Knowledge and Skills

- Use geometric vocabulary to describe angles, polygons, and circles.
  **TEKS 6.6**

**Key Vocabulary**

- angle (p. 430)
- degree (p. 430)
- quadrilateral (p. 454)
- vertex (p. 430)

**Real-World Link**

**Roller Coasters** Riders of the Millennium Force roller coaster in Ohio travel at a 45° angle up the first hill and then down a 300-foot-long drop at an 80° angle.

**Foldables™ Study Organizer**

**Geometry: Angles and Polygons** Make this Foldable to help organize information about angles and polygons. Begin with seven half-sheets of notebook paper.

1. **Fold** a sheet in half lengthwise. Then cut a 1” tab along the left edge through one thickness.

2. **Glue** the 1” tab down. Write the word *Geometry* on this tab and the lesson title on the front tab.

3. **Write** Definitions and Examples under the tab.

4. **Repeat** Steps 1–3 for each lesson using the remaining paper. Staple them to form a booklet.
Option 1
Take the Quick Quiz below. Refer to the Quick Review for help.

### Quick Quiz

Solve each equation. \( (\text{Used in Lesson 9-3}) \)

1. \( x + 44 = 90 \)  
2. \( 68 + x = 90 \)  
3. \( x + 122 = 180 \)  
4. \( 87 + x = 180 \)  
5. **BASKETBALL** In the first two season games, Lee scored a total of 40 points. If he scored 21 points in the second game, how many points did he score in the first game?

### Quick Review

#### Example 1

Solve \(54 + x = 180\).

\[
54 + x = 180 \\
54 + 126 = 180
\]

**THINK** What number added to \(54\) equals \(180\)?

**You know that** \(54 + 126 = 180\).

The solution is \(126\).

#### Example 2

Solve \(61 + x + 22 = 180\).

\[
61 + x + 22 = 180 \\
83 + x = 180
\]

**Add 61 and 22.**

**THINK** What number added to \(83\) equals \(180\)?

**You know that** \(83 + 97 = 180\).

The solution is \(97\).

#### Example 3

Tell whether the triangles below have the same size and shape.

Yes, the triangles have the same size and same shape.
Measuring Angles

Main IDEA
Measure and classify angles.

Targeted TEKS 6.6
The student uses geometric vocabulary to describe angles, polygons, and circles.
(A) Use angle measurements to classify angles as acute, obtuse, or right. 6.8 The student solves application problems involving estimation and measurement of length, area, time, temperature, volume, weight, and angles.
(C) Measure angles. Also addresses TEKS 6.11(D).

NEW Vocabulary
angle
side
vertex
degree
right angle
acute angle
obtuse angle
straight angle

READING in the Content Area
For strategies in reading this lesson, visit tx.msmath1.com.

GARDENING  The circle graph shows what Mai-Lin planted in her garden this spring.

1. Mai-Lin planted the most of which food? Of which did she plant the least? Explain your reasoning.
2. The percents 30%, 25%, 20%, 15%, and 10% correspond to the sections in the graph. Explain how you would match each percent with its corresponding section.

Each section of the circle graph above shows an angle. Angles have two sides that share a common endpoint called the vertex of the angle. An angle is often named by the label at the vertex. Angle C can be written as \( \angle C \).

The most common unit of measure for angles is the degree. A circle can be separated into 360 equal-sized parts. Each part would make up a one-degree (\(^\circ\)) angle.

Measure Angles

Use a protractor to find the measure of each angle.

Make sure one side of the angle passes through zero on the protractor.

Read the measure on the protractor where the other side of the angle crosses the protractor.

Align the center of the protractor with the vertex of the angle.

The angle measures 150°.
Degrees The measurement 0° is read zero degrees.

The angle measures 75°.

Find the measure of each angle.

a. 

b. 

Angles can be classified according to their measures.

Classify Angles

Classify each angle as acute, obtuse, right, or straight.

The angle is 90°.
So, it is a right angle.

The angle is larger than a right angle, but smaller than a straight angle. So, it is an obtuse angle.

Classify each angle as acute, obtuse, right, or straight.

MathOnline Extra Examples at tx.msmath1.com
Use a protractor to find the measure of each angle. Then classify each angle as **acute, obtuse, right, or straight**.

1. 
2. 
3. 

4. **HOCKEY** The lie of a hockey stick is the angle between the blade and the shaft. Classify this angle.

**Signs** Classify the angles in each road sign.

5. 
6. 
7. 

8. 
9. 
10. 

Find the measure of the indicated angle in each figure.

11. 
12. 

13. 
14. 
15. 

16. **Flags** What is the measure of $\angle F$ on the Ohio flag shown at the right?

17. **Airplanes** What type of angle does the path of an airplane make with the ground when it is landing?
24. **SCIENCE** The melting point of sulfur is 246°F. Its boiling point is 833°F. How much does the temperature need to increase for sulfur to go from melting to boiling?  
   **Lesson 8-8**

25. Find the sum of 13 hours 45 minutes and 27 hours 50 minutes.  
   **Lesson 8-7**

### H.O.T. Problems

18. **OPEN ENDED** Select three objects in your classroom or at home. Measure the angles found in the object, and then classify each angle.

19. **FIND THE ERROR** Lorena and Dave are measuring angles. Who is correct? Explain your reasoning.

![Lorena and Dave measuring angles](image)

20. **CHALLENGE** Measure $\angle Z$ to the nearest degree. Then describe the method you used to find the measure.

21. **WRITING IN MATH** Without measuring, explain in your own words how you can classify an angle as acute, obtuse, or right.

### Test Practice

22. Find the measure of $\angle FGK$ to the nearest degree.

```
A  130°  C  85°  B  95°  D  45°
```

23. What type of angle is at each vertex of the regular hexagon below?

```
F  Acute  H  Right
G  Obtuse  J  Straight
```

### Spiral Review

24. **SCIENCE** The melting point of sulfur is 246°F. Its boiling point is 833°F. How much does the temperature need to increase for sulfur to go from melting to boiling?  
   **Lesson 8-8**

25. Find the sum of 13 hours 45 minutes and 27 hours 50 minutes.  
   **Lesson 8-7**

### Get Ready for the Next Lesson

**PREREQUISITE SKILL** Use a ruler to draw a diagram that shows how the hands on a clock appear at each time.  
   **Page 659**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>26.</td>
<td>9:00</td>
<td>27.</td>
<td>12:10</td>
</tr>
<tr>
<td>28.</td>
<td>3:45</td>
<td>29.</td>
<td>2:30</td>
</tr>
</tbody>
</table>
Main IDEA
Estimate measures of angles and draw angles.

Targeted TEKS 6.8
The student solves application problems involving estimation and measurement of length, area, time, temperature, volume, weight, and angles.
(A) Estimate measurements (including circumference) and evaluate reasonableness of results.

Estimating and Drawing Angles

To estimate the measure of an angle, use angles that measure 45°, 90°, and 180°.

STEP 1
Fold a paper plate in half to find the center of the plate.

STEP 2
Cut wedges as shown. Then measure and label each angle.

1. Use the wedges to estimate the measure of each angle shown.

2. How did the wedges help you to estimate each angle?

3. Explain how the 90° and 45° wedges can be used to estimate the angle at the right. What is a reasonable estimate for the angle?

4. How would you estimate the measure of any angle without using the wedges?

To estimate the measure of an angle, compare it to an angle whose measure you know.

Estimate Angle Measures

1. Estimate the measure of the angle.

   The angle is a little less than a 90° angle. So, a reasonable estimate is about 80°.

Check Your Progress

Estimate the measure of each angle.

a. 

b.
**Example 2**

Use a protractor and a straightedge to draw angles having the following measurements.

- **6.** 25°  
- **7.** 140°  
- **8.** 60°

**Check for Reasonableness**

You can check whether you have used the correct scale by comparing your angle with an estimate of its size.

**Example 1**

Estimate the measure of each angle.

1.  
2.  
3.  

**BICYCLE** For Exercises 4 and 5, refer to the bicycle diagram.

4. Estimate the measure of the head angle.
5. Estimate the measure of the seat angle.

**Example 2**

Use a protractor and a straightedge to draw angles having the following measurements.

- **6.** 25°  
- **7.** 140°  
- **8.** 60°

---

**Straightedge**

A straightedge is an object, like a ruler, used to draw straight lines.

**Check Your Progress**

Use a protractor and a straightedge to draw angles having the following measurements.

