city is $5\frac{1}{5}$ inches across on the map, what is the actual distance across the actual city? Use estimation to check your answer.

24. **TREES** A model of a tree is made using a scale of 1 inch : 25 feet. What is the height of the actual tree if the height of the model is $4\frac{3}{8}$ inches?

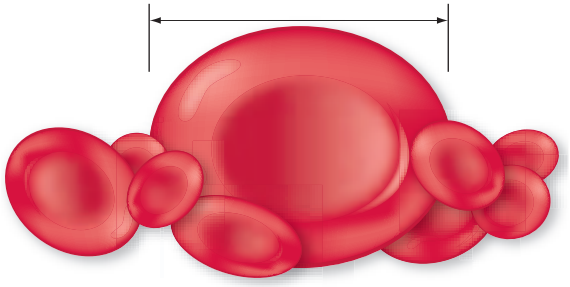
25. **SCULPTURES** Refer to the information at the left. Find the scale factor and the actual height of George Washington's face on the sculpture if the height of the model's face is 5 feet.

26. **GEOGRAPHY** Lexington and Elizabethtown, Kentucky, are 79 miles apart. If the distance on the map is $2\frac{1}{2}$ inches, find the scale of the map.

27. **RESEARCH** Find the dimensions of any U.S. presidential monument. Give an appropriate scale that can be used to make a scale model of the monument. State the dimensions of the model using your scale.

28. **IGLOOS** The diameter of the circular floor of an igloo is 10 feet. If you were to make a model of the igloo with sugar cubes, which might be an appropriate scale: 1 in. = 1 ft or 1 in. = 50 ft? Explain your reasoning.

29. **LIFE SCIENCE** A scale drawing of a red blood cell is shown below. If the blood cell's actual diameter is 0.008 millimeter, use a ruler to find the scale factor of the drawing.



H.O.T. Problems

30. **OPEN ENDED** On grid paper, create a scale drawing of a room in your home. Include the scale that you used.

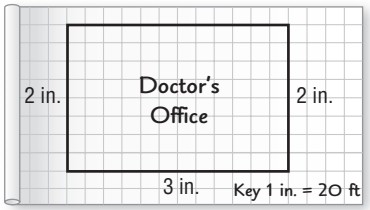
31. **CHALLENGE** Montoya constructed three models, A, B, and C, of the same figure, with scales of 0.5 cm = 1 mm, 1.5 mm = 4 cm, and 0.25 cm = 2.5 mm, respectively.
    - **a.** Which model is larger than the actual figure? Justify your answer.
    - **b.** Which model is smaller than the actual figure? Justify your answer.
    - **c.** Which model is the same size as the actual figure? Justify your answer.

32. **REASONING** Compare and contrast the terms *scale* and *scale factor*. Include an example in your comparison.

33. **WRITING IN MATH** Explain how you could use estimation to find the actual distance between San Diego, California, and Seattle, Washington, on a map.

TEST PRACTICE

34. A scale drawing of a doctor's office is shown.



What are the actual dimensions of the doctor's office?

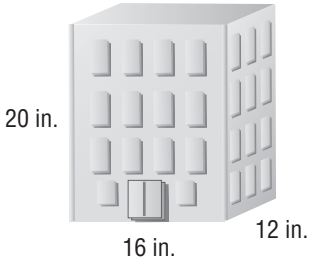
A 24 feet by 48 feet

B 30 feet by 52 feet

C 40 feet by 60 feet

D 37.5 feet by 65 feet

35. GRIDDABLE A scale model of a new office building is shown. If the scale is 5 inches = 18 feet, how tall is the actual building in feet?



36. Ernesto drew a map of his school. He used a scale in which 1 inch equals 50 feet. What distance on Ernesto's map should represent the 625 feet between the cafeteria and the science lab?

F 8 in. **H** 12.5 in.

G 10.5 in. **J** 15 in.

Spiral Review

37. BIRDS In a certain pair of binoculars, objects 35 feet away appear to be only 1 foot away. If a goldfinch is 368 feet away, what will the distance appear to be in the binoculars? Round to the nearest tenth. (Lesson 7-4)

Find each unit rate. (Lesson 7-2)

38. 200 miles in 5 hours **39.** 99¢ for 30 ounces **40.** 150 meters in 12 seconds

41. JOGGING The table shows the number of miles Tonya jogged each week for the past several weeks. Estimate the total number of miles she jogged. (Lesson 6-1)

Week	Miles
1	$7\frac{1}{6}$
2	$8\frac{3}{4}$
3	10
4	$12\frac{1}{4}$
5	$6\frac{2}{3}$

Find each product. (Lesson 3-3)

42. $4.5 \times 1{,}000$ **43.** 0.361×100

44. $17.89 \times 10{,}000$ **45.** $0.08 \times 100{,}000$

46. $1.17 \times 1{,}000$ **47.** 53×100

GET READY for the Next Lesson

PREREQUISITE SKILL **Divide. Write in simplest form.** (Lesson 6-7)

48. $2\frac{3}{4} \div 10$ **49.** $4\frac{1}{3} \div 10$ **50.** $30\frac{2}{3} \div 100$ **51.** $87\frac{1}{2} \div 100$

Extend 7-6

Spreadsheet Lab
Scale Drawings

Main IDEA

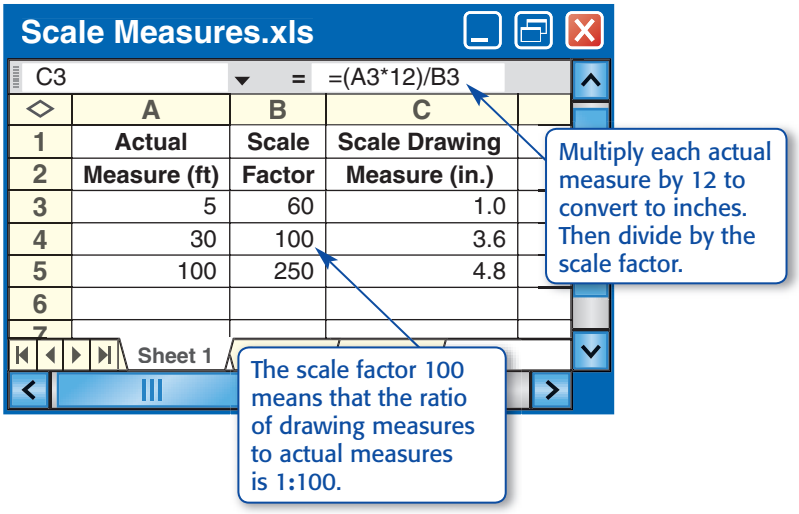
Use a spreadsheet to calculate measurements for scale drawings.

