CHAPTER 12

Algebra: Nonlinear Functions and Polynomials

Knowledge and Skills

- Understand that a function represents a dependence of one quantity on another and can be described in a variety of ways. **TEKS A.1**

Key Vocabulary

monomial (p. 642)
nonlinear function (p. 630)
polynomial (p. 642)
quadratic function (p. 637)

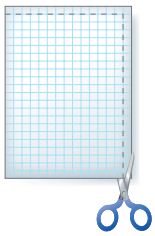
Real-World Link

Fountains Many real-world situations, such as the shape of a fountain, cannot be modeled by linear functions. These can be modeled using nonlinear functions.

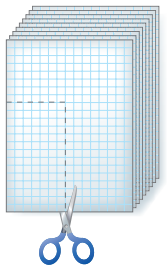
FOLDABLES Study Organizer

Algebra: Nonlinear Functions and Polynomials Make this Foldable to help you organize your notes. Begin with eight sheets of grid paper.

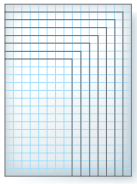
1. **Cut** off one section of the grid paper along both the long and short edges.



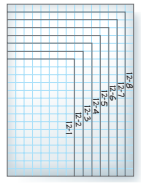
2. **Cut** off two sections from the second sheet, three sections from the third sheet, and so on to the 8th sheet.



3. **Stack** the sheets from narrowest to widest.



4. **Label** each of the right tabs with a lesson number.



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GET READY for Chapter 12

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

MathFolio Take the Online Readiness Quiz at tx.msmath3.com.

Option 1

Take the Quick Quiz below. Refer to the Quick Review for help.

QUICK Quiz

Identify the like terms in each expression. (Used in Lessons 12-3 through 12-5)

1. $3x + 5 - x$
2. $2 - 4n + 1 + 6n$

Rewrite each expression using parentheses so that the like terms are grouped together. (Used in Lessons 12-3 through 12-5)

3. $(a + 2b) + (2a - 5b)$
4. $(8w + 7x) + (3w + 9x)$

Rewrite each expression as an addition expression by using the additive inverse. (Used in Lessons 12-3 through 12-5)

5. $3 - 5y$
6. $2m - 7n$

Write each expression using exponents. (Used in Lessons 12-6 and 12-7)

7. $6 \cdot 6 \cdot 6 \cdot 6$
8. $3 \cdot 7 \cdot 7 \cdot 3 \cdot 7$
9. **FUND-RAISER** The students at Hampton Middle School raised $8 \cdot 8 \cdot 2 \cdot 8 \cdot 2$ dollars to help build a new community center. How much money did they raise?

QUICK Review

Example 1

Identify the like terms in $8m + 3 - 4m + 2$.

$8m$ and $-4m$ are like terms.
3 and 2 are like terms.

Example 2

Rewrite $(x + 6y) + (4x - 3y)$ using parentheses so that the like terms are grouped together.

$(x + 6y) + (4x - 3y)$
$= (x + 4x) + (6y - 3y)$ x and $4x$ are like terms as well as $6y$ and $-3y$.

Example 3

Rewrite $6 - 8p$ as an addition expression by using the additive inverse.

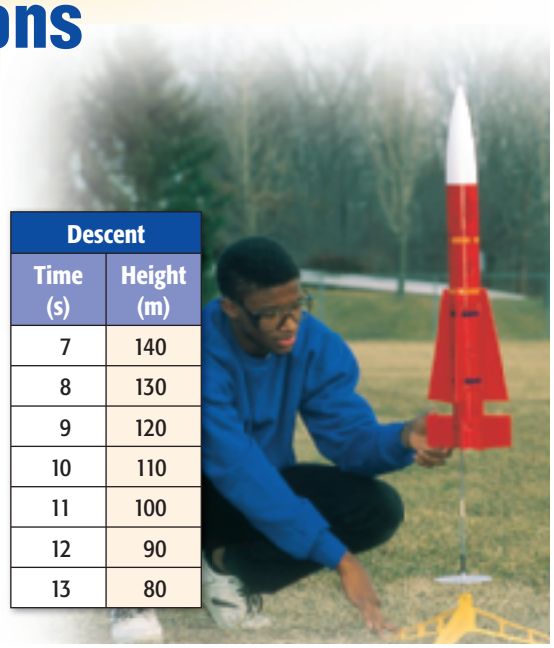
$6 - 8p = 6 + (-8p)$ Subtracting $8p$ is the same as adding $-8p$.

Example 4

Write $5 \cdot 4 \cdot 5 \cdot 4 \cdot 5$ using exponents.

5 is multiplied by itself 3 times and 4 is multiplied by itself 2 times.
So, $5 \cdot 4 \cdot 5 \cdot 4 \cdot 5 = 5^3 \cdot 4^2$.

12-1 Linear and Nonlinear Functions



Main IDEA

Determine whether a function is linear or nonlinear.

Preparation for TEKS A.5 The student understands that linear functions can be represented in different ways and translates among their various represent ations. **(A) Determine whether or not given situations can be represented by linear functions.**

NEW Vocabulary

nonlinear function

REVIEW Vocabulary

constant rate of change occurs when the rate of change between any two data points is proportional. (Lesson 4-10)

GET READY for the Lesson

ROCKETRY The tables show the flight data for a model rocket launch. The first table gives the rocket's height at each second of its ascent, or upward flight. The second table gives its height as it descends back to Earth using a parachute.

Ascent	
Time (s)	Height (m)
0	0
1	38
2	74
3	106
4	128
5	138
6	142

Descent	
Time (s)	Height (m)
7	140
8	130
9	120
10	110
11	100
12	90
13	80

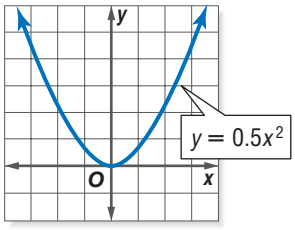
1. During its ascent, did the rocket travel the same distance each second? Justify your answer.
2. During its descent, did the rocket travel the same distance each second? Justify your answer.
3. Graph the ordered pairs (time, height) for the rocket's ascent and descent on separate axes. Connect the points with a straight line or smooth curve. Then compare the graphs.

In Lesson 11-2, you learned that linear functions have graphs that are straight lines. These graphs represent constant rates of change. **Nonlinear functions** are functions that do not have constant rates of change. Therefore, their graphs are not straight lines.

EXAMPLES Identify Functions Using Graphs

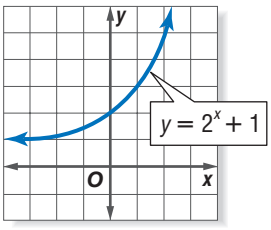
Determine whether each graph represents a *linear* or *nonlinear* function. Explain.