- **c.** 68°  
- **d.** 105°  
- **e.** 85°

**Online Personal Tutor at tx.msmath1.com**

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**Example 1**

(p. 434)

**Example 2**

(p. 435)
Estimate the measure of each angle.

9.  
10.  
11.  
12.  
13.  
14.  

Use a protractor and a straightedge to draw angles having the following measurements.

15.  $75^\circ$  
16.  $50^\circ$  
17.  $45^\circ$  
18.  $20^\circ$  
19.  $115^\circ$  
20.  $175^\circ$  
21.  $133^\circ$  
22.  $79^\circ$  

23. **TIME** Estimate the measure of the angle formed by the hands on the clock shown.

24. **GOLF** Estimate the measure of the golfer’s spine angle in the photo at the left.

Estimate the measure of each angle. Explain your reasoning.

25.  
26.  
27.  

28. **RESEARCH** Use the Internet or another source to find a photo of a humpback whale. Draw an example of the angle formed by the two tail fins. Then give a reasonable estimate for the measure of this angle.

29. **LADDERS** To be considered safe, a ladder should be leaned at an angle of about fifteen degrees formed by the top of the ladder and the vertical wall. Estimate the measure of the angles formed by each ladder below and determine which ladders would be considered safe.

a.  
   b.  
   c.  

**Real-World Link** While swinging a golf club, it is important to keep your spine angle constant throughout your entire swing.

Source: golftipsmag.com
**TREES** For Exercises 30–32, use the following information and the diagram at the right.

When pruning apple trees, if the angle formed by a branch growing off a main trunk is less than 60°, the branch should be pruned, or trimmed, as the branch will become weak.

**Estimate the measure of each angle labeled \( C \). Then determine whether the branch should be pruned.**

30. 
31. 
32. 

**33. SCIENCE** Most globes show that Earth’s axis inclines 23.5° from vertical. Use the data below to draw diagrams that show the inclination of axis of each planet listed.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Uranus</th>
<th>Neptune</th>
<th>Pluto</th>
<th>Venus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclination of Axis</td>
<td>97.9°</td>
<td>29.6°</td>
<td>122°</td>
<td>177.3°</td>
</tr>
</tbody>
</table>

**34. TRIANGLES** Use a protractor and straightedge to draw a triangle with angle measures of 70°, 60°, and 50°. Label each angle with its measure.

**35. CHALLENGE** Estimate the measure of each angle in the figure below. Then analyze any relationships you observe in these angle measures.

**36. REASONING** Mr. Morales is a physical therapist. At each visit with a patient recovering from knee surgery, he determines the angle at which the patient can bend his or her knee. Do you think Mr. Morales should use estimation to track his patient’s progress? Explain your reasoning.

**37. OPEN ENDED** Choose a capital letter that is made up of straight line segments, at least two of which meet to form an acute angle. Draw this letter using a straightedge. Label the angle \( \angle 1 \). Then estimate the measure of this angle.

**38. WRITING IN MATH** Describe a situation in which drawing a diagram with approximate angle measures would be appropriate and useful.
39. Which angle has an approximate measure of 50°?

A        C

B        D

40. Below is the shape of a kite that Jermain made.

Estimate the measure of \( \angle T \).

F 45°  H 100°
G 80°  J 140°

---

Spiral Review

Use a protractor to find the measure of each angle. Then classify each angle as acute, obtuse, right, or straight. (Lesson 9-1)

41. 

42. 

43. 

Choose the more reasonable temperature for each. (Lesson 8-8)

44. air temperature on a snowy day: 35°C or 0°C
45. bread just out of the oven: 70°C or 140°C
46. ice cream sandwich: 15°F or 50°F

OCEANS For Exercises 47 and 48, refer to the graph at the right. (Lesson 3-5)

47. What is the combined area of the two smallest oceans?
48. How many millions of miles greater is the Pacific Ocean than the Atlantic Ocean?

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Source: Time Almanac for Kids

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GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation mentally. (Lesson 1-8)

49. \( x + 45 = 180 \)  
50. \( 25 + x = 90 \)  
51. \( 130 + x = 180 \)  
52. \( x + 50 = 90 \)
Main IDEA
Classify and apply angle relationships.

Preparation for TEKS 7.6 The student compares and classifies two- and three-dimensional figures using geometric vocabulary and properties. (A) Use angle measurements to classify pairs of angles as complementary or supplementary. Also addresses TEKS 6.12(A), 6.13(B).

NEW Vocabulary
vertical angles
congruent angles
supplementary angles
complementary angles

MATH Symbols
\( \cong \) is congruent to

Copy the figure below on dot paper.

Use a protractor to find the measure of each angle.

1. What do you notice about the measures of \( \angle 1 \) and \( \angle 3? \ \angle 2 \) and \( \angle 4? \)

2. MAKE A CONJECTURE Describe the relationship between opposite angles formed by intersecting lines.

3. Find the sum of the measures of \( \angle 3 \) and \( \angle 4 \) and of \( \angle 2 \) and \( \angle 3 \).

4. What type of angle is formed by \( \angle 3 \) and \( \angle 4? \ \angle 2 \) and \( \angle 3? \)

5. MAKE A CONJECTURE Describe the relationship between the angles that form a straight angle.

When two lines intersect, they form two pairs of opposite angles called vertical angles. Vertical angles have the same measure. Angles with the same measure are congruent angles.

\[ \angle 1 \cong \angle 4 \]
\[ \angle 2 \cong \angle 3 \]

The symbol \( \cong \) is used to show that the angles are congruent.

**EXAMPLE**

**Find a Missing Angle Measure**

Find the value of \( x \) in the figure.

The angle labeled \( x^\circ \) and the angle labeled \( 140^\circ \) are vertical angles. Therefore, they are congruent.

So, the value of \( x \) is 140.

**CHECK Your Progress**

Find the value of \( x \) in each figure.

a. \[ x^\circ \]

b. \[ x^\circ \]

\[ 17^\circ \]

\[ 95^\circ \]
Pairs of angles can also have other relationships. In the Mini Lab, you found pairs of angles that have a sum of 180°. Two angles are **supplementary** if the sum of their measures is 180°. Two angles are **complementary** if the sum of their measures is 90°.

### Key Concept

**Pairs of Angles**

**Words**  
Two angles that have a sum of 180° are supplementary angles.

**Models**

\[ m\angle 1 = 120^\circ, m\angle 2 = 60^\circ, m\angle 1 + m\angle 2 = 180^\circ \]

**Words**  
Two angles that have a sum of 90° are complementary angles.

**Models**

\[ m\angle 1 = 30^\circ, m\angle 2 = 60^\circ, m\angle 1 + m\angle 2 = 90^\circ \]

You can use the definitions of complementary and supplementary to classify angles.

### Examples

**Classify Pairs of Angles**

Classify each pair of angles as **complementary**, **supplementary**, or **neither**.

2. \[ 45^\circ \quad 45^\circ \]
   
   \[ 45^\circ + 45^\circ = 90^\circ \]
   
   Since the sum of their measures is 90°, the angles are complementary.

3. \[ 110^\circ \quad 75^\circ \]
   
   \[ 110^\circ + 75^\circ = 185^\circ \]
   
   Since the sum of their measures is not 90° or 180°, the angles are neither complementary nor supplementary.

### Check Your Progress

Classify each pair of angles as **complementary**, **supplementary**, or **neither**.

C. \[ 45^\circ \quad 135^\circ \]

D. \[ 80^\circ \quad 10^\circ \]
Find Missing Angle Measures

Find the value of $x$ in each figure.

4. Since the angles form on a straight line, they are supplementary.

\[120° + x° = 180°\]

Definition of supplementary angles

\[120° + 60° = 180°\]

So, the value of $x$ is 60.

5. Since the angles form a right angle, they are complementary.

\[x° + 20° = 90°\]

Definition of complementary angles

\[70° + 20° = 90°\]

THINK What measure added to $20°$ equals $90°$?

So, the value of $x$ is 70.

Examples 2, 3 (p. 440)
Examples 1, 4, 5 (pp. 439–441)
Example 4 (p. 441)

Classify each pair of angles as complementary, supplementary, or neither.

1. 2. 3.

Find the value of $x$ in each figure.

4. 5. 6.

7. TREES What is the value of $x$ in the maple leaf at the right?
Classify each pair of angles as complementary, supplementary, or neither.