Targeted TEKS
7.3 The student solves problems involving direct proportional relationships. **(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.**

A computer spreadsheet is a useful tool for calculating measures for scale drawings. You can change the scale factors and the dimensions, and the spreadsheet will automatically calculate the new values.

ACTIVITY

Suppose you want to make a scale drawing of your school. Set up a spreadsheet like the one shown below. In this spreadsheet, the actual measures are in feet, and the scale drawing measures are in inches.



	A	B	C
1	Actual	Scale	Scale Drawing
2	Measure (ft)	Factor	Measure (in.)
3	5	60	1.0
4	30	100	3.6
5	100	250	4.8
6			

Analyze the Results

1. The length of one side of the school building is 100 feet. If you use a scale factor of 1:250, what is the length on your scale drawing?
2. The length of a classroom is 30 feet. What is the scale factor if the length of the classroom on a scale drawing is 3.6 inches?
3. Calculate the length of a 30-foot classroom on a scale drawing if the scale factor is 1:10.
4. The width of a hallway is 20 feet. What is the scale factor if the width of the hallway on a scale drawing is 2.5 inches?
5. Suppose the actual measures of your school are given in meters. Describe how you could use a spreadsheet to calculate the scale drawing measures in centimeters using a scale factor of 1:50.
6. Choose three rooms in your home and use a spreadsheet to make scale drawings. First, choose an appropriate scale and calculate the scale factor. Include a sketch of the furniture drawn to scale in each room.

7-7 Fractions, Decimals, and Percents

Main IDEA

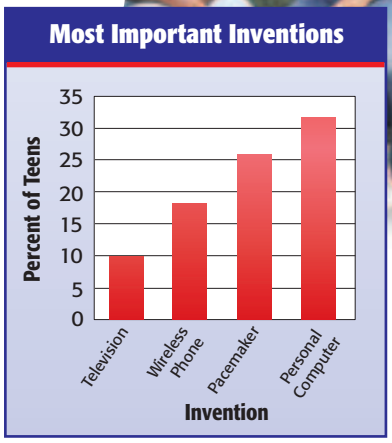
Write percents as fractions, and decimals and vice versa.

Targeted TEKS 7.1 The student represents and uses numbers in a variety of equivalent forms. **(B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator.** *Also addresses TEKS 7.1(A).*

GET READY for the Lesson

SURVEYS The graph shows the results of a survey in which teens were asked to name the most important invention of the 20th century.

1. What percent of the teens said that the personal computer was the most important invention?
2. How is this percent written as a ratio?
3. Simplify the ratio.



Source: Lemelson-MIT Program

REVIEW Vocabulary

percent a ratio that compares a number to 100 (Lesson 5-6)

In Lesson 5-6, you wrote percents like 32% as fractions by writing fractions with denominators of 100 and then simplifying. You can use the same method to write percents like 16.8% and $8\frac{1}{3}\%$ as fractions.

EXAMPLES Percents as Fractions

1 **HOCKEY** **In men's college hockey, 16.8% of the players are from Ontario, Canada. What fraction is this? Write in simplest form.**

$16.8\% = \frac{16.8}{100}$ Write a fraction with a denominator of 100.

$= \frac{16.8}{100} \cdot \frac{10}{10}$ Multiply by $\frac{10}{10}$ to eliminate the decimal in the numerator.

$= \frac{168}{1{,}000}$ or $\frac{21}{125}$ Simplify.

So, out of every 125 players, 21 are from Ontario.

2 **Write $8\frac{1}{3}\%$ as a fraction in simplest form.**

$8\frac{1}{3}\% = \frac{8\frac{1}{3}}{100}$ Write a fraction.

$= 8\frac{1}{3} \div 100$ Divide.

$= \frac{25}{3} \div 100$ Write $8\frac{1}{3}$ as an improper fraction.

$= \frac{25}{3} \cdot \frac{1}{100}$ Multiply by the reciprocal of 100, which is $\frac{1}{100}$.

$= \frac{25}{300}$ or $\frac{1}{12}$ Simplify.

CHECK Your Progress

Write each percent as a fraction in simplest form.

a. 15% b. $17\frac{1}{2}\%$ c. $33\frac{1}{3}\%$

To write a fraction like $\frac{8}{25}$ as a percent, multiply the numerator and the denominator by a number so that the denominator is 100. If the denominator is not a factor of 100, you can write fractions as percents by using a proportion.

EXAMPLES Fractions as Percents

3 **TESTS** **On a math test, Amalia got 7 questions correct out of 8. Find her grade as a percent.**

To find Amalia's grade, write $\frac{7}{8}$ as a percent.

Estimate $\frac{6}{8} = \frac{3}{4}$ or 75%.

So, $\frac{7}{8}$ is greater than 75%.

$\frac{7}{8} = \frac{n}{100}$ Write a proportion.

$700 = 8n$ Find the cross products.

$\frac{700}{8} = \frac{8n}{8}$ Divide each side by 8.

$87\frac{1}{2} = n$ Simplify.

So, $\frac{7}{8} = 87\frac{1}{2}\%$ or 87.5%.

Check for Reasonableness $87.5\% > 75\%$ ✔

STUDY TIP

Choose the Method
To write a fraction as a percent,

- use *multiplication* when a fraction has a denominator that is a factor of 100,
- use a *proportion* for any type of fraction.