1



The graph is a curve, not a straight line. So it represents a nonlinear function.

2



This graph is also a curve. So it represents a nonlinear function.

Doug Martin

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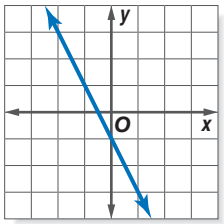
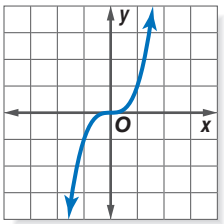
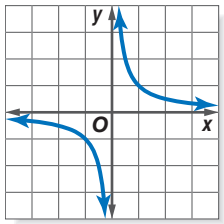
CHECK Your Progress **Determine whether each graph represents a *linear* or *nonlinear* function. Explain.**

a. 

b. 

c. 

STUDY TIP

Identifying Linear Equations Always examine an equation after it has been solved for y to see that the power of x is 1 or 0. Then check to see that x does not appear in the denominator.

Recall that the equation for a linear function can be written in the form $y = mx + b$, where m represents the constant rate of change.

EXAMPLES Identify Functions Using Equations

Determine whether each equation represents a *linear* or *nonlinear* function. Explain.

3 $y = x + 4$

Since the equation can be written as $y = 1x + 4$, this function is linear.

4 $y = \frac{6}{x}$

The equation cannot be written in the form $y = mx + b$. So this function is nonlinear.

CHECK Your Progress

d. $y = 2x^3 + 1$ e. $y = 3x$ f. $y = \frac{x}{5}$

A nonlinear function does not increase or decrease at the same rate.

EXAMPLES Identify Functions Using Tables

Determine whether each table represents a *linear* or *nonlinear* function. Explain.

5

	x	y	
+2	2	50	−15
+2	4	35	−15
+2	6	20	−15
	8	5	

As x increases by 2, y decreases by 15 each time. The rate of change is constant, so this function is linear.

6

	x	y	
+3	1	1	+15
+3	4	16	+33
+3	7	49	+51
	10	100	

As x increases by 3, y increases by a greater amount each time. The rate of change is not constant, so this function is nonlinear.

CHECK Your Progress

g.

x	0	5	10	15
y	20	16	12	8

h.

x	0	2	4	6
y	0	2	8	18

Real-World Link
The NCAA women's basketball tournament begins with 64 teams and consists of 6 rounds of play.

Real-World EXAMPLE

7 **BASKETBALL** **Use the table to determine whether the number of teams is a linear function of the number of rounds of play.**

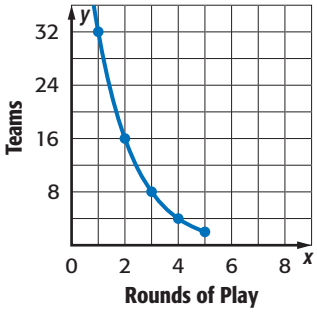
Round(s) of play	Teams
1	32
2	16
3	8
4	4
5	2

Examine the differences between the number of teams for each round.

$16 - 32 = -16$ $\quad 8 - 16 = -8$
$4 - 8 = -4$ $\quad 2 - 4 = -2$

While there is a pattern in the differences, they are not the same. Therefore, this function is nonlinear.

Check Graph the data to verify the ordered pairs do not lie on a straight line.



CHECK Your Progress

i. **TICKETS** Tickets to the school dance cost \$5 per student. Are the ticket sales a linear function of the number of tickets sold? Explain.

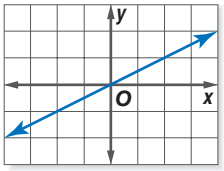
Number of Tickets Sold	1	2	3
Ticket Sales	\$5	\$10	\$15

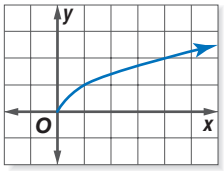
Personal Tutor at tx.msmath3.com

CHECK Your Understanding

Examples 1–4 (pp. 630–631)

Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

1. 

2. 

3. $y = \frac{x}{3}$

4. $y = 2x^2$

Examples 5, 6 (p. 631)

5.

x	0	1	2	3
y	1	3	6	10

6.

x	0	3	6	9
y	−3	9	21	33

Example 7 (p. 632)

7. **GEOMETRY** The table shows the measures of the sides of several rectangles. Are the widths of the rectangles a linear function of the lengths? Explain.

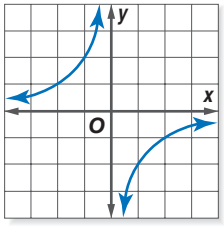
Length (in.)	1	4	8	10
Width (in.)	64	16	8	6.4

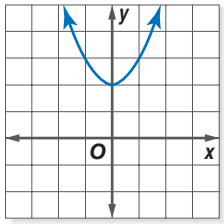
Elise Amendola/AP/Wide World Photos

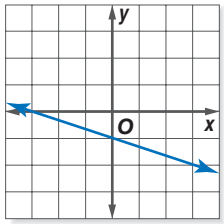
Exercises

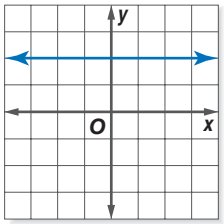
HOMEWORK HELP	
For Exercises	See Examples
8–13	1, 2
14–19	3, 4
20–25	5, 6
26–29	7

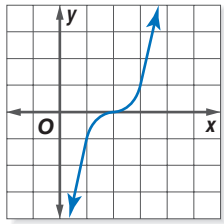
Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

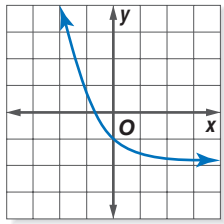
8. 

9. 

10. 

11. 

12. 

13. 

14. $y = x^3 - 1$

15. $y = 4x^2 + 9$

16. $y = 0.6x$

17. $y = \frac{3x}{2}$

18. $y = \frac{4}{x}$

19. $y = \frac{8}{x} + 5$

20.

x	3	6	9	12
y	12	10	8	6

21.

x	1	2	3	4
y	1	4	9	16

22.

x	5	10	15	20
y	13	28	43	58

23.

x	1	3	5	7
y	−2	−18	−50	−98

24.

x	2	4	6	8
y	10	12	16	24

25.

x	4	8	12	16
y	3	0	−3	−6

26. **TRAVEL** The Guzman family drove from San Antonio to Fort Stockton, Texas. Use the table to determine whether the distance driven is a linear function of the hours traveled. Explain.

Time (h)	1	2	3	4
Distance (mi)	65	130	195	260

27. **BUILDINGS** The table shows the height of several buildings in Chicago, Illinois. Use the table to determine whether the height of the building is a linear function of the number of stories. Explain.