8. 
\[ \begin{align*}
50^\circ & \quad 40^\circ \\
\end{align*} \]

9. 
\[ \begin{align*}
50^\circ & \quad 130^\circ \\
\end{align*} \]

10. 
\[ \begin{align*}
40^\circ & \quad 40^\circ \\
\end{align*} \]

11. 
\[ \begin{align*}
60^\circ & \quad 30^\circ \\
\end{align*} \]

12. 
\[ \begin{align*}
75^\circ & \quad 15^\circ \\
\end{align*} \]

13. 
\[ \begin{align*}
85^\circ & \quad 95^\circ \\
\end{align*} \]

Find the value of \( x \) in each figure.

14. 
\[ \begin{align*}
\quad 128^\circ \\
\quad x^\circ \\
\end{align*} \]

15. 
\[ \begin{align*}
\quad 88^\circ \\
\quad x^\circ \\
\end{align*} \]

16. 
\[ \begin{align*}
\quad 140^\circ \\
\quad x^\circ \\
\end{align*} \]

17. 
\[ \begin{align*}
\quad x^\circ \\
\quad 25^\circ \\
\end{align*} \]

18. 
\[ \begin{align*}
\quad x^\circ \\
\quad 25^\circ \\
\end{align*} \]

19. 
\[ \begin{align*}
\quad x^\circ \\
\quad 55^\circ \\
\end{align*} \]

20. **HORSES** What is the value of \( x \) in the hurdle shown at the right?

21. **BRIDGES** A truss bridge is made of many short straight beams as shown in the diagram below. Create a problem that can be solved by referring to the angles labeled 1–4 in the diagram.
22. Angles $G$ and $H$ are complementary. Find $m\angle H$ if $m\angle G = 40^\circ$.

23. Angles $J$ and $K$ are supplementary. Find $m\angle J$ if $m\angle K = 65^\circ$.

**ARCHITECTURE** For Exercises 24–28, refer to the photo of the Renaissance Tower in Dallas at the right.

Classify each pair of angles.

24. $\angle 1$ and $\angle 2$

25. $\angle 2$ and $\angle 4$

26. $\angle 3$ and $\angle 4$

27. $\angle 1$ and $\angle 3$

28. If the $m\angle 3$ is 46°, what is the measure of $\angle 2$?

Determine whether each statement is sometimes, always, or never true. Explain your reasoning.

29. Vertical angles have the same angle measure.

30. Two right angles are complementary.

31. Two obtuse angles are supplementary.

32. Two vertical angles are complementary.

For Exercises 33–35, refer to the figure below.

33. What is the value of $x$?

34. What is the value of $y$?

35. How many pairs of complementary angles are in the figure? Describe each pair.

36. **OPEN ENDED** Draw and label an angle in which you can find both a complementary angle and supplementary angle to the angle drawn. Classify the type of angle that must be drawn. Explain your reasoning.

37. **REASONING** Answer each of the following questions.
   a. What kind of angle is the supplement of an acute angle?
   b. What kind of angle is the supplement of a right angle?
   c. Can two acute angles be supplementary? Justify your answer.

38. **CHALLENGE** Refer to the figure at the right. If $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$, what can you conclude about the sum of $m\angle 1$ and $m\angle 3$? Justify your answer.

39. **WRITING IN MATH** Two angles are supplementary to the same angle. What is true about the measures of the two angles? Explain your reasoning.
40. If $\angle A$ and $\angle B$ are complementary and the measure of $\angle A$ is 60°, what is the measure of $\angle B$?
   A 30°  
   B 60°  
   C 90°  
   D 120°

41. GRIDDABLE What is the value of $x$ in the figure below?

42. Which two angle pairs are NOT supplementary?
   F
   G
   H
   J

43. 75°
44. 25°
45. 110°

46. Use a protractor and a straightedge to draw angles having the following measurements. (Lesson 9-2)

47. Use a protractor to find the measure of each angle. Then classify each angle as acute, obtuse, right, or straight. (Lesson 9-1)

48. BICYCLING Use the following information for Exercises 49 and 50. (Lesson 6-7)
Devon rides his bike at an average rate of 12 miles per hour.
49. Write an equation to find $d$, the distance Devon bikes in $h$ hours.
50. How many miles can Devon bike in 3 hours?

ALGEBRA Evaluate each expression. (Lesson 5-5)
51. $m + n$ if $m = \frac{11}{16}$ and $n = \frac{1}{4}$
52. $c - d$ if $c = \frac{7}{8}$ and $d = \frac{3}{5}$

PREREQUISITE SKILL Find the value of each expression. (Lesson 1-4)
53. 180 − (45 + 60)
54. 180 − (70 + 70)
55. 180 − (37 + 83)
Main IDEA
Explore the relationship among the angles of a triangle.

Targeted TEKS 6.6
The student uses geometric vocabulary to describe angles, polygons, and circles. (B) Identify relationships involving angles in triangles and quadrilaterals. Also addresses TEKS 6.6(A), 6.8(C).

Geometry Lab
Angles in Triangles

Triangle means three angles. In this lab, you will explore how the three angles of a triangle are related.

ACTIVITY

Draw a triangle similar to the one shown below on notebook or construction paper.

Label the corners 1, 2, and 3. Then tear each corner off.

Rearrange the torn pieces so that the corners all meet at one point as shown.

Repeat steps 1 and 2 with two differently-shaped triangles.

ANALYZE THE RESULTS

1. What does each torn corner represent?

2. The point where these three corners meet is the vertex of another angle as shown. Classify this angle as right, acute, obtuse, or straight. Explain.

3. What is the measure of this angle?

4. MAKE A CONJECTURE What is the sum of the measures of angles 1, 2, and 3 for each of your triangles? Verify your conjecture by measuring each angle using a protractor. Then find the sum of these measures for each triangle.

5. MAKE A CONJECTURE What is the sum of the measures of the angles of any triangle?
Main IDEA

Classify triangles and find missing angle measures in triangles.

Targeted TEKS 6.6
The student uses geometric vocabulary to describe angles, polygons, and circles.
(B) Identify relationships involving angles in triangles and quadrilaterals. 6.8 The student solves application problems involving estimation and measurement of length, area, time, temperature, volume, weight, and angles.
(C) Measure angles.

NEW Vocabulary
acute triangle  
right triangle  
obtuse triangle  
line segment  
congruent segments  
scalene triangle  
isosceles triangle  
equilateral triangle

MINI Lab

Draw the triangle shown at the right on dot paper. Then cut it out.

Measure each angle of the triangle and label each angle with its measure.

The triangle shown above has two acute angles. Since the third angle is obtuse, the triangle is an obtuse triangle.

1. Repeat the activity with nine other triangles.
2. Sort your triangles into three groups based on the third angle measures. Name the groups acute, right, and obtuse.

All triangles have at least two acute angles. As you discovered in the Mini Lab above, a triangle can be classified according to the angle measure of its third angle.

KEY CONCEPT
Classify Triangles Using Angles

<table>
<thead>
<tr>
<th>Acute Triangle</th>
<th>Right Triangle</th>
<th>Obtuse Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>all acute angles</td>
<td>1 right angle</td>
<td>1 obtuse angle</td>
</tr>
</tbody>
</table>

EXAMPLES
Classify a Triangle by Its Angles

Classify each triangle as acute, right, or obtuse.

1. \(40^\circ, 95^\circ, 45^\circ\)
   - The \(95^\circ\) angle is obtuse.
   - So, the triangle is an obtuse triangle.

2. \(60^\circ, 63^\circ, 57^\circ\)
   - All the angles are acute.
   - So, the triangle is an acute triangle.
Classify each triangle as *acute*, *right*, or *obtuse*.

a. 

b. 

In the Lab on page 445, you discovered the following relationship.

**KEY CONCEPT**

**Sum of Angle Measures in a Triangle**

<table>
<thead>
<tr>
<th>Words</th>
<th>The sum of the measures of the angles in a triangle is 180°.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>![Model Diagram]</td>
</tr>
<tr>
<td>Symbols</td>
<td>$x + y + z = 180°$</td>
</tr>
</tbody>
</table>

You can find a missing angle measure by using the fact that the sum of the measures of the angles is 180°.

**Find Angle Measures**

**FLAGS** Find the value of $x$ in the Antigua and Barbuda flag.

The three angles marked are the angles of a triangle. Since the sum of the angle measures in a triangle is 180°, $x + 55 + 90 = 180$. Use mental math to solve the equation.

\[
x + 55 + 90 = 180 \quad \text{Write the equation.}
\]

\[
x + 145 = 180 \quad \text{Add 55 and 90.}
\]

**THINK** What measure added to 145 equals 180?

\[
35 + 145 = 180 \quad \text{You know that 35 + 145 = 180.}
\]

So, the value of $x$ is 35.