4 **Write $\frac{4}{15}$ as a percent. Round to the nearest hundredth.**

Estimate $\frac{4}{15}$ is about $\frac{4}{16}$, which equals $\frac{1}{4}$ or 25%.

$\frac{4}{15} = \frac{n}{100}$ Write a proportion.

$400 = 15n$ Find the cross products.

$\frac{400}{15} = \frac{15n}{15}$ Divide each side by 15.

400 ÷ 15 ENTER 26.66666667 Use a calculator to simplify.

So, $\frac{4}{15}$ is about 26.67%.

Check for Reasonableness $26.67\% \approx 25\%$ ✔

Personal Tutor Personal Tutor for MAC

CHECK Your Progress

Write each fraction as a percent. Round to the nearest hundredth if necessary.

d. $\frac{2}{15}$ e. $\frac{7}{16}$ f. $\frac{17}{25}$

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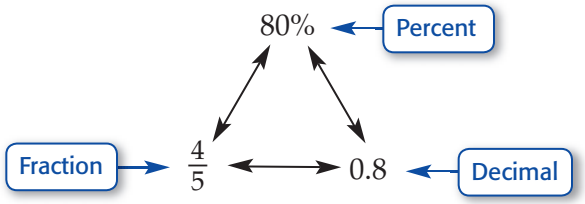
STUDY TIP

Look Back
You can review **writing fractions as decimals** in Lesson 5-3.

In this lesson, you have written percents as fractions and fractions as percents. In Chapter 5, you wrote percents and fractions as decimals. You can also write a fraction as a percent by first writing the fraction as a decimal and then writing the decimal as a percent.



Percents, fractions, and decimals are all different names that represent the same number.

EXAMPLES Fractions as Percents

5 **Write $\frac{5}{6}$ as a percent. Round to the nearest hundredth.**

$\frac{5}{6} = 0.833333333\ldots$ Write $\frac{5}{6}$ as a decimal.

$\approx 83.33\%$ Multiply by 100 and add the %.

6 **BOOKS** **Bryce has read $\frac{3}{5}$ of a book. What percent of the book has he read?**

$\frac{3}{5} = 0.6$ Write the fraction as a decimal.

$= 60\%$ Multiply by 100 and add the %.

So, Bryce has read 60% of the book.

CHECK Your Progress

Write each fraction as a percent. Round to the nearest hundredth if necessary.

g. $\frac{5}{16}$ h. $\frac{7}{12}$ i. $\frac{2}{9}$

j. **LAWNS** Mika is mowing lawns to earn extra money. She has mowed 6 out of 13 lawns. What percent of the lawns has she mowed?

Some fractions whose denominators are not factors of 100 are used often in everyday situations. It is helpful to memorize these fractions and their equivalent decimals and percents. These common equivalents are shown below.

KEY CONCEPT Common Equivalents

Fraction	Decimal	Percent	Fraction	Decimal	Percent
$\frac{1}{3}$	$0.\overline{3}$	$33\frac{1}{3}\%$	$\frac{3}{8}$	0.375	$37\frac{1}{2}\%$
$\frac{2}{3}$	$0.\overline{6}$	$66\frac{2}{3}\%$	$\frac{5}{8}$	0.625	$62\frac{1}{2}\%$
$\frac{1}{8}$	0.125	$12\frac{1}{2}\%$	$\frac{7}{8}$	0.875	$87\frac{1}{2}\%$

CHECK Your Understanding

Examples 1, 2 (pp. 366–367)

Write each percent as a fraction in simplest form.

1. 13.5%
2. 18.75%
3. $7\frac{1}{2}\%$
4. $66\frac{2}{3}\%$

5. **FOOD** Steven and Rebecca ate 62.5% of a pizza. What fraction of the pizza did they eat?

Examples 3–5 (pp. 367–368)

Write each fraction as a percent. Round to the nearest hundredth if necessary.

6. $\frac{3}{4}$
7. $\frac{4}{25}$
8. $\frac{4}{11}$
9. $\frac{1}{9}$

Example 6 (p. 368)

10. **SCHOOL** Moses has finished 11 out of 15 homework questions. To the nearest hundredth, what percent of the homework is complete?

Exercises

HOMEWORK HELP

For Exercises	See Examples
11–14, 19	1
15–18, 20	2
21–32	3–6
33–34	3

Write each percent as a fraction in simplest form.

11. 62.5%
12. 6.2%
13. 28.75%
14. 56.25%
15. $33\frac{1}{3}\%$
16. $16\frac{2}{3}\%$
17. $93\frac{3}{4}\%$
18. $78\frac{3}{4}\%$

19. **POPULATION** According to a recent census, 6.6% of all people living in Florida are 10–14 years old. What fraction of Florida's population is this?

20. **ATTENDANCE** At last year's spring dance, $78\frac{1}{3}\%$ of the student body attended. What fraction of the student body is this?

Write each fraction as a percent. Round to the nearest hundredth if necessary.

21. $\frac{11}{20}$
22. $\frac{18}{25}$
23. $\frac{3}{8}$
24. $\frac{21}{40}$
25. $\frac{29}{30}$
26. $\frac{8}{9}$
27. $\frac{5}{7}$
28. $\frac{1}{16}$
29. $\frac{1}{80}$
30. $\frac{57}{200}$
31. $\frac{5}{12}$
32. $\frac{7}{15}$

33. **LANGUAGES** In Virginia, 1 person out of every 40 speaks an Asian language at home. What percent of people in Virgina is this?

34. **PETS** In a class, 28 out of 32 students had a pet. What percent is this?

Replace each ● with >, <, or = to make a true statement.

35. $0.86 \bullet \frac{7}{8}$
36. $\frac{9}{20} \bullet 45\%$
37. $5\% \bullet 0.004$

Order each set of numbers from least to greatest.