Building	Stories	Height (ft)
Harris Bank III	35	510
One Financial Place	40	515
Kluczynski Federal Building	45	545
Mid Continental Plaza	50	582
North Harbor Tower	55	556

Source: *The World Almanac*

GEOMETRY **For Exercises 28 and 29, use the following information.**
Recall that the circumference of a circle is equal to pi times its diameter and that the area of a circle is equal to pi times the square of its radius.

28. Is the circumference of a circle a linear or nonlinear function of its diameter? Explain your reasoning.

29. Is the area of a circle a linear or nonlinear function of its radius? Explain your reasoning.

For Exercises 30–34, determine whether each equation or table represents a *linear* or *nonlinear* function. Explain.

30. $y - x = 1$

31. $xy = -9$

32. $y = 2^x$

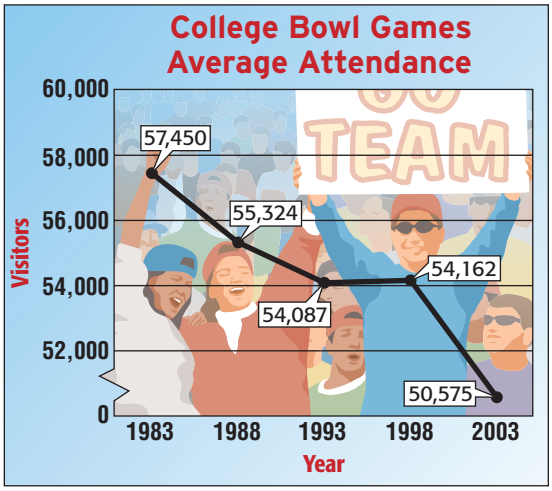
33.

x	0.5	1	1.5	2
y	15	8	1	−6

34.

x	−4	0	4	8
y	2	1	−1	−4

35. **FOOTBALL** The graphic shows the decrease in the average attendance at college bowl games from 1983 to 2003. Would you describe the decline as linear or nonlinear? Explain.



Source: USA Today

36. **GEOMETRY** Make a graph showing the area of a square as a function of its perimeter. Explain whether the function is linear.

EXTRA PRACTICE
See pages 725, 739.
Self-Check Quiz at tx.msmath3.com

H.O.T. Problems

37. **GRAPHING** Water is poured at a constant rate into the vase at the right. Draw a graph of the water level as a function of time. Is the water level a linear or nonlinear function of time? Explain.

38. **CHALLENGE** *True* or *false*? All graphs of straight lines are linear functions. Explain your reasoning or provide a counterexample.

39. **Which One Doesn't Belong?** Identify the function that is not linear. Explain your reasoning.

$y = 2x$	$y = x^2$	$y - 2 = x$	$x - y = 2$

40. **OPEN ENDED** Give an example of a nonlinear function using a table of values.

41. **WRITING IN MATH** Describe two methods for determining whether a function is linear given its equation.

TEST PRACTICE

42. Which equation describes the data in the table?

x	−7	−5	−3	0	4
y	50	26	10	1	17

A $5x + 1 = y$

B $xy = 68$

C $x^2 + 1 = y$

D $-2x^2 + 8 = y$

43. Which equation represents a nonlinear function?

F $y = 3x + 1$

G $y = \frac{x}{3}$

H $2xy = 10$

J $y = 3(x - 5)$

Spiral Review

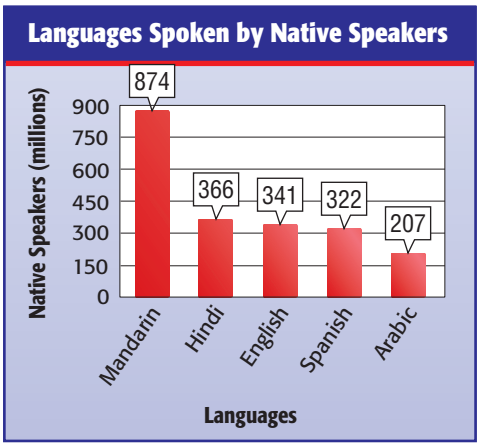
Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship. (Lesson 11-8)

44. grade on a test and amount of time spent studying

45. age and number of siblings

46. number of Calories burned and length of time exercising

47. LANGUAGES The graph shows the top five languages spoken by at least 100 million native speakers worldwide. What conclusions can you make about the number of Mandarin native speakers and the number of English native speakers? (Lesson 11-7)



Source: *The Wolrd Almanac For Kids*

Solve each equation. Check your solution. (Lesson 10-5)

48. $1 - 3c = 9c + 7$

49. $7k + 12 = 8 - 9k$

50. $13.4w + 17 = 5w - 4$

51. $8.1a + 2.3 = 5.1a - 3.1$

52. MAGAZINES The numbers of pages in a magazine in the last nine issues were 196, 188, 184, 200, 168, 176, 192, 160, and 180. Find the range, median, upper and lower quartiles, and the interquartile range of the data. (Lesson 9-5)

53. VENTILATION The cylindrical air duct of a large furnace has a diameter of 30 inches and a height of 120 feet. If it takes 15 minutes for the contents of the duct to be expelled into the air, what is the volume of the substances being expelled each hour? (Lesson 7-7)

GET READY for the Next Lesson

PREREQUISITE SKILL **Graph each equation.** (Lesson 11-2)

54. $y = 2x$

55. $y = x + 3$

56. $y = 3x - 2$

57. $y = \frac{1}{3}x + 1$

Explore 12-2

Graphing Calculator Lab
Families of Quadratic Functions

Main IDEA

Use a graphing calculator to graph families of quadratic functions.

Preparation for TEKS A.2
The student uses the properties and attributes of functions. **(A) Identify and sketch the general forms** of linear ($y = x$) and **quadratic ($y = x^2$) parent functions.**

Families of nonlinear functions share a common characteristic based on a parent function. The parent function of a family of quadratic functions is $y = x^2$. You can use a TI-83/84 Plus graphing calculator to investigate families of quadratic functions.

ACTIVITY

Graph $y = x^2$, $y = x^2 + 5$, and $y = x^2 - 3$ on the same screen.

STEP 1 Clear any existing equations from the Y= list by pressing Y= CLEAR.

STEP 2 Enter each equation. Press
X,T,θ, x^2 ENTER,
X,T,θ, x^2 + 5 ENTER, and
X,T,θ, x^2 − 3 ENTER.

STEP 3 Graph the equations in the standard viewing window. Press ZOOM 6.