**Alternative Method**

If you know two of the angle measures in a triangle, you can find the third angle measure by subtracting the two known measures from 180°. The value of $x$ is $180 - 55 - 90$ or 35.

**CHECK** Your Progress

Find the value of $x$ in each triangle below.

c. 

d.
You can also classify triangles by their sides. Each side of a triangle is a **line segment**, or a straight path between two points. Line segments that have the same length are called **congruent segments**. On a figure, congruent sides are indicated by the tick marks on the sides of the figure.

Since an isosceles triangle is defined as having *at least* two congruent sides, all equilateral triangles are also isosceles.

**EXAMPLES**

**Classify a Triangle by Its Sides**

Classify each triangle as **scalene**, **isosceles**, or **equilateral**.

4. Only two of the sides are congruent. 
   So, the triangle is an isosceles triangle.

5. None of the sides are congruent. 
   So, the triangle is a scalene triangle.

**Real-World Link**

In music, a “triangle” is generally an equilateral triangle but earlier forms of the instrument were often isosceles triangles. The side lengths of each modern triangle are usually 4 inches, 6 inches, or 9 inches.

**CHECK Your Progress**

Classify each triangle as **scalene**, **isosceles**, or **equilateral**.

e. f.
Classify each triangle as acute, right, or obtuse.

1. \(40^\circ, 115^\circ, 25^\circ\)
2. \(55^\circ, 65^\circ, 60^\circ\)

Find the value of \(x\) in each triangle.

3. \(75^\circ, 60^\circ, x^\circ\)
4. \(45^\circ, 30^\circ, x^\circ\)

5. SAILING What is the value of \(x\) in the sail of the sailboat at the right?

Classify each triangle as scalene, isosceles, or equilateral.

6. \(\ldots\)
7. \(\ldots\)

Classify each triangle drawn or having the given angle measures as acute, right, or obtuse.

8. \(48^\circ, 42^\circ, 90^\circ\)
9. \(85^\circ, 45^\circ, 50^\circ\)
10. \(60^\circ, 72^\circ, 48^\circ\)
11. \(100^\circ, 45^\circ, 35^\circ\)
12. \(90^\circ, 75^\circ, 15^\circ\)
13. \(114^\circ, 33^\circ, 33^\circ\)

Find the value of \(x\) in each triangle drawn or having the given angle measures.

14. \(\ldots\)
15. \(\ldots\)
16. \(\ldots\)
17. \(70^\circ, 60^\circ, x^\circ\)
18. \(x^\circ, 60^\circ, 25^\circ\)
19. \(x^\circ, 35^\circ, 25^\circ\)
20. **SKYSCRAPERS** The diagram below shows the view of the top of Fountain Place in Dallas. What is the value of $x$?

![Diagram showing the view of Fountain Place](image)

21. **PARKS** An A-frame picnic shelter at George Rogers Clark Historic Park in Ohio is shown below. What is the value of $x$?

![Diagram of a picnic shelter](image)

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Real-World Link

Fountain Place is a 60-story skyscraper in downtown Dallas. It is a unique building with a 10-sided exterior structure.

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Classify each triangle drawn or described as *scalene*, *isosceles*, or *equilateral*.

22. ![Triangle](image)

23. ![Triangle](image)

24. ![Triangle](image)

25. sides: 9 in., 11 in., 13 in.

26. sides: 5 cm, 6 cm, 5 cm

27. What is the measure of the third angle of a triangle if one angle measures $25^\circ$ and the second angle measure $50^\circ$?

28. What is the measure of the third angle of a right triangle if one of the angles measures $30^\circ$?

29. What is the relationship between the two acute angles of a right triangle?

30. **ART** The boat below was created using *origami*, a paper-folding technique. Measure the angles of the triangles labeled 1, 2, and 3. Then classify the triangles as *acute*, *right*, or *obtuse*.

![Boat with triangles labeled 1, 2, 3](image)

Measure the angles and sides of each triangle. Classify each triangle as *acute*, *right*, or *obtuse*. Then classify each triangle as *scalene*, *isosceles*, or *equilateral*.

31. ![Triangle](image)

32. ![Triangle](image)

33. ![Triangle](image)
34. **OPEN ENDED** Draw an obtuse scalene triangle using a ruler and protractor. Label each side and angle with its measure.

35. **CHALLENGE** Apply what you know about the sum of the measures of the angles of a triangle to find the values of $x$ and $y$ in the figure below. Justify your answer.

![Triangle with angles labeled]

36. **WRITING IN MATH** Explain why a triangle must always have at least two acute angles. Include drawings in your explanation.

37. A triangle has angles measuring $25^\circ$ and $60^\circ$. What is the measure of the triangle’s third angle?
   - A $15^\circ$
   - B $85^\circ$
   - C $95^\circ$
   - D $115^\circ$

38. **GRIDDABLE** Triangle $ABC$ is isosceles. If the measure of $\angle B$ is $48^\circ$ and the measures of $\angle A$ and $\angle C$ are equal, what is the measure of $\angle A$ in degrees?

39. Angles $A$ and $B$ are complementary. Find $m\angle A$ if $m\angle B = 35^\circ$. *(Lesson 9-3)*

**Use a protractor and a straightedge to draw angles having the following measurements.** *(Lesson 9-2)*

- 40. $85^\circ$
- 41. $20^\circ$
- 42. $125^\circ$

43. **VEHICLES** The Thrust SuperSonic Car is the fastest vehicle in the world and can reach a speed of 763 miles per hour. It is 54 feet long. What is the length of this vehicle in yards? *(Lesson 8-1)*

44. **SCHOOL** Twenty-seven percent of the students in Ms. Malan’s class are female. Identify the complement of selecting a female student at random from the class. Then find its probability. *(Lesson 7-4)*

**Spiral Review**

40. Find $m\angle A$ if $m\angle B = 35^\circ$. *(Lesson 9-3)*

**BED: A protractor and a straightedge to draw angles having the following measurements.** *(Lesson 9-2)*

- 42. $125^\circ$

43. **VEHICLES** The Thrust SuperSonic Car is the fastest vehicle in the world and can reach a speed of 763 miles per hour. It is 54 feet long. What is the length of this vehicle in yards? *(Lesson 8-1)*

44. **SCHOOL** Twenty-seven percent of the students in Ms. Malan’s class are female. Identify the complement of selecting a female student at random from the class. Then find its probability. *(Lesson 7-4)*

**PREREQUISITE SKILL** Draw an example of each figure.

- 45. rectangle
- 46. parallelogram
- 47. triangle
Use a protractor to find the measure of each angle. Then classify each angle as *acute, obtuse, right,* or *straight.* (Lesson 9-1)

1. 

2. 

3. **TEST PRACTICE** Find the measure of \( \angle HJE \) to the nearest degree. (Lesson 9-1)

4. Estimate the measure of \( \angle R \) on the leaf rake shown. (Lesson 9-2)

5. **TEST PRACTICE** Which angle measures between 45° and 90°? (Lesson 9-2)

6. 35°
7. 110°
8. 80°

Classify each pair of angles as *complementary, supplementary,* or *neither.* (Lesson 9-3)

9. 

10. 

11. **TEST PRACTICE** If \( \angle P \) and \( \angle Q \) are supplementary and the measure of \( \angle P \) is 41°, what is the measure of \( \angle Q \)? (Lesson 9-3)

   A 49°  C 139°
   B 59°  D 149°

12. Find the value of \( x \). (Lesson 9-3)

Classify each triangle as *acute, right,* or *obtuse.* (Lesson 9-4)

13. 

14. 

15. Find the value of \( x \). (Lesson 9-4)
Explore the relationship among the angles of different quadrilaterals.

**Main IDEA**
Explore the relationship among the angles of different quadrilaterals.

**Targeted TEKS 6.6**
The student uses geometric vocabulary to describe angles, polygons, and circles. (B) Identify relationships involving angles in triangles and quadrilaterals. Also addresses TEKS 6.8(C).

**Geometry Lab**
**Angles in Quadrilaterals**

Quadrilateral means *four sides*. Four-sided figures also have four angles. In this lab, you will explore how the angles of different quadrilaterals are related.