38. $\frac{1}{4}$, 22%, 0.3, 0.02
39. 0.48, $\frac{1}{2}\%$, 0.5, $\frac{2}{5}$

40. **GEOGRAPHY** Use the information at the left. What percent of the states in the United States do *not* border an ocean or the Gulf of Mexico?

Real-World Link . . . . .
Out of the 50 states, 23 states border an ocean or the Gulf of Mexico.
Source: theus50.com

EXTRA PRACTICE
See pages 735, 761.
Math Online
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41. **FIND THE DATA** Refer to the Texas Data File on pages 16–19. Choose some data and write a real-world problem in which you would write a fraction as a percent.

CARS For Exercises 42 and 43, use the table, which shows the percent of people in a recent survey who kept the listed items in their car.

Items in Car	Percent of People
Pen/Pencil	73.0%
Cassette Tapes/CDs	66.1%
First-aid Kit	38.2%
Sports Equipment	28.9%

42. What fraction of people kept a first-aid kit in their car?

43. Approximately 26 out of 125 people surveyed kept a hairbrush in their car. Is this greater or less than the percent who kept sports equipment? Explain.

H.O.T. Problems

44. **CHALLENGE** For what value of x does $\frac{1}{x} = x\%$?

45. **WRITING IN MATH** Describe two ways to write a fraction as a percent.

TEST PRACTICE

46. Ms. Gallagher made 64 ounces of punch. The punch contained 17 ounces of apple juice. Which equation can be used to find x, the percent of apple juice in the punch?

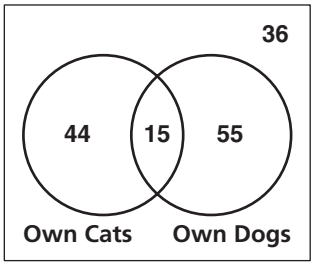
A $\frac{x}{100} = \frac{64}{17}$

B $\frac{x}{17} = \frac{64}{100}$

C $\frac{x}{64} = \frac{17}{100}$

D $\frac{x}{100} = \frac{17}{64}$

47. A group of 150 students were asked if they own a pet. The results are shown.



What percent of those surveyed own dogs?

F 50%
G 46.7%
H 36.7%
J 30%

Spiral Review

48. **MODELS** On a scale model of a building, 2 in. = 15 ft. If the model is 9 inches tall, how tall is the actual building? (Lesson 7-6)

Solve each proportion. (Lesson 7-4)

49. $\frac{6}{9} = \frac{4}{x}$

50. $\frac{3}{t} = \frac{9}{5}$

51. $\frac{n}{5} = \frac{2.8}{7}$

52. $\frac{7}{12} = \frac{r}{30}$

GET READY for the Next Lesson

PREREQUISITE SKILL Write each fraction in simplest form. (Lesson 5-4)

53. $\frac{8}{10}$

54. $\frac{45}{100}$

55. $\frac{450}{100}$

56. $\frac{175}{100}$

7-8 Percents Greater Than 100% and Percents Less Than 1%

Main IDEA

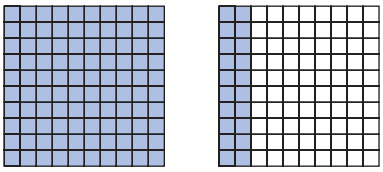
Write percents greater than 100% and percents less than 1% as fractions and as decimals, and vice versa.

Targeted TEKS 7.1 The student represents and uses numbers in a variety of equivalent forms. **(B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator.**

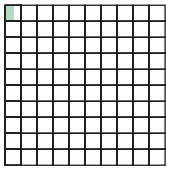
MINI Lab

- Draw three 10 × 10 squares on a piece of grid paper. Each large square represents 100%. Each small square represents 1%.
- For the first model, use two grids to shade 120 small squares.
- For the second model, shade half of one small square on a grid.

Model 1



Model 2



1. What percent does Model 1 represent? Model 2?

Shade grids to represent each percent.

2. 150%
3. 215%
4. $\frac{1}{4}$%

Percents greater than 100% or less than 1% can be written as decimals. They can also be written as mixed numbers or fractions.

EXAMPLES Percents as Decimals or Fractions

1 **Write 0.2% as a decimal and as a fraction in simplest form.**

$0.2\% = 00.2$ Divide by 100 and remove % symbol.

$= 0.002$ Decimal form

$= \frac{2}{1{,}000}$ or $\frac{1}{500}$ Fraction form

A percent less than 1% equals a number less than 0.01 or $\frac{1}{100}$.

2 **GEOGRAPHY** **The Amazon River is about 170% as long as the Mississippi River. Write 170% as a mixed number and as a decimal in simplest form.**

$170\% = \frac{170}{100}$ Definition of percent

$= 1\frac{70}{100}$ or $1\frac{7}{10}$ Mixed number form

$= 1.7$ Decimal form

A percent greater than 100% equals a number greater than 1.

So, the Amazon is 1.7 times as long as the Mississippi River.

CHECK Your Progress

Write each percent as a decimal and as a mixed number or fraction in simplest form.

a. 0.25%
b. 300%
c. 530%

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READING Math

Percents For percents less than 1%, interpret using fractional form. For example, 0.05% of the students means $\frac{1}{2{,}000}$ or 1 out of every 2,000 students.

Since you divide by 100 and remove the % symbol to write a percent as a decimal, to write decimals as percents, do the opposite. So, to write decimals less than 0.01 and decimals greater than 1 as percents, multiply by 100 and add a % symbol. Recall that you can multiply by 100 mentally by moving the decimal point two places to the right.

EXAMPLES Decimals as Percents

Write each decimal as a percent.

3 **1.68**

$1.68 = 1.68$ Multiply by 100.

$= 168\%$ Add % symbol.

4 **0.0075**

$0.0075 = 0.0075$ Multiply by 100.

$= 0.75\%$ Add % symbol.

5 **BASEBALL** **A baseball player's batting average is defined as the total number of hits divided by the total number of at bats. If a baseball player's batting average is 0.285, what percent of the player's at bats were hits?**

$0.285 = 0.285$ Multiply by 100.