Analyze The Results

1. Compare and contrast the three equations you graphed.
2. Describe how the graphs of the three equations are related.
3. **MAKE A CONJECTURE** How does changing the value of c in the equation $y = x^2 + c$ affect the graph?
4. Use a graphing calculator to graph $y = 0.5x^2$, $y = x^2$, and $y = 2x^2$.
5. Compare and contrast the three equations you graphed in Exercise 4.
6. Describe how the graphs of the three equations are related.
7. **MAKE A CONJECTURE** How does changing the value of a in the equation $y = ax^2$ affect the graph?
8. List a family of three quadratic functions. Describe the common characteristic of their graphs.

12-2 Graphing Quadratic Functions

Main IDEA

Graph quadratic functions.

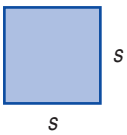
Preparation for TEKS A.2 The student uses the properties and attributes of functions. **(A) Identify and sketch the general forms of** linear $(y = x)$ and **quadratic $(y = x^2)$ parent functions.**

NEW Vocabulary

quadratic function

MINI Lab

You know that the area A of a square is equal to the length of a side s squared, $A = s^2$.



s	s^2	(s, A)
0	0	(0, 0)
1	1	(1, 1)
2		
3		
4		
5		
6		

STEP 1 Copy and complete the table.

STEP 2 Graph the ordered pairs from the table. Connect them with a smooth curve.

1. Is the relationship between the side length and the area of a square linear or nonlinear? Explain.
2. Describe the shape of the graph.

A **quadratic function**, like $A = s^2$, is a function in which the greatest power of the variable is 2. Its graph is U-shaped, opening upward or downward.

STUDY TIP

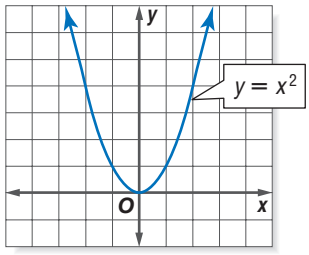
Quadratic Fuctions The graph of a quadratic function is called a *parabola.*

EXAMPLES Graph Quadratic Functions

1 Graph $y = x^2$.

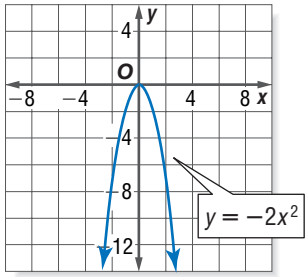
To graph a quadratic function, make a table of values, plot the ordered pairs, and connect the points with a smooth curve.

x	x^2	y	(x, y)
−2	$(-2)^2 = 4$	2	(−2, 4)
−1	$(-1)^2 = 1$	1	(−1, 1)
0	$(0)^2 = 0$	0	(0, 0)
1	$(1)^2 = 1$	1	(1, 1)
2	$(2)^2 = 4$	4	(2, 4)



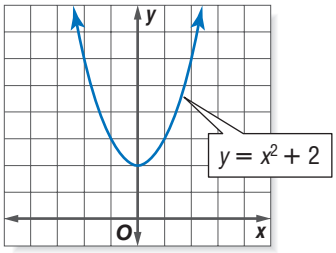
2 Graph $y = -2x^2$.

x	$-2x^2$	y	(x, y)
−2	$-2(-2)^2 = -8$	−8	(−2, −8)
−1	$-2(-1)^2 = -2$	−2	(−1, −2)
0	$-2(0)^2 = 0$	0	(0, 0)
1	$-2(1)^2 = -2$	−2	(1, −2)
2	$-2(2)^2 = -8$	−8	(2, −8)



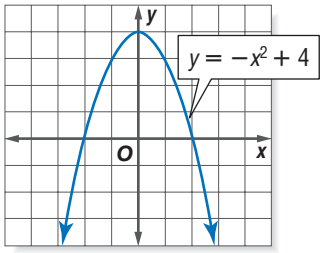
3 Graph $y = x^2 + 2$.

x	$x^2 + 2$	y	(x, y)
-2	$(-2)^2 + 2 = 6$	6	$(-2, 6)$
-1	$(-1)^2 + 2 = 3$	3	$(-1, 3)$
0	$(0)^2 + 2 = 2$	2	$(0, 2)$
1	$(1)^2 + 2 = 3$	3	$(1, 3)$
2	$(2)^2 + 2 = 6$	6	$(2, 6)$



4 Graph $y = -x^2 + 4$.

x	$-x^2 + 4$	y	(x, y)
-2	$-(-2)^2 + 4 = 0$	0	$(-2, 0)$
-1	$-(-1)^2 + 4 = 3$	3	$(-1, 3)$
0	$-(0)^2 + 4 = 4$	4	$(0, 4)$
1	$-(1)^2 + 4 = 3$	3	$(1, 3)$
2	$-(2)^2 + 4 = 0$	0	$(2, 0)$



CHECK Your Progress **Graph each function.**

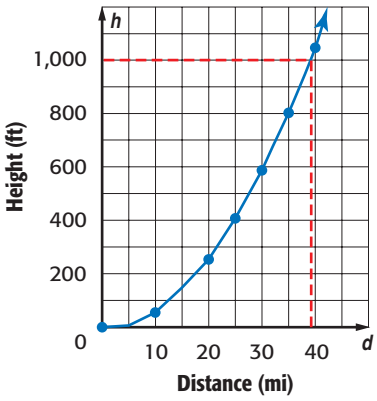
a. $y = 6x^2$ b. $y = x^2 - 2$ c. $y = -2x^2 - 1$

Real-World EXAMPLE

5 **MONUMENTS** **The function $h = 0.66d^2$ represents the distance d in miles you can see from a height of h feet. Graph this function. Then use your graph and the information at the left to estimate how far you could see from the top of the Eiffel Tower.**

Distance cannot be negative, so use only positive values of d.

d	$h = 0.66d^2$	(d, h)
0	$0.66(0)^2 = 0$	$(0, 0)$
10	$0.66(10)^2 = 66$	$(10, 66)$
20	$0.66(20)^2 = 264$	$(20, 264)$
25	$0.66(25)^2 = 412.5$	$(25, 412.5)$
30	$0.66(30)^2 = 594$	$(30, 594)$
35	$0.66(35)^2 = 808.5$	$(35, 808.5)$
40	$0.66(40)^2 = 1{,}056$	$(40, 1{,}056)$



At a height of 986 feet, you could see approximately 39 miles.

Real-World Link
The Eiffel Tower in Paris, France, opened in 1889 as part of the World Exposition. It is about 986 feet tall.
Source: structurae.de

CHECK Your Progress

d. **TOWERS** The outdoor observation deck of the Space Needle in Seattle, Washington, is 520 feet above ground level. Estimate how far you could see from the observation deck.

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CONTENTS

CHECK Your Understanding

Examples 1–4 (pp. 637–638)

Graph each function.