**Activity**

**Step 1**
Draw the quadrilaterals shown on grid paper.

**Step 2**
Use a protractor to measure the angles of each figure. Record your results in a table like the one shown.

<table>
<thead>
<tr>
<th>Quadrilateral</th>
<th>$m\angle A$</th>
<th>$m\angle B$</th>
<th>$m\angle C$</th>
<th>$m\angle D$</th>
<th>Sum of Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Analyze the Results**

1. Describe any patterns you see in the angle measurements of Quadrilateral 1 and Quadrilateral 2.
2. Describe any patterns you see in the angle measurements of Quadrilaterals 1–4.
3. **MAKE A CONJECTURE** Are any of the patterns found in Quadrilaterals 1–4 present in Quadrilateral 5? If not, make a conjecture as to what makes Quadrilateral 5 different from Quadrilaterals 1–4.
The figure below is a **quadrilateral**, since it has four sides and four angles.

**STEP 1** Draw a quadrilateral.

**STEP 2** Pick one vertex and draw the diagonal to the opposite vertex.

1. Name the shape of the figures formed when you drew the diagonal. How many figures were formed?

2. **MAKE A CONJECTURE** Use the relationship among the angles measures in a triangle to find the sum of the angle measures in a quadrilateral. Explain.

3. Find the measure of each angle of your quadrilateral. Compare the sum of these measures to the sum you found in Exercise 2.

The angles of a quadrilateral have a special relationship.

**KEY CONCEPT** **Angles of a Quadrilateral**

Words  The sum of the measures of the angles of a quadrilateral is 360°.

Model  

![Model Diagram]

Symbols  \( w + x + y + z = 360 \)

**EXAMPLE**

Find the value of \( x \) in the quadrilateral shown.

Since the sum of the angle measures in a quadrilateral is 360°, \( x + 65 + 85 + 90 = 360 \).

\[
\begin{align*}
x + 65 + 85 + 90 &= 360 \\
x + 240 &= 360
\end{align*}
\]

**Write the equation.**

**Add 65, 85, and 90.**

**THINK** What measure added to 240 equals 360?

\[120 + 240 = 360\]  

You know that \(120 + 240 = 360\).

So, the value of \( x \) is 120.
Find the value of $x$ in each quadrilateral.

a. $\begin{align*}
\text{Angle Sum of Triangle:} & \quad 140^\circ + 116^\circ + 40^\circ = 296^\circ \\
\text{Find } x: & \quad x = 180^\circ - 140^\circ = 40^\circ
\end{align*}$

b. $\begin{align*}
\text{Angle Sum of Quadrilateral:} & \quad x + 19^\circ = 180^\circ \\
\text{Find } x: & \quad x = 180^\circ - 19^\circ = 161^\circ
\end{align*}$

The table shows the characteristics of 5 special quadrilaterals.

<table>
<thead>
<tr>
<th>Quadrilateral</th>
<th>Figure</th>
<th>Characteristics</th>
</tr>
</thead>
</table>
| Rectangle     | ![Rectangle](image) | - Opposite sides congruent  
- All angles are right angles  
- Opposite sides parallel |
| Square        | ![Square](image) | - All sides congruent  
- All angles are right angles  
- Opposite sides parallel |
| Parallelogram | ![Parallelogram](image) | - Opposite sides congruent  
- Opposite sides parallel  
- Opposite angles congruent |
| Rhombus       | ![Rhombus](image) | - All sides congruent  
- Opposite sides parallel  
- Opposite angles congruent |
| Trapezoid     | ![Trapezoid](image) | - Exactly one pair of opposite sides parallel |

**READING Math**

**Congruent Angles** The red arcs show congruent angles.

**Parallel** Parallel means that if you extend the lengths of the sides, the opposite sides will never intersect or meet.

**Perpendicular** Lines that meet to form a right angle are perpendicular lines.

**Real-World EXAMPLE**

**QUILTS** Classify the quadrilaterals labeled 1 and 2 in the quilt piece.

Figure 1 is a square. Figure 2 is a rhombus.

**c. LOGOS** Classify the quadrilaterals used in the logo below.
GRIDDABLE What is the value of $x$ in the parallelogram at the right?

Read the Test Item
You need to find the value of $x$.

Solve the Test Item
You know that in a parallelogram, opposite angles are congruent. Since the angle opposite the missing measure has a measure of $70^\circ$, $x = 70$.

Check
$70^\circ + 110^\circ + 70^\circ + 110^\circ = 360^\circ$ ✔

d. GRIDDABLE A rhombus is shown at the right. Find the measure of $\angle P$.

Example 1
Find the value of $x$ in each quadrilateral.

1. $105^\circ 60^\circ$ 2. 

Example 2
3. SIGNS Classify each quadrilateral.

Example 3
4. TEST PRACTICE Find the value of $x$ in the parallelogram at the right.
Find the value of $x$ in each quadrilateral.

5. $\begin{align*}
80^\circ & \quad 120^\circ \\
120^\circ & \quad x^\circ \\
65^\circ & \quad x^\circ
\end{align*}$

6. $\begin{align*}
60^\circ & \quad 70^\circ \\
x^\circ & \quad 110^\circ \\
110^\circ & \quad 110^\circ
\end{align*}$

7. $\begin{align*}
98^\circ & \quad 105^\circ \\
98^\circ & \quad x^\circ
\end{align*}$

8. $\begin{align*}
95^\circ & \quad 55^\circ \\
x^\circ & \quad 110^\circ \\
55^\circ & \quad 110^\circ
\end{align*}$

9. $\begin{align*}
60^\circ & \quad 105^\circ \\
x^\circ & \quad 60^\circ \\
105^\circ & \quad 60^\circ
\end{align*}$

10. $\begin{align*}
75^\circ & \quad 115^\circ \\
75^\circ & \quad x^\circ \\
115^\circ & \quad x^\circ
\end{align*}$

Classify each quadrilateral.

11.

12.

13.

14.

15.

16.

17. **FLAGS** Many aircraft display the shape of the American flag slightly distorted to indicate motion. Classify each quadrilateral.

18. **SIGNS** Classify each quadrilateral.

19. **TANGRAM** Triangles and quadrilaterals are polygons. A *polygon* is a simple closed figure formed by three or more sides. Refer to the seven tangram pieces shown at the left. Classify the polygons numbered 3 and 5.

Find the value of $x$ in each quadrilateral.

20. $\begin{align*}
100.4^\circ & \quad 90.3^\circ \\
78.5^\circ & \quad x^\circ
\end{align*}$

21. $\begin{align*}
122.8^\circ & \quad 122.8^\circ \\
122.8^\circ & \quad x^\circ \\
78.5^\circ & \quad 78.5^\circ
\end{align*}$

22. $\begin{align*}
2^\circ & \quad 2^\circ \\
2^\circ & \quad 2^\circ \\
2^\circ & \quad 2^\circ
\end{align*}$

**Real-World Link.** A tangram is an ancient Chinese puzzle consisting of 7 geometric shapes.

*Source: tangrams.ca/*
23. **SORTING** Grace sorted a set of quadrilaterals into two categories according to a certain rule. The shapes that followed the rule were put in Set A, and the shapes that did not follow the rule were put in Set B.

<table>
<thead>
<tr>
<th>Set A</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>square</td>
<td>rhombus</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Set B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangle</td>
<td>parallelogram</td>
</tr>
</tbody>
</table>

What rule did Grace use to sort the quadrilaterals?

24. **RESEARCH** Use the Internet or another source to look up the meaning of the term *isosceles trapezoid*. Explain how an isosceles trapezoid is related to an isosceles triangle. Then draw an example of an isosceles trapezoid.

25. **OPEN ENDED** Describe two different real-world items that are shaped as quadrilaterals. Then classify those quadrilaterals.

26. **NUMBER SENSE** Three of the angle measures of a quadrilateral are congruent. Without calculating, determine if the measure of the fourth angle in each of the following situations is greater than, less than, or equal to $90^\circ$. Explain your reasoning.
   
a. The three congruent angles each measure $89^\circ$.
   b. The three congruent angles each measure $90^\circ$.
   c. The three congruent angles each measure $91^\circ$.

27. **CHALLENGE** Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
   
   27. A rhombus is a square.
   28. A quadrilateral is a parallelogram.
   29. A rectangle is a square.
   30. A square is a rectangle.

28. **SELECT A TOOL** Quentin is designing a company logo for his lawn care company. He wants to use polygons in the logo. Which of the following tools might Quentin use to design several sample company logos? Justify your selection(s). Use the tool(s) to create a sample logo using polygons. Then classify each polygon you used in the sample logo.

   - real objects
   - paper/pencil
   - technology

29. **WRITING IN MATH** Make a diagram that shows the relationship between each of the following shapes: rectangle, parallelogram, square, rhombus, quadrilateral, and trapezoid. Then write a few sentences that explain your diagram.
33. The drawing below shows the shape of Hinto’s patio.