$= 28.5\%$ Add % symbol.

The baseball player hit 28.5% of the at bats.

CHECK Your Progress **Write each decimal as a percent.**

d. 2.5 **e.** 0.004 **f.** 0.0016

g. **BASEBALL** Find the percent of at bats that were hit by a baseball player if his or her batting average was 0.179.

CHECK Your Understanding

Examples 1, 2 (p. 371) **Write each percent as a decimal and as a mixed number or fraction in simplest form.**

1. 325%
2. 200%
3. 480%
4. 0.7%
5. 0.15%
6. 0.5%

Example 1 (p. 371)

7. **MONEY** A company had a 0.9% increase in sales last month. Write 0.9% as a decimal and as a mixed number in simplest form.

Examples 3, 4 (p. 372) **Write each decimal as a percent.**

8. 1.8
9. 0.0015
10. 0.0025
11. 2.75
12. 0.0005
13. 3.25

Example 5 (p. 372)

14. **MANUFACTURING** A manufacturer of light bulbs finds that 0.0019 of the light bulbs they manufacture are defective. Write this as a percent.

Exercises

HOMEWORK HELP

For Exercises	See Examples
19–22, 23	1
15–18, 24	2
25–32	3, 4
33, 34	5

Write each percent as a decimal and as a mixed number or fraction in simplest form.

15. 350%
16. 475%
17. 600%
18. 400%
19. 0.6%
20. 0.05%
21. 0.55%
22. 0.04%

23. **ENVIRONMENT** Freshwater from lakes only accounts for 0.1% of the world's water supply. Write this percent as a decimal and as a mixed number or fraction in simplest form.

24. **TOYS** A collectible action figure sold for 193% of its original price. Write this percent as a decimal and as a mixed number or fraction in simplest form.

Real-World Link . . . . .
Each year, consumers spend about $1 billion on action figures and accessories.
Source: *Toy Industry Association, Inc*

Write each decimal as a percent.

25. 8.5
26. 35
27. 2.64
28. 1.07
29. 0.009
30. 0.003
31. 0.0034
32. 0.0077

33. **FOOD** The size of a large milk shake is 1.4 times of the size of the medium milk shake. Write 1.4 as a percent.

34. **CARS** A car's tire pressure decreased by 0.098 of its original pressure. Write 0.098 as a percent.

Write each mixed number or fraction as a percent.

35. $3\frac{1}{2}$
36. $9\frac{3}{4}$
37. $\frac{2}{500}$
38. $\frac{1}{400}$

Write each percent as a decimal.

39. $\frac{1}{8}\%$
40. $\frac{3}{4}\%$
41. $\frac{3}{25}\%$
42. $\frac{11}{20}\%$

43. **POPULATION** The U.S. Census Bureau reported that 0.3% of the population in the United States in 2000 was Japanese. Write this percent as a decimal and as a fraction. Then interpret its meaning as a ratio of the United States population.

44. **SCHOOL** Adrienne answered all 21 multiple choice questions correctly on her science test. If her teacher decided to let one of the questions count as a bonus, worth the same number of points as the other problems on the test, what was Adrienne's test score? Write your answer as a decimal and as a percent.

EXTRA PRACTICE
See pages 735, 761.
Math Online
Self-Check Quiz at tx.msmath2.com

LIFE SCIENCE **For Exercises 45–46, refer to the table at the right.**

45. Write the percent of magnesium found in the human body as a decimal.
46. Which element makes up $\frac{1}{400}$ of the human body?

Elements in the Human Body

Element	Percent
Magnesium	0.05
Potassium	0.35
Sodium	0.15
Sulfur	0.25

Source: chemistry.about.com

47. **TIME** One hour is what percent of one week?

48. **PHOTOGRAPHY** A photograph developed from 35 mm film is 428% larger than the negative. Express this percent as a decimal and as a mixed number or fraction in simplest form. Then interpret its meaning.

H.O.T. Problems

49. **FIND THE ERROR** Mikasi and Julie are writing $\frac{3}{2{,}000}$ as a percent. Who is correct? Explain your reasoning.

Mikasi

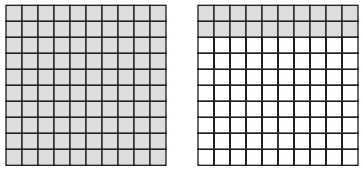
Julie

50. **CHALLENGE** The speed of a giraffe is 250% of the speed of a squirrel. If a squirrel speed is 12 miles per hour, find the speed of a giraffe.

51. **WRITING IN MATH** Write a real-world problem involving a percent greater than 100%. Then solve the problem.

TEST PRACTICE

52. What percent of one 10 by 10 grid is modeled below?



A 80%
B 100%
C 120%
D 130%

53. A certain stock increased its value by 467% over 10 years. Which number is NOT equivalent to 467%?

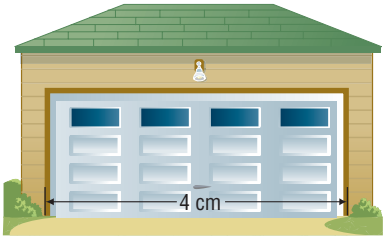
F 4.67
G 0.467
H $4\frac{67}{100}$
J $\frac{467}{100}$

Spiral Review

Write each percent as a fraction in simplest form. (Lesson 7-7)

54. 7.5%
55. 1.2%
56. $6\frac{1}{4}\%$
57. $92\frac{1}{2}\%$

58. **ANALYZE DRAWINGS** A garage door is 18 feet wide. A scale drawing of this door is shown. Find the scale of the drawing. (Lesson 7-6)



GET READY for the Next Lesson

PREREQUISITE SKILL **Write each percent as a decimal.** (Lesson 5-7)

59. 85%
60. 6.5%
61. 36.9%
62. 12.3%

CHAPTER 7

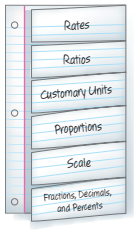
Study Guide and Review

Download Vocabulary Review from tx.msmath2.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Ratios and Rates (Lessons 7-1 and 7-2)

- A ratio is a comparison of two quantities by division.
- A rate is a ratio comparing two quantities with different kinds of units.