1. $y = 3x^2$
2. $y = -5x^2$
3. $y = -4x^2$
4. $y = -x^2 + 1$
5. $y = x^2 - 3$
6. $y = -2x^2 + 2$

Example 5 (p. 638)

7. **CARS** The function $d = 0.006s^2$ represents the braking distance d in meters of a car traveling at a speed s in kilometers per second. Graph this function. Then use your graph to estimate the speed of the car if its braking distance is 12 meters.

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–11	1, 2
12–19	3, 4
20, 21	5

Graph each function.

8. $y = 4x^2$
9. $y = 5x^2$
10. $y = -3x^2$
11. $y = -6x^2$
12. $y = x^2 + 6$
13. $y = x^2 - 4$
14. $y = -x^2 + 2$
15. $y = -x^2 - 5$
16. $y = 2x^2 - 1$
17. $y = 2x^2 + 3$
18. $y = -4x^2 - 1$
19. $y = -3x^2 + 2$

20. **RACING** The function $d = \frac{1}{2}at^2$ represents the distance d that a race car will travel over an amount of time t given the rate of acceleration a. Suppose a car is accelerating at a rate of 5 feet per second every second. Graph this function. Then use your graph to find the time it would take the car to travel 125 feet.

21. **WATERFALLS** The function $d = -16t^2 + 182$ models the distance d in feet a drop of water falls t seconds after it begins its descent from the top of the 182-foot high American Falls in New York. Graph this function. Then use your graph to estimate the time it will take the drop of water to reach the river at the base of the falls.

Graph each function.

22. $y = 0.5x^2 + 1$
23. $y = 1.5x^2$
24. $y = 4.5x^2 - 6$
25. $y = \frac{1}{3}x^2 - 2$
26. $y = \frac{1}{2}x^2$
27. $y = -\frac{1}{4}x^2 + 1$

GEOMETRY **For Exercises 28 and 29, write a function for each of the following. Then graph the function in the first quadrant.**

28. The volume V of a cube is a function of the edge length a. Use your graph to estimate the edge length of a cube whose volume is 125 cubic centimeters.
29. The volume V of a rectangular prism with a square base and a fixed height of 5 inches is a function of the base edge length s. Use your graph to estimate the base edge length of a prism whose volume is 180 cubic inches.

EXTRA PRACTICE

See pages 725, 739.

Self-Check Quiz at tx.msmath3.com

H.O.T. Problems

CHALLENGE The graphs of quadratic functions may have exactly one highest point, called a *maximum*, or exactly one lowest point, called a *minimum*. Graph each quadratic equation. Determine whether each graph has a maximum or a minimum. If so, give the coordinates of each point.

30. $y = 2x^2 + 1$ **31.** $y = -x^2 + 5$ **32.** $y = x^2 - 3$

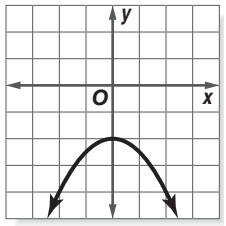
33. OPEN ENDED Write and graph a quadratic function that opens upward and has its minimum at $(0, -3.5)$.

34. WRITING IN MATH Write a quadratic function of the form $y = ax^2 + c$ and explain how to graph it.

TEST PRACTICE

35. Which graph represents the function $y = -0.5x^2 - 2$?

A



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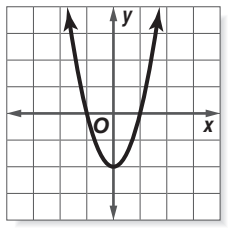
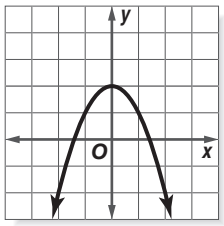
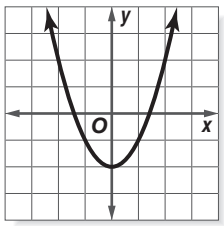


C



D



Spiral Review

Determine whether each equation represents a *linear* or *nonlinear* function. (Lesson 12-1)

36. $y = x - 5$ **37.** $y = 3x^3 + 2$ **38.** $x + y = -6$ **39.** $y = -2x^2$

CRANES For Exercises 40–42, use the information below and the table at the right. (Lesson 11-8)

The table shows the population of the Texas whooping crane at the Aransas National Wildlife Refuge.

Year	Population
2000	172
2001	171
2002	181
2003	194
2004	197

Source: Texas Parks and Wildlife Department

40. Draw a scatter plot of the data and draw a line of fit.

41. Does the scatter plot show a *positive*, *negative*, or *no* relationship?

42. Use your graph to estimate the population of the Texas whooping crane at the refuge in 2005.

43. SAVINGS Anna's parents put $750 into a college savings account. After 6 years, the investment had earned $540. Write an equation that you could use to find the simple interest rate. Then find the simple interest rate. (Lesson 5-9)

GET READY for the Next Lesson

PREREQUISITE SKILL Identify the like terms in each expression. (Lesson 10-1)

44. $4a + 1 - 2a$ **45.** $2x + 3x + 5 - 1$ **46.** $-1 - 2d + 3 + d$ **47.** $x + 2 - 7x + 8$

Explore 12-3

Algebra Lab
Modeling Expressions with Algebra Tiles

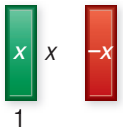
Main IDEA

Model expressions using algebra tiles.

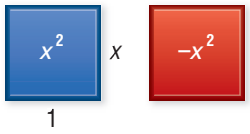
Preparation for TEKS A.5 The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. **(A) Use symbols to represent unknowns and variables.**

In a set of algebra tiles, the integer 1 is represented by a tile that is 1 unit by 1 unit. Notice that the area of this tile is 1 square unit. The opposite of 1, -1, is represented by a red tile with the same shape and size.

The variable x is represented by a tile that is 1 unit by x units. Notice that the area of this tile is x square units. The opposite of x, $-x$, is represented by a red tile with the same shape and size.



Similarly, the expression x^2 is represented by a tile that is x units by x units. A red tile with the same shape and size is used to represent $-x^2$.



You can use these tiles to model expressions like $2x^2 + 5x - 6$.

ACTIVITY

Use algebra tiles to model $2x^2 + 5x - 6$.

$2x^2 + 5x - 6$

CHECK Your Progress

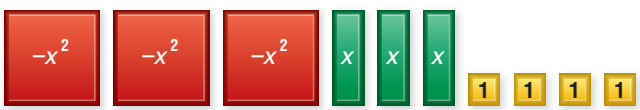
Use algebra tiles to model each expression.

a. $4x^2$ **b.** $-3x^2$ **c.** $3x^2 - 4x$ **d.** $-x^2 + 2x$

e. $x^2 - x + 1$ **f.** $-2x^2 + x - 5$ **g.** $2x^2 - 3x + 2$ **h.** $-4x^2 + 3x + 8$

Analyze the Results

1. Name the expression modeled below.



2. **MAKE A CONJECTURE** What might a model of the expression x^3 look like?