Find the measure of \( \angle A \).
A 75°  
B 105°  
C 165°  
D 195°

34. A parallelogram is shown below. Find the measure of \( \angle M \) to the nearest degree.

\[ \begin{array}{c}
F 30°  
G 60°  
H 120°  
J 150°  
\end{array} \]

### Spiral Review

Find the value of \( x \) in each triangle. (Lesson 9-4)

35.

36.

37.

Classify each pair of angles as complementary, supplementary, or neither. (Lesson 9-3)

38.

39.

40.

41. **AIRPLANES** An airplane is flying at 34,848 feet from the ground. How many miles from the ground is the airplane? (Lesson 8-1)

42. **BAKING** Mallory needs \( \frac{4}{4} \) cups of flour for a bread recipe. Write \( \frac{4}{4} \) as an improper fraction. (Lesson 4-3)

### GET READY for the Next Lesson

**PREREQUISITE SKILL** Tell whether each pair of figures appear to have the same size and shape.

43.

44.

45.
Problem-Solving Investigation

MAIN IDEA: Solve problems by drawing a diagram.

TARGETED TEKS 6.11 The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. (C) Select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture...to solve a problem. Also addresses TEKS 6.11(B).

EMAIL: DRAW A DIAGRAM

YOUR MISSION: Draw a diagram to solve the problem.

THE PROBLEM: How many planting beds can the science club make to fit in the school courtyard?

RORY: The science club will plant cacti in the courtyard, which is 46 feet by 60 feet. The planting beds will be 6 feet across, 8 feet apart, and 6 feet from any walls.

EXPLORE

You know all the dimensions. You need to find how many beds will fit.

PLAN

Draw a diagram.

SOLVE

The diagram shows that 12 cactus beds will fit inside the courtyard.

CHECK

The total distances across and down match the dimensions given.

1. Explain why you think the students drew a diagram to solve the problem.
2. (WRITING IN MATH) Write and solve a problem that can be solved by drawing a diagram.
Solve Exercises 3–5. Use the *draw a diagram* strategy.

3. **DRIVING** The downtown section of a city is rectangular, 4 blocks by 3 blocks. How many ways are there to drive from one corner of downtown to the opposite corner, if you must make exactly two turns?

4. **DECORATING** For the spring dance, there are 5 columns arranged in the shape of a pentagon. One large streamer is hung from each column to every other column. How many streamers are there in all?

5. **BROWNIES** Colin is arranging brownies on a rectangular sheet 9 inches by 12 inches. How many brownies will fit if each is 1 inch square and needs to be placed 2 inches apart from another brownie?

Use any strategy to solve Exercises 6–9. Some strategies are shown below.

**Problem-Solving Strategies**
- Use the four-step plan.
- Make an organized list.
- Look for a pattern.
- Draw a diagram.

6. **FOOD** A sandwich shop offers 4 different kinds of bread and 6 different kinds of meat. How many different sandwiches can be made using one kind of bread and two different kinds of meat?

7. **PATTERNS** Draw the next two figures in the pattern.

8. **CAMPING** Rita bought a tent for camping. Each of the four sides of the tent needs three stakes to secure it properly to the ground. How many stakes are needed?

9. **CLUBS** Refer to the Venn diagram. If there are 28 students in the class, determine the number of students that are in both the ecology club and the honor society.

For Exercises 10–12, select the appropriate operation(s) to solve the problem. Justify your selection(s) and solve the problem.

10. **GEOMETRY** How many times greater is the length of the longest side in the triangle below than the length of the shortest side?

11. **BASKETBALL** The table gives the frequency of three–point shots made by a team over the course of five games. Find the mean number of three–point shots made by the team for games 1–5.

<table>
<thead>
<tr>
<th>Game</th>
<th>Tally</th>
<th>Three-Point Shots</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

12. **TRAVEL** Jeffrey drove a total of 285 miles to visit his sister. He drove 55 miles per hour for the first 165 miles and then 60 miles per hour for the rest of the trip. How many hours did it take him to complete the trip?
Similar and Congruent Figures

Main IDEA
Identify similar and congruent figures.

Targeted TEKS 6.3
The student solves problems involving direct proportional relationships. (A) Use ratios to describe proportional situations. (B) Represent ratios and percents with concrete models, fractions, and decimals. Also addresses TEKS 6.13(A).

NEW Vocabulary
similar figures
congruent figures
corresponding sides

Vocabulary Link
Similar
Everyday Use nearly, but not exactly, the same or alike
Math Use figures that have the same shape but not necessarily the same size

GET READY for the Lesson

PATTERNS The triangle at the right is called Sierpinski’s triangle. Notice how the pattern is made up of various equilateral triangles.

1. How many different-sized triangles are in the pattern?
2. Compare the size and shape of these triangles.

Figures that have the same shape but not necessarily the same size are called similar figures.

<table>
<thead>
<tr>
<th>Similar</th>
<th>Not Similar</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Similar Fig 1" /></td>
<td><img src="image2.png" alt="Not Similar Fig 1" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Similar Fig 2" /></td>
<td><img src="image4.png" alt="Not Similar Fig 2" /></td>
</tr>
</tbody>
</table>

Figures that have the same size and shape are congruent figures.

<table>
<thead>
<tr>
<th>Congruent</th>
<th>Not Congruent</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5.png" alt="Congruent Fig 1" /></td>
<td><img src="image6.png" alt="Not Congruent Fig 1" /></td>
</tr>
<tr>
<td><img src="image7.png" alt="Congruent Fig 2" /></td>
<td><img src="image8.png" alt="Not Congruent Fig 2" /></td>
</tr>
</tbody>
</table>

Identify Similar and Congruent Figures

Tell whether each pair of figures is congruent, similar, or neither.

1. The figures have the same size and shape. They are congruent.
2. The figures have the same shape but not the same size. They are similar.
Tell whether each pair of figures is congruent, similar, or neither.

a. 

b. 

c. 

The sides of similar or congruent figures that match are called corresponding sides. In similar figures, these matching sides are proportional. In congruent figures, these sides are congruent.

The triangles above are similar with corresponding sides $FG$ and $XY$, $GH$ and $YZ$, and $HF$ and $ZX$.

**Real-World Example** Identify Corresponding Sides

**Photography** The photographs below are similar rectangles.

What side of rectangle $ABCD$ corresponds to $QR$?

Corresponding sides represent the same side of similar figures. So, $BC$ corresponds to $QR$.

d. **Photography** Determine which side corresponds to $DC$ in the photographs in Example 3.

Personal Tutor at tx.msmath1.com
In a rectangle, all angles are right angles and therefore congruent. So, to determine if two rectangles are similar, you need to only see if corresponding sides are proportional. Recall from Lesson 6-3 that quantities are said to be proportional if they have a constant ratio.

**EXAMPLE**

**Identify Similar Figures**

4. Which rectangle(s) below is similar to rectangle $ABCD$?

Examine the ratios of corresponding sides to see if they have a constant ratio.

**READING Math**

*Segments $BC$ is read as the measure of side $BC$.*

**Study Tip**

*Right Triangles Two right triangles are similar if the ratios of two pairs of corresponding sides are proportional.*

**e.** Which triangle(s) below is similar to triangle $DEF$? Assume corresponding angles are congruent.

So, rectangle $JKLM$ is similar to rectangle $ABCD$. 

**CHECK Your Progress**

<table>
<thead>
<tr>
<th>Rectangle $EFGH$</th>
<th>Rectangle $JKLM$</th>
<th>Rectangle $NOPQ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BC = \frac{6}{8} \text{ or } \frac{3}{4}$</td>
<td>$BC = \frac{6}{3} \text{ or } \frac{2}{1}$</td>
<td>$BC = \frac{6}{4} \text{ or } \frac{3}{2}$</td>
</tr>
<tr>
<td>$FG = \frac{6}{8} \text{ or } \frac{3}{4}$</td>
<td>$KL = \frac{3}{3} \text{ or } \frac{2}{1}$</td>
<td>$OP = \frac{6}{4} \text{ or } \frac{3}{2}$</td>
</tr>
<tr>
<td>$CD = \frac{10}{12} \text{ or } \frac{5}{6}$</td>
<td>$CD = \frac{10}{5} \text{ or } \frac{2}{1}$</td>
<td>$CD = \frac{10}{6} \text{ or } \frac{5}{3}$</td>
</tr>
<tr>
<td>$GH = \frac{10}{12} \text{ or } \frac{5}{6}$</td>
<td>$LM = \frac{5}{5} \text{ or } 1$</td>
<td>$PQ = \frac{10}{6} \text{ or } \frac{5}{3}$</td>
</tr>
<tr>
<td>Not similar</td>
<td>Similar</td>
<td>Not similar</td>
</tr>
</tbody>
</table>
Tell whether each pair of figures is congruent, similar, or neither.