Changing Customary Units (Lesson 7-3)

- To convert from larger units to smaller units, multiply by the appropriate unit ratio.
- To convert from smaller units to larger units, multiply by the reciprocal of the appropriate unit ratio.

Proportions (Lesson 7-4)

- Proportions are equations that state two ratios or rates are equivalent.

Scale Drawings (Lesson 7-6)

- Scale drawings represent something that is too large or too small to be drawn at actual size.

Fractions, Decimals, and Percents (Lesson 7-7)

- Common equivalents:

Fraction	Decimal	Percent
$\frac{1}{3}$	$0.\overline{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.\overline{6}$	$66\frac{2}{3}\%$
$\frac{1}{8}$	0.125	$12\frac{1}{2}\%$
$\frac{3}{8}$	0.375	$37\frac{1}{2}\%$
$\frac{5}{8}$	0.625	$62\frac{1}{2}\%$
$\frac{7}{8}$	0.875	$87\frac{1}{2}\%$

Key Vocabulary

cross products (p. 348)	scale (p. 358)
equivalent ratios (p. 331)	scale drawing (p. 358)
proportion (p. 348)	scale factor (p. 360)
proportional (p. 348)	scale model (p. 358)
rate (p. 335)	unit rate (p. 335)
ratio (p. 330)	unit ratio (p. 343)

Vocabulary Check

Choose the term from the list above that best matches each phrase.

1. a comparison of two quantities by division
2. two ratios that have the same value
3. a ratio of two measurements with different units
4. an equation that shows that two ratios or rates are equivalent
5. used to represent something that is too large or too small for an actual-size drawing
6. the ratio of the distance on a map to the actual distance
7. a scale written as a ratio in simplest form
8. product of the numerator of one ratio in a proportion and the denominator of the other ratio
9. a rate that is simplified so that it has a denominator of 1
10. two quantities that have a constant rate or ratio

CHAPTER 7 Study Guide and Review

Lesson-by-Lesson Review

7-1 Ratios (pp. 330–334)

Write each ratio as a fraction in simplest form.

11. 16 dogs : 12 cats
12. 5 ft : 25 ft
13. 50 boys : 75 girls
14. 36 ft : 6 ft

Determine whether the ratios are equivalent. Explain.

15. 18 out of 24 people agree
    5 out of 20 people agree
16. 27 goldfish to 15 frogs
    9 goldfish to 5 frogs

17. **MUSEUM** A museum requires two tour guides for every 28 visitors. There are 168 visitors and 12 tour guides at the museum. Is this the correct number of tour guides based on the requirement? Justify your answer.

Example 1 **Write the ratio 32 to 18 as a fraction in simplest form.**

$32 \text{ to } 18 = \frac{32}{18}$ Write the ratio as a fraction.

$= \frac{16}{9}$ Simplify.

Example 2 **Determine whether 5:6 and 15:18 are equivalent.**

$5:6 = \frac{5}{6}$ $\quad 15:18 = \frac{15}{18}$ or $\frac{5}{6}$

The ratios in simplest form both equal $\frac{5}{6}$. So, 5:6 and 15:18 are equivalent.

7-2 Rates (pp. 335–340)

Find each unit rate.

18. $23.75 for 5 pounds
19. 810 miles in 9 days
20. 14 laps in 4 minutes

21. **SHOPPING** Which bottle of laundry detergent shown at right costs the least per ounce?

Bottle	Price
12 oz	$2.16
32 oz	$5.60
64 oz	$10.24

Example 3 **Find the unit price of a 16-ounce box of pasta that is on sale for 96 cents.**

$16\text{-ounce box for 96 cents} = \frac{96 \text{ cents}}{16 \text{ ounces}}$

$= \frac{96 \text{ cents} \div 16}{16 \text{ ounces} \div 16}$

$= \frac{6 \text{ cents}}{1 \text{ ounce}}$

The unit price is 6 cents per ounce.

7-3 Measurement: Changing Customary Units (pp. 342–347)

Complete.

22. 4 qt = ■ pt
23. 6 gal = ■ qt
24. 48 oz = ■ lb
25. 9 c = ■ pt

26. **SCARF** Kaneko's scarf is 48 inches long. How many feet long is the scarf?

Example 4 Complete: 32 qt = ■ gal

Since 1 gallon = 4 quarts, the unit ratio is $\frac{1 \text{ gal}}{4 \text{ qt}}$.

$32 \text{ qt} = 32 \text{ qt} \cdot \frac{1 \text{ gal}}{4 \text{ qt}}$ Multiply by $\frac{1 \text{ gal}}{4 \text{ qt}}$.

$= 32 \text{ qt} \cdot \frac{1 \text{ gal}}{4 \text{ qt}}$ Divide out common factors and units.

$= 8 \text{ gal}$ Multiply.

7-4 Algebra: Solving Proportions (p. 348–353)

Solve each proportion.

27. $\frac{x}{10} = \frac{3}{5}$
28. $\frac{4}{9} = \frac{24}{m}$
29. $\frac{2}{t} = \frac{8}{50}$
30. $\frac{15}{w} = \frac{35}{21}$
31. $\frac{12}{8} = \frac{a}{6}$
32. $\frac{7}{18} = \frac{d}{6}$

33. **WEIGHT** If 3 televisions weigh 240.6 pounds, how much do 9 of the same televisions weigh?

Example 5 Solve $\frac{6}{9} = \frac{n}{12}$.

$\frac{6}{9} = \frac{n}{12}$ Write the proportion.

$6 \cdot 12 = 9 \cdot n$ Find the cross products.

$72 = 9n$ Multiply.

$\frac{72}{9} = \frac{9n}{9}$ Divide each side by 9.

$8 = n$ Simplify.