12-3 Simplifying Polynomials

Main IDEA

Simplify polynomials.

Preparation for TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions,** transform and solve equations, and factor **as necessary in problem situations. (B) Use the commutative, associative, and distributive properties to simplify algebraic expressions.**

NEW Vocabulary

monomial
polynomial

REVIEW Vocabulary

like terms terms that contain the same variable (Lesson 10-1)

READING in the Content Area

For strategies in reading this lesson, visit tx.msmath3.com.

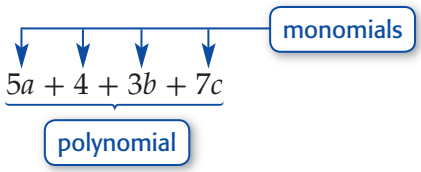
GET READY for the Lesson

MONEY Suppose you need money to buy a drink and a snack. The table shows the number and type of coins you find in your backpack and in your pocket.

Coin Type	Number in Backpack	Number in Pocket
Quarter	3	0
Dime	5	2
Nickel	2	3
Penny	4	0

1. Let q, d, n, and p represent the value of a quarter, a dime, a nickel, and a penny, respectively. Write an expression for the total amount of money in your backpack.
2. Write an expression for the total amount of money in your pocket.
3. Write an expression for the total amount of money in all.

A **monomial** is a number, a variable, or a product of numbers and/or variables. An algebraic expression that is the sum or difference of one or more monomials is called a **polynomial**.



You have already learned how to simplify polynomials such as $3x + 4 + 2x - 8$ by using the Distributive Property to combine like terms. You can use the same process to simplify polynomials containing more than one variable.

EXAMPLE 1 Simplify a Polynomial

Simplify $-5d + 2n + 4d - 3n$.

The like terms in this expression are $-5d$ and $4d$, and $2n$ and $-3n$.

$-5d + 2n + 4d - 3n$	Write the polynomial.
$= -5d + 2n + 4d + (-3n)$	Definition of subtraction
$= (-5d + 4d) + [2n + (-3n)]$	Group like terms.
$= -1d + (-1n)$ or $-d - n$	Simplify by combining like terms.

CHECK Your Progress **Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.**

a. $4f - 3g - f - 9g$ b. $6h + k - 8$ c. $-7x + 5y + 2 - y$

CONTENTS

Concepts in MOtion
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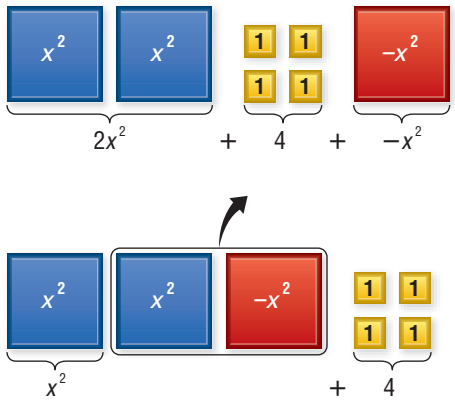
The expression $2x^2$ is another example of a monomial, since it is the product of 2, x, and x. You can simplify expressions like $2x^2 + 4 - x^2$ using algebra tiles.

EXAMPLE Simplify Polynomials

2 **Simplify $2x^2 + 4 - x^2$.**

Use the definition of subtraction to write this polynomial as $2x^2 + 4 + (-x^2)$. Then group and add like terms.

$$\begin{aligned} &2x^2 + 4 + (-x^2) \\ &= [2x^2 + (-x^2)] + 4 \\ &= [2x^2 + (-1x^2)] + 4 \\ &= 1x^2 + 4 \\ &= x^2 + 4 \end{aligned}$$



STUDY TIP

Look Back To review **zero pairs,** see Lesson 1-4.

CHECK Your Progress

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.

d. $-3c^2 + 7 - c^2 + 9$ **e.** $x^2 + 3 - 2x^2 - 6$ **f.** $8a^2 - 4$

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STUDY TIP

Standard Form
When simplifying polynomials, it is customary to write the result in *standard form*; that is, with the powers of the variable decreasing from left to right.

$-5x^2 + 3x + 2$, *not* $3x - 5x^2 + 2$

From these examples, you can see that like terms must have the same variable and the same power. Thus, $2x^2$ and $3x^2$ are like terms, while $4x^2$ and $5x$ are not.

EXAMPLE Simplify Polynomials

3 **Simplify $x^2 - 1 - x + 3 + 2x$.**

$x^2 + (-1) + (-1x) + 3 + 2x$	Write the polynomial.
$= x^2 + (-1x + 2x) + (-1 + 3)$	Group like terms.
$= x^2 + 1x + 2$	Simplify by combining like terms.
$= x^2 + x + 2$	$1x = x$

CHECK Your Progress

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.

g. $4x + 8x^2 + x$ **h.** $-2m^2 - 3m + 7$ **i.** $y^2 - 5y - 6y^2 + 9$

CHECK Your Understanding

Examples 1–3 (pp. 642–643) **Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form.***

1. $9g - 9h + 3g + 1$
2. $2x^2 - 2 + x^2$
3. $-5w^2 + 3w^2 - 8w$

Example 1 (p. 642)

4. **GEOMETRY** The perimeter of a rectangle can be found by using the polynomial $\ell + \ell + w + w$, where ℓ is the length and w is the width. Simplify the polynomial.

Exercises

HOMEWORK HELP

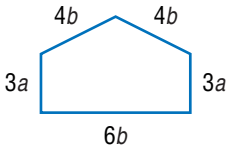
For Exercises	See Examples
5–10, 17, 18	1
11–16	2, 3

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form.*

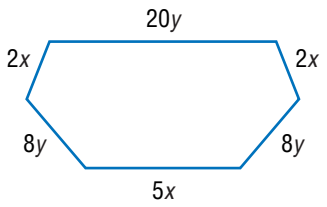
5. $6a + 8b - 7a + b$
6. $5x + 7y + 8 - z$
7. $-n + 4p + 5 - 6n$
8. $3f - 2g - 9g + 5f$
9. $-8c - d + 4c + 2$
10. $2j + 7 + k - 9$
11. $m^2 + m - 3$
12. $a + 5a^2 - 7a$
13. $4 - 3x^2 + 6x + x^2$
14. $2w^2 - 6w - w + 1$
15. $3k^2 + 4 - 8k + k - 2$
16. $y^2 + 8y + 1 + 7y^2 - 4$

GEOMETRY Write the perimeter of each figure in simplest form.