1. 
2. 
3. 

**Example 3**

**BILLIARDS** For Exercises 4 and 5, refer to the similar billiards tables below.

4. What side of rectangle $WXYZ$ corresponds to $TU$?
5. What side of rectangle $STUV$ corresponds to $WX$?

**Example 4**

State whether each rectangle is similar to rectangle $WXYZ$.

6. 
7. 
8. 

**Exercises**

Tell whether each pair of figures is congruent, similar, or neither.

9. 
10. 
11. 
12. 
13. 
14. 

**QUILTING** For Exercises 15 and 16, refer to the quilt at the right.

The quilt is made up of many congruent quadrilaterals.

15. What side of quadrilateral $DEFG$ corresponds to $KL$?
16. What side of quadrilateral $KLMN$ corresponds to $EF$?
TELESCOPE  For Exercises 17 and 18, refer to the photo of the Hobby-Eberly Telescope Dome and the following information.

The triangles that form the outside of the telescope dome are similar triangles.

17. What side of triangle $LMN$ corresponds to $\overline{AC}$?
18. What side of triangle $ABC$ corresponds to $\overline{MN}$?

State whether each triangle is similar to triangle $ABC$.

19. $\triangle LMN$
20. $\triangle ABC$
21. $\triangle LMN$

State whether each rectangle is similar to rectangle $JKLM$.

22. $JKLM$
23. $JKLM$
24. $JKLM$
25. $JKLM$

STATE FAIR  A plot of land will be separated into ten congruent rectangles with rope as shown for the judging of cows. Find the total length of rope needed. Then find the total length of rope needed to section the same plot of land into five congruent rectangles.

Each pair of figures is similar. Find the value of $x$.

26. $\triangle LMN$
27. $\triangle ABC$

JEWELRY  Flora made the earrings shown out of colored wire. She wants the smaller earring to be similar to the larger earring. Find the total length of colored wire needed to make the smaller earring.
29. OPEN ENDED Draw a pair of similar triangles and a pair of congruent quadrilaterals. Label the vertices of the figures. Using these labels, generalize a statement about each pair of figures.

REASONING Tell whether each statement is sometimes, always, or never true. Explain your reasoning.

30. Two rectangles are similar.  31. Two squares are similar.

32. CHALLENGE Analyze the figure to determine the number of congruent triangles and the number of similar triangles there are in the figure. (It may be helpful to copy the figure and outline the triangles.)

33. WRITING IN MATH Compare and contrast similarity and congruence.

34. Mrs. Daisy’s garden contains pathways which form equilateral triangles.

How many meters of pathways are there in her garden?

A 18 m  C 96 m
B 66 m  D 180 m

35. Which of the following is NOT true about the triangles below, given they are similar?

F Side XY is congruent to side XW.
G Triangle WXY has the same shape as triangle RST.
H Side RS is congruent to side WX.
J Triangle WXY and triangle RST are both isosceles triangles.

36. SATELLITES A main satellite sends a signal to each of two smaller satellites. If each of those two satellites sends a signal to each other and a signal back to the main satellite, draw a diagram to determine the number of signals sent. (Lesson 9-6)

Find the value of $x$ in each quadrilateral. (Lesson 9-5)

37. 38. 39.

40. WEATHER The probability that it will snow tomorrow is forecasted at 60%. Describe the complement of this event and find its probability. (Lesson 7-4)
A pattern formed by repeating figures that fit together without gaps or overlaps is a **tessellation**. Tessellations are formed using slides, flips, or turns of congruent figures.

**ACTIVITY**

Select the three pattern blocks shown.

Choose one of the blocks and trace it on your paper. Choose a second block that will fit next to the first without any gaps or overlaps and trace it.

Trace the third pattern block into the tessellation.

Continue the tessellation by expanding the pattern.

Create a tessellation using the pattern blocks shown.

**ANALYZE THE RESULTS**

1. Tell if a tessellation can be created using a square and an equilateral triangle. Justify your answer with a drawing.

2. **MAKE A CONJECTURE** What is the sum of the measures of the angles where the vertices of the figures meet? Is this true for all tessellations?

3. Name two figures that cannot be used to create a tessellation. Use a drawing to justify your answer.
Key Concepts

Types of Angles  (Lesson 9-1)
- An acute angle measures less than 90°.
- An obtuse angle measures between 90° and 180°.
- A right angle measures exactly 90°.
- A straight angle measures exactly 180°.

Classifying Triangles  (Lesson 9-4)
- An acute triangle has all acute angles.
- A right triangle has one right angle.
- An obtuse triangle has one obtuse angle.

Sum of Angle Measures  (Lessons 9-4 and 9-5)
- The sum of the angle measures in a triangle is 180°.
- The sum of the measures of the angles in a quadrilateral is 360°.

Similar and Congruent Figures  (Lesson 9-7)
- Figures that have the same shape but not necessarily the same size are similar figures.
- Figures that have the same shape and size are congruent figures.

Key Vocabulary

- acute angle  (p. 431)
- acute triangle  (p. 446)
- angle  (p. 430)
- complementary angles  (p. 440)
- congruent angles  (p. 439)
- congruent figures  (p. 462)
- congruent segments  (p. 448)
- corresponding sides  (p. 463)
- degree  (p. 430)
- equilateral triangle  (p. 448)
- isosceles triangle  (p. 448)
- line segment  (p. 448)
- obtuse angle  (p. 431)
- obtuse triangle  (p. 446)
- parallelogram  (p. 455)
- quadrilateral  (p. 454)
- rectangle  (p. 455)
- rhombus  (p. 454)
- right angle  (p. 431)
- right triangle  (p. 446)
- scalene triangle  (p. 448)
- side  (p. 430)
- similar figures  (p. 462)
- square  (p. 455)
- straight angle  (p. 431)
- supplementary angles  (p. 440)
- tessellation  (p. 468)
- trapezoid  (p. 455)
- vertex  (p. 430)
- vertical angles  (p. 439)

Vocabulary Check

State whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

1. When two lines intersect, they form two pairs of opposite angles called supplementary angles.
2. A four-sided figure with exactly one pair of opposite sides parallel is a trapezoid.
3. All triangles have at least two obtuse angles.
4. The sum of the angle measures in a quadrilateral is 360°.
5. Congruent figures are also similar figures.
Lesson-by-Lesson Review

9-1 Measuring Angles (pp. 430–432)

Use a protractor to find the measure of each angle. Then classify each angle as acute, obtuse, right, or straight.

6. [Angle Diagram]

7. [Angle Diagram]

8. ROLLER COASTERS

Use a protractor to measure the angle formed at the top of the roller coaster. Then classify it as right, acute, obtuse, or straight.

Example 1

Use a protractor to find the measure of the angle. Then classify the angle as acute, obtuse, right, or straight.

The angle measures 75°. Since it is less than 90°, it is an acute angle.

9-2 Estimating and Drawing Angles (pp. 434–438)

Use a protractor and a straightedge to draw angles having the following measurements.

9. 36°  10. 127°
11. 180°  12. 90°

Estimate the measure of each angle.

13. [Angle Diagram]
14. [Angle Diagram]

15. AIRPLANES Estimate the measure of the angle shown below.

Example 2

Use a protractor and a straightedge to draw a 47° angle.

Draw one side of the angle. Align the center of the protractor and the 0° with the line. Find 47°. Make a mark.

Draw the other side of the angle.
9-3 **Angle Relationships** (pp. 439–444)

Classify each pair of angles as complementary, supplementary, or neither.

16. \( \begin{align*} \angle 59^\circ & \quad \angle 31^\circ \\
\end{align*} \)

17. \( \begin{align*} \angle 77^\circ \quad \angle \quad \angle \\
\end{align*} \)

18. \( \begin{align*} \angle 112^\circ & \quad \angle 68^\circ \\
\end{align*} \)

Find the value of \( x \) in each figure.

19. \( \begin{align*} \angle x^\circ & \quad \angle 136^\circ \\
\end{align*} \)

20. \( \begin{align*} \angle x^\circ & \quad \angle 29^\circ \\
\end{align*} \)

21. **TRAFFIC** Find the value of \( x \) in the intersection shown.