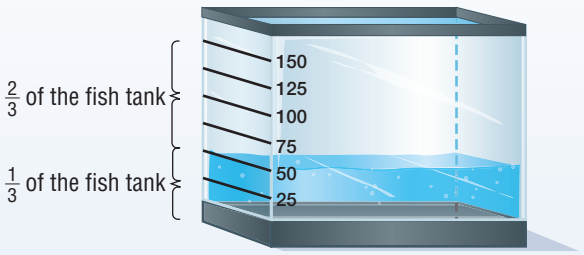
7-5 PSI: Draw A Diagram (pp. 356–357)

Solve each problem by drawing a diagram.

34. **PAINTING** Marian is painting a fence that is 72 feet long. She has already painted $\frac{5}{8}$ of the fence. How many feet of fence does she have left to paint?

35. **COOKIES** A cookie jar contains three types of cookies: oatmeal, chocolate chip, and sugar. 60 percent are chocolate chip. Half of the remaining cookies are oatmeal. If there are 9 oatmeal cookies, how many cookies are in the jar?

Example 6 Ramiro has filled $\frac{1}{3}$ or 50 gallons of his fish tank. Find the total capacity of the fish tank.



If $\frac{1}{3}$ of the fish tank is 50 gallons, then $\frac{2}{3}$ of the fish tank is 100 gallons. So, the missing two-thirds must be 100 gallons. The total capacity of the fish tank is 50 + 100, or 150 gallons.

CHAPTER 7

Study Guide and Review

7-6 Scale Drawings (pp. 358–364)

36. **MAPS** Washington, D.C, and Baltimore, Maryland, are 2 inches apart on a map. If the scale is $\frac{1}{2}$ inch:6 miles, what is the actual distance between the cities?

37. **MODELS** A Boeing 747 jet is 70.5 meters long and has a wingspan of 60 meters. A model of the 747 has a wingspan of 80 centimeters. What is the length of the model?

Example 7 On a map, the distance between two cities is 10.9 centimeters. If the scale is 1 centimeter = 250 kilometers, what is the actual distance?

	Scale		Distance	
map →	$\frac{1 \text{ cm}}{250 \text{ km}}$	$=$	$\frac{10.9 \text{ cm}}{n \text{ km}}$	← map
actual →				← actual

$1 \cdot n = 250 \cdot 10.9$

$n = 2{,}725$

The actual distance is 2,725 kilometers.

7-7 Fractions, Decimals, and Percents (pp. 366–370)

Write each percent as a fraction in simplest form.

38. 27.5% 39. 5.4% 40. $45\frac{1}{4}\%$

Write each fraction as a percent. Round to the nearest hundredth if necessary.

41. $\frac{1}{8}$ 42. $\frac{5}{6}$ 43. $\frac{7}{40}$

44. **COINS** A quarter is made of $\frac{1}{12}$ nickel, and the rest is copper. Write the portion of a quarter that is copper as a percent. Round to the nearest hundredth if necessary.

Example 8 Write 82.5% as a fraction in simplest form.

$82.5\% = \frac{82.5}{100}$ Write a fraction with a denominator of 100.

$= \frac{825}{1{,}000}$ Multiply 82.5 and 100 by 10 to eliminate the decimal.

$= \frac{33}{40}$ Simplify.

Example 9 Write $\frac{14}{15}$ as a percent. Round to the nearest hundredth if necessary.

$\frac{14}{15} = 0.933\ldots$ Write $\frac{14}{15}$ as a decimal.

$\approx 93.33\%$ Multiply by 100 and add the %.

7-8 Percents Greater than 100% and Percents Less than 1% (pp. 371–374)

Write each percent as a decimal and as a mixed number or fraction in simplest form.

45. 125% 46. 0.75% 47. 0.5%

Write each decimal as a percent.

48. 0.002 49. 4.75 50. 0.0095

51. **MUSEUM** A museum's attendance increased by 135%. Write 135% as a decimal and as a mixed number in simplest form.

Example 10 Write 248% as a decimal and as a mixed number in simplest form.

$248\% = \frac{248}{100}$ Definition of percent

$= 2.48$ Write as a decimal.

$= 2\frac{48}{100}$ Write as a mixed number.

$= 2\frac{12}{25}$ Simplify.

Example 11 Write 0.008 as a percent.

0.008×100 Multiply by 100.

$= 0.8\%$

CHAPTER 7

Practice Test

LAWN CARE **For Exercises 1 and 2, use the following information to write each ratio as a fraction in simplest form.**

A bag of fertilizer nutrients contains 18 pounds of nitrogen, 6 pounds of phosphorus, and 12 pounds of potassium.

1. nitrogen : potassium
2. phosphorus : nitrogen

Find each unit rate. Round to the nearest hundredth if necessary.

3. 24 greeting cards for $4.80
4. 330 miles on 15 gallons of gasoline

5. **TEST PRACTICE** The population of bacteria in 4 different-sized lab dishes are given. Which dish has the lowest density of bacteria or bacteria per square inch?

Dish	Bacteria	Dish Area
1	100	205 sq in.
2	50	125 sq in.
3	35	75 sq in.
4	180	300 sq in.

A Dish 1 **C** Dish 3
B Dish 2 **D** Dish 4

6. **TYPING** Peggy can word process 12 words in 15 seconds. At this rate, how many words can she word process in 3 minutes?

Complete.

7. 9 yd = ■ ft
8. 11,000 lb = ■ T
9. 4 lb = ■ oz
10. 15 qt = ■ gal

11. **GROCERIES** If 1 can of tomato paste contains 6 ounces, how many pounds do 10 cans contain?

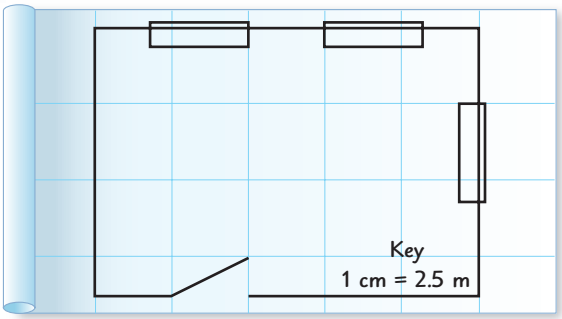
Solve each proportion.