17.



18.



19. **FUND-RAISING** The table shows the number of items Orlando and Emma bought from the Music Boosters. If c represents the cost of cookies, p the cost of popcorn, and w the cost of wrapping paper, write an expression in simplest form for the total amount spent by Orlando and Emma.

Name	Cookies	Popcorn	Wrapping Paper
Orlando	2	1	0
Emma	0	2	3

20. **BASKETBALL** In the 2004–2005 regular season, Yao Ming of the Houston Rockets made 538 field goals (2 points) and 389 free throws (1 point). His teammate, Dikembe Mutombo, made 108 field goals and 106 free throws. If g represents the number of points for a field goal and t the number of points for a free throw, write an expression in simplest form for the total number of points scored by Ming and Mutombo during the 2004–2005 season.

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form.*

21. $b^2 + 6b - 9 + b^2 - b + 3$
22. $-5t^3 - 8t^2 + 4t - 6 + 7t^3 + 3t$
23. $1.4x^2 - 3.8x + 1.2x^2 + 4.5x$
24. $\frac{3}{4}y^2 - 5y - \frac{1}{4}y + 5y$

EXTRA PRACTICE

See pages 725, 739.

Self-Check Quiz at tx.msmath3.com

25. **SAVINGS** Shanté receives $50 each birthday from her aunt. Her parents put this money in a savings account with an interest rate of r. The table gives the account balance after each birthday. Find the balance of Shanté's account after her third birthday if the interest rate is 4%.

Birthday	Balance ($)
1	50
2	$(50r + 50) + 50$
3	$(50r^2 + 100r + 50) + (50r + 50) + 50$

H.O.T. Problems

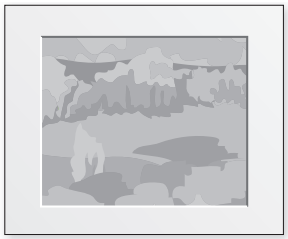
26. **CHALLENGE** Determine whether $2x^2 + 3x = 5x^2$ is *sometimes*, *always*, or *never* true for all x. Explain your reasoning.

27. **OPEN ENDED** Write a polynomial with four terms that simplifies to $5a - 9b$.

28. **WRITING IN MATH** Are $6x$ and $3x^2$ like terms? Explain why or why not.

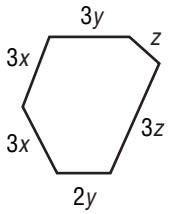
TEST PRACTICE

29. The area of the mat and the picture is $5x^2 - 2$ square inches. The area of the picture is $3x^2$ square inches. What is the area of the mat?



A $8x^2 - 2$ in^2
B $7x^2 - 3$ in^2
C $2x^2 - 2$ in^2
D $x^2 - 1$ in^2

30. Write the perimeter of the figure in simplest form.



F $14xyz$
G $6x + 5y + 3z$
H $15xyz$
J $6x + 5y + 4z$

Spiral Review

Graph each function. (Lesson 12-2)

31. $y = 5x^2$
32. $y = x^2 + 5$
33. $y = x^2 - 4$
34. $y = -x^2 - 3$

35. **BIOLOGY** The table shows how long it took for the first 400 bacteria cells to grow in a petri dish. Is the growth of the bacteria a linear function of time? Explain. (Lesson 12-1)

Time (min)	46	53	57	60
Number of Cells	100	200	300	400

36. **CARDS** Two cards are drawn from a deck of ten cards numbered 1 through 10. Once a card is selected, it is not returned. Find the probability of drawing two even cards in a row. (Lesson 8-4)

GET READY for the Next Lesson

PREREQUISITE SKILL **Rewrite each expression using parentheses so that the like terms are grouped together.** (Lessons 1-2 and 10-1)

37. $(a + 2) + (3a + 4)$
38. $(c + d) + (7c - 2d)$
39. $(x^2 + 4x) + (6x^2 - 8x)$

12-4 Adding Polynomials

Main IDEA

Add polynomials.

Preparation for TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions,** transform and solve equations, and factor **as necessary in problem situations. (B) Use the commutative, associative, and distributive properties to simplify algebraic expressions.**

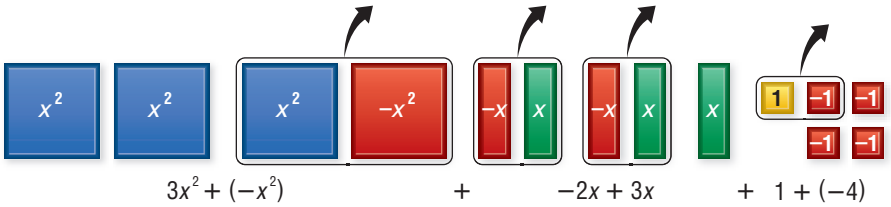
MINI Lab

Consider the polynomials $3x^2 - 2x + 1$ and $-x^2 + 3x - 4$ modeled below.

Follow these steps to add the polynomials.

STEP 1 Combine the tiles that have the same shape.

STEP 2 Remove any zero pairs.



1. Write the polynomial for the tiles that remain.
2. Use algebra tiles to find $(x^2 + x - 2) + (6x^2 - 5x - 1)$.

You can add polynomials horizontally or vertically by combining like terms.

EXAMPLES Add Polynomials

1 **Find $(4x + 1) + (2x + 3)$.**

Add horizontally.

$(4x + 1) + (2x + 3) = (4x + 2x) + (1 + 3)$ Associative and Commutative Properties

$= 6x + 4$

2 **Find $(3x^2 + 5x - 9) + (x^2 + x + 6)$.**

Add vertically.

$$\begin{array}{r} 3x^2 + 5x - 9 \\ (+)\ x^2 + x + 6 \\ \hline 4x^2 + 6x - 3 \end{array}$$

STUDY TIP

Check for Reasonableness Check Example 2 by substituting 1 for x in the addends and the sum. When $x = 1$, $(3x^2 + 5x - 9) + (x^2 + x + 6) = -1 + 8$ or 7 and $4x^2 + 6x - 3 = 7$. Therefore, the answer is correct.

3 **Find $(7y^2 + 2y) + (-5y + 8)$.**

$(7y^2 + 2y) + (-5y + 8) = 7y^2 + (2y - 5y) + 8$ Group like terms.

$= 7y^2 - 3y + 8$ Simplify.

The sum is $7y^2 - 3y + 8$.

4 **Find $(6x^2 - x + 5) + (2x^2 - 3)$.**

$$\begin{array}{r} 6x^2 - x + 5 \\ (+)\ 2x^2 \quad\ - 3 \\ \hline 8x^2 - x + 2 \end{array}$$

Leave a space because there is no other term like $-x$.