**Example 3**

\[ \begin{align*} 56^\circ + 134^\circ &= 190^\circ \\
\text{The angles are neither complementary nor supplementary.} \\
\end{align*} \]

**Example 4**

\[ \begin{align*} 23^\circ + 67^\circ &= 90^\circ \\
\text{The angles are complementary.} \\
\end{align*} \]

**Example 5** Find the value of \( x \) in the figure.

The angle labeled \( x^\circ \) and the angle labeled 63° are vertical angles. Therefore, they are congruent. So, the value of \( x \) is 63.
**Example 6** Classify the triangle shown as **acute, right, or obtuse**.

Since the triangle has one right angle, it is a right triangle.

**Example 7** Find the value of $x$ in the triangle shown.

The sum of the angle measures in a triangle is $180^\circ$. So, $x + 139 + 21 = 180$.

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + 139 + 21 = 180$</td>
</tr>
<tr>
<td>$x + 160 = 180$</td>
</tr>
<tr>
<td>$20 + 160 = 180$</td>
</tr>
</tbody>
</table>

You know that $20 + 160 = 180$.

So, the value of $x$ is 20.

**Example 8** Classify the triangle shown as **scalene, isosceles, or equilateral**.

Since the triangle has all three congruent sides, it is an equilateral triangle. It is also an isosceles triangle since it has at least two congruent sides.

**ARCHITECTURE** The top of the R.R. Donnelly Building in Chicago, Illinois, contains a triangle. Classify the triangle as **scalene, isosceles, or equilateral**.
**9-5 Quadrilaterals** (pp. 454–459)

Find the value of \( x \) in each quadrilateral.

30. \( x \)

\[ \begin{align*}
62° & \quad 113° \\
85° & \quad 62°
\end{align*} \]

31. \( x \)

\[ \begin{align*}
162° & \quad 79° \\
83° & \quad 162°
\end{align*} \]

**Example 9** Find the value of \( x \) in the quadrilateral shown.

The sum of the angle measures in a quadrilateral is 360°.

\[ x + 91° + 78° + 83° = 360° \]

\[ x + 252° = 360° \]

\[ 108° + 252° = 360° \]

You know that 108° + 252° = 360°.

So, the value of \( x \) is 108.

**Example 10** Classify the quadrilateral shown.

The quadrilateral has exactly one pair of parallel sides, so it is a trapezoid.

Classify each quadrilateral.

32.

33.

34. **TABLES** Identify the quadrilateral outlined.

**9-6 PSI: Draw a Diagram** (pp. 460–461)

Solve by drawing a diagram.

35. **ART** Gina is painting a design. She will paint six dots in a circle. Each dot is to be connected to every other dot by a straight line. How many straight lines will Gina need to draw?

36. **WINDOWS** A contractor is designing one side of an office building 48 feet in length. The windows are each 5 feet long, and will be placed 6 feet apart and 5 feet from the end of the building wall. How many windows can the contractor design?

**Example 11** Five friends are seated in a circle at a restaurant. How many ways can any two friends split a meal?

Draw five dots to represent the friends. Connect all dots to represent all possible choices for splitting a meal.

Count the number of lines that were drawn, 10. So, there are 10 ways for any two friends to split a meal.
Similar and Congruent Figures (pp. 462–468)

37. Tell whether the pair of figures shown below is congruent, similar, or neither.

The triangles shown are congruent.

38. What side of triangle \( XYZ \) corresponds to \( AC \)?

39. Which rectangle below is similar to rectangle \( MNOP \)?

40. **ART** Manuela created the design below out of ribbon for an art display. If the design consists of 10 equilateral triangles, how much ribbon was used?

Examine the ratios of the corresponding sides to see if they form a constant ratio.

**Triangle** \( DEF \)  \( \text{Triangle} \) \( SRT \)

\[
\frac{EF}{ST} = \frac{21}{18} \text{ or } \frac{7}{6} \quad \frac{DF}{RS} = \frac{14}{9}
\]

So, the triangles are not similar.
Use a protractor to measure each angle and classify as **acute, obtuse, right, or straight**.

1.  
2.  
3.  

4. **HILLS** Estimate the measure of $\angle x$.

Classify each pair of angles as **complementary, supplementary, or neither**.

5.  
6.  

7. Find the value of $x$ in the triangle at the right.

Classify each triangle as **scalene, isosceles, or equilateral**.

8.  
9.  

10. **TEST PRACTICE** Find the measure of $\angle R$ in the trapezoid below.

11. **PATIOS** Classify each quadrilateral outlined in the brick patio.

12. **TEST PRACTICE** Refer to the shapes.

Which statement is NOT true?

F  Each shape is a quadrilateral.
G  Each shape is a polygon.
H  Each shape is a parallelogram.
J  The sum of the angle measures of each shape is 360°.

13. **SPORTS** To block off the boundaries for a game of Capture the Flag, Jonas plans to use 5 orange cones on each side of the rectangular field. This includes one cone at each corner. How many cones are needed?

14. The quadrilaterals shown are similar. What side of quadrilateral $ABCD$ corresponds to $\overline{GH}$?

Tell whether each pair of figures is **congruent, similar, or neither**.

15.  
16.  

Texas Test Practice
Cumulative, Chapters 1–9

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. At a concession stand, each pizza was cut into 8 equal-sized pieces. Dave sold 7 pieces, Sally sold 5 pieces, and Lori sold 6 pieces. Find the total portion of pizza sold by these 3 people.
   A 18
   B \(2\frac{3}{4}\)
   C \(2\frac{1}{4}\)
   D \(2\frac{9}{10}\)

2. GRIDDABLE The ratio of boys to girls at a rock concert was about 7 to 6. If there were 1,092 boys at the rock concert, about how many girls were at the concert?

3. Find the measure of \(\angle P\) of the parallelogram shown to the nearest degree.
   F 127°
   G 96°
   H 28°
   J 53°

4. The table shows the scores at a gymnastics competition. Who had the highest overall score?

<table>
<thead>
<tr>
<th>Gymnast</th>
<th>Overall Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amber</td>
<td>8.95</td>
</tr>
<tr>
<td>Camille</td>
<td>8.82</td>
</tr>
<tr>
<td>Jacy</td>
<td>8.73</td>
</tr>
<tr>
<td>Luisa</td>
<td>8.99</td>
</tr>
</tbody>
</table>

   A Amber  C Jacy
   B Camille  D Luisa

5. Sarah needed to select two sweaters to take on a vacation. She had 1 red sweater, 1 yellow sweater, 1 green sweater, and 1 blue sweater to choose from. Which diagram shows all the possible outcomes?

   F
   G
   H
   J

6. Each day after school, Bernardo runs for 10 minutes. If he runs an average of 900 feet per minute, how can you find the number of feet he runs per day?
   A Divide his average running speed in feet per minute by the number of minutes he runs each day
   B Multiply his average running speed in feet per minute by the number of minutes he runs each day
   C Subtract the number of minutes he runs each day from his average running speed in feet per minute
   D Add his average running speed in feet per minute to the number of minutes he runs each day

7. Which fraction is NOT equivalent to \(\frac{2}{3}\)?
   F \(\frac{4}{6}\)
   G \(\frac{6}{9}\)
   H \(\frac{9}{12}\)
   J \(\frac{10}{15}\)
8. A rectangular garden has a width of 5.3 feet. The length of the garden is 1.8 feet more than the width. Which of the following represents the length of the garden?
   A 2.9 ft
   B 3.5 ft
   C 7.1 ft
   D 8.4 ft

9. The drawing below shows the shape of the laundry room in a house. Find the measure of \( \angle H \).
   - F 26°
   - G 138°
   - H 180°
   - J 212°

10. The length of Rashid’s bedroom is 432 centimeters. What is the length of his bedroom in meters?
    - A 43.2 m
    - B 4.32 m
    - C 0.432 m
    - D 0.0432 m

11. In which of the pictures below does \( \angle M \) appear to be an acute angle?
    - F
    - G
    - H
    - J

12. **GRIDDABLE** A recipe for a batch of cookies calls for one \( 5\frac{3}{4} \)-ounce package of coconut. About how many ounces of coconut are needed for 2 batches of cookies?

   **Record your answers on a sheet of paper.**
   **Show your work.**

13. Use triangle \( ABC \) to answer the following questions.
   a. Classify triangle \( ABC \) by its angles and by its sides.
   b. Find the measure of \( \angle A \).
   c. What type of angle is \( \angle C \)?