12. $\frac{2}{3} = \frac{x}{42}$
13. $\frac{t}{21} = \frac{15}{14}$

14. **NUTRITION** If an 8-ounce glass of orange juice has 72 milligrams of vitamin C, how much vitamin C is in a 7-ounce glass?

15. **SWIMMING** Solana swims 3 laps in 12 minutes. At this same rate, how many laps will she swim in 30 minutes?

BLUEPRINTS **For Exercises 16–18, use the following blueprint of a room.**



16. Use a centimeter ruler to find the length of the wall with 2 windows.
17. How wide would a 1.4-meter wide dresser appear on this drawing?
18. Find the scale factor of the drawing.

Write each fraction as a percent. Round to the nearest hundredth if necessary.

19. $\frac{5}{8}$
20. $\frac{7}{15}$

21. **TEST PRACTICE** A company test marketed 7 new flavors of gum last year. Only 2 of these received favorable ratings. Which represents the percent of flavors that did NOT receive favorable ratings?

F 71% **H** 5%
G 29% **J** 2%

Write each percent as a fraction or mixed number in simplest form.

22. $83\frac{1}{3}\%$
23. 0.45%
24. 135%

25. Write 0.0095 as a percent.

CHAPTER 7

Texas Test Practice
Cumulative, Chapters 1–7

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Mrs. Stewart painted the door to her deck. The door is a rectangle with length 8 feet and width 4 feet. In the middle of the door, there is a rectangular panel of glass that measures 5 feet by 2 feet. Find the painted area of the door in square feet.

   A 10 ft^2

   B 22 ft^2

   C 24 ft^2

   D 32 ft^2

2. The table shows the prices of 3 different boxes of cereal. Which box of cereal has the highest price per ounce?

Cereal Box Size (ounces)	Price ($)
48	5.45
32	3.95
20	3.10

   F The 20-ounce box

   G The 32-ounce box

   H The 48-ounce box

   J All three boxes have the same price per ounce.

3. A bakery sells 6 bagels for for a total of $2.99 and 4 muffins for a total of $3.29. If you bought 4 dozen bagels and 16 muffins, what is the total cost of the bagels and muffins, not including tax?

   A $64.60

   B $37.08

   C $31.10

   D $26.50

4. Mrs. Black is making 2 pasta salads for a picnic. The first pasta salad requires $4\frac{2}{3}$ cups of pasta, and the second pasta salad requires $\frac{1}{3}$ cup more than the first. Which of the following equations can be used to find n, the number of cups of pasta needed for the second recipe?

   F $n = 4\frac{2}{3} \div \frac{1}{3}$

   G $n = 4\frac{2}{3} + \frac{1}{3}$

   H $n = 4\frac{2}{3} - \frac{1}{3}$

   J $n = 4\frac{2}{3} \times \frac{1}{3}$

5. **GRIDDABLE** Simplify the expression below.

$$8 + 3(15 - 5) - 3^2$$

6. A shoe store had to increase prices. The table shows the regular price, r, and the new price, n, of several shoes. Which of the following formulas can be used to calculate the new price?

Shoe	Regular Price (r)	New Price (n)
A	$25.00	$27.80
B	$30.00	$32.80
C	$35.00	$37.80
D	$40.00	$42.80

   A $n = r - 2.80$

   B $n = r + 2.80$

   C $n = r \times 0.1$

   D $n = r \div 0.1$

7. Annika can run 2 miles in 15 minutes. At this rate, about how long will it take her to run $3\frac{1}{2}$ miles?

   F 26 minutes

   G 32 minutes

   H 36 minutes

   J 45 minutes

8. A TV station estimates that 59.2% of its viewers will watch TV at 8:00 P.M. Which number is NOT equivalent to 59.2%?

   A $\frac{592}{1000}$

   B 0.592

   C 0.00592×10^2

   D 5.92

9. You can drive your car 21.75 miles with one gallon of gasoline. How many miles can you drive with 13.2 gallons of gasoline?

   F 13.2

   G 21.75

   H 150.2

   J 287.1

10. The table shows the number of yards of material Leah used each day last week. What was the total number of yards Leah used last week?

Day	Material (yd)
Monday	2.3
Tuesday	$1\frac{3}{4}$
Wednesday	2.8
Thursday	3.1
Friday	$3\frac{1}{4}$
Saturday	1.7
Sunday	$4\frac{1}{2}$

   A 19.4 yd

   B 17 yd

   C 16 yd

   D 16.5 yd

11. JoAnne has a doll collection containing 192 dolls. She decided to give $\frac{5}{8}$ of her collection to her little sister. What percent of her dolls did JoAnne give to her sister?

   F $12\frac{1}{2}\%$

   G $62\frac{1}{2}\%$

   H 63%

   J $63\frac{1}{3}\%$

TEST-TAKING TIP

Question 12 When a question involves information from a previous part of a question, make sure to check that information before you move on.

Pre-AP

Record your answers on a sheet of paper. Show your work.

12. Jessie built a model of her town using items from her family's recycling bin.

   a. One centimeter in Jessie's model equals 10 kilometers in the real town. If Lansdowne Road is 12 centimeters long, on the model, how long is the actual Lansdowne Road?

   b. Jessie cut up milk cartons to construct $\frac{4}{5}$ of her model town. What percent of the model town was constructed using milk cartons?

   c. Jessie used a total of 70 recycled items to build her model town. Explain how to find the number of items that were milk cartons. Then find the number of items that were milk cartons.

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	4-6	7-2	3-5	6-2	1-4	1-10	7-2	5-7	3-2	6-2	5-6	7-7
For Help with Test Objective...	2	2	1	6	1	2	1	1	1	1	1	1