The sum is $8x^2 - x + 2$.

CHECK Your Progress **Add.**

a. $(4x + 3) + (x - 1)$

b. $(10a^2 + 5a + 7) + (a^2 - 3)$

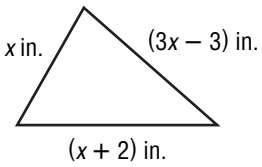
STUDY TIP

Adding Vertically
When adding vertically, be sure to correctly identify the terms of each polynomial. For example, the last term of the polynomial $2x^2 - 3$ is -3, not 3.

Polynomials are often used to represent measures of geometric figures.

TEST EXAMPLE

5 Which expression best describes the perimeter of the triangle?



A $(2x + 2)$ in.

B $(3x - 3)$ in.

C $(5x - 1)$ in.

D $(5x + 5)$ in.

Test-Taking Tip

Facts and Formulas
Many standardized tests provide a list of common geometry facts and formulas. Be sure to find this list before the test begins so you can refer to it easily.

Read the Test Item

The figure is a triangle. The perimeter of a triangle is the sum of the measures of its sides.

Solve the Test Item

Add the polynomials to find the perimeter.

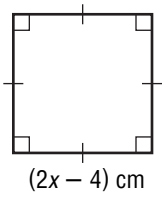
$x + (3x - 3) + (x + 2) = (x + 3x + x) + (-3 + 2)$ Group like terms.

$= 5x + (-1)$ Simplify.

The sum is $(5x - 1)$ inches. The answer is C.

CHECK Your Progress

c. Which expression best represents the perimeter of the square?



F $(2x - 4)$ cm

G $(8x + 16)$ cm

H $(8x - 4)$ cm

J $(8x - 16)$ cm

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CHECK Your Understanding

Add.

Examples 1, 2 (p. 646)

1. $\begin{array}{r} h + 3 \\ (+)\ 2h + 1 \\ \hline \end{array}$

2. $\begin{array}{r} 2b^2 + 6b + 9 \\ (+)\ b^2 + 2b - 7 \\ \hline \end{array}$

3. $\begin{array}{r} 6g^2 - g + 3 \\ (+)\ -2g^2 - 3g + 1 \\ \hline \end{array}$

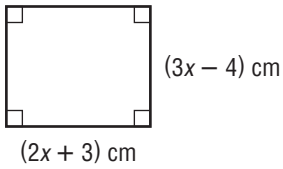
Examples 3, 4 (p. 647)

4. $(8f^2 - 3f) + (5f - 9)$

5. $(4t^2 + t + 1) + (3t^2 - 5)$

Example 5 (p. 647)

6. **TEST PRACTICE** Which expression best represents the perimeter of the rectangle?



A $(2x + 3)$ cm **C** $(5x - 1)$ cm

B $(5x + 7)$ cm **D** $(10x - 2)$ cm

Exercises

HOMEWORK HELP

For Exercises	See Examples
7–10, 15, 16, 21, 22	1, 2
11–14, 17–20	3, 4
32, 33	5

Add.

7. $\begin{array}{r} 5y + 6 \\ (+)\ 2y + 4 \\ \hline \end{array}$

8. $\begin{array}{r} 5p^2 + 3 \\ (+)\ 8p^2 + 1 \\ \hline \end{array}$

9. $\begin{array}{r} s^2 - s - 4 \\ (+)\ 4s^2 + 2s - 5 \\ \hline \end{array}$

10. $\begin{array}{r} k^2 + 6k - 2 \\ (+)\ 7k^2 - 3k - 1 \\ \hline \end{array}$

11. $\begin{array}{r} 4m^2 + m - 5 \\ (+)\ 3m^2 \quad + 9 \\ \hline \end{array}$

12. $\begin{array}{r} 8x^2 - 6x - 7 \\ (+)\ -4x^2 - 6x \\ \hline \end{array}$

13. $(4n^2 + 8) + (2n^2 - 5n)$

14. $(-6r - 2) + (r^2 + 9r)$

15. $(7j^2 + j + 1) + (j^2 - 5j - 2)$

16. $(4q^2 - 2q - 1) + (q^2 + 5q + 1)$

17. $(-2b^2 - 3b - 7) + (5b + 2)$

18. $(-3a^2 - 2a - 9) + (-3a^2 - 5a)$

19. $(5v^2 - v + 1) + (v^2 + v)$

20. $(6x^2 - 5x - 4) + (-x^2 - 9)$

21. **JOBS** Wei-Ling works at a grocery store after school and baby-sits on weekends. She makes the same hourly wage for both jobs. During one week, Wei-Ling worked 18 hours at the grocery store, and $9 was deducted for taxes. She worked 7 hours baby-sitting, and no taxes were deducted. Let x represent her hourly pay. Write a polynomial expression to represent Wei-Ling's total weekly take-home pay.

22. **BOARD GAMES** In a word game, players take turns forming words using tiles. A player receives a score based upon the point value of each letter and any extra points depending on the placement of the word on the board. Suppose you spell "sit" on your first turn and earn a double word score of 6 points. On your second turn, you spell "love" with a double letter score of 2 points. Each letter of "sit" and "love" are the same value. Let y represent the value of each letter. Write a polynomial expression that represents your total score after your second turn.

Add. Then evaluate each sum if $x = 6$, $y = 3$, and $z = -5$.

23. $(-3x + 4z) + (5y - 2z)$

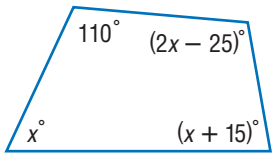
24. $(4x - 6y - 13z) + (-3x - 4y + 11z)$

25. $(5y^2 - 2) + (4y + 6)$

26. $(-6x^2 - 10z) + (6x^2 + 7z)$

EXTRA PRACTICE
See pages 726, 739.
Self-Check Quiz at tx.msmath3.com

GEOMETRY **For Exercises 27 and 28, use the figure at the right.**



27. Write an expression in simplest form for the sum of the measures of the angles.

28. The sum of the measures of the angles in any quadrilateral is 360°. Find the measure of each angle.

H.O.T. Problems

29. **CHALLENGE** If $(3a - 5b) + (2a + 3b) = 5a - 2b$, then what is $(5a - 2b) - (3a - 5b)$? Explain your reasoning.

30. **OPEN ENDED** Find two polynomials whose sum is $4x - 5y$.

31. **WRITING IN MATH** Which of the following methods would you use to add $5a^2 - 7a$ and $3a^2 + 2a - 1$? Explain your reasoning.

Horizontal	Vertical
$(5a^2 - 7a) + (3a^2 + 2a - 1)$	$5a^2 - 7a$ $(+)\ 3a^2 + 2a - 1$

TEST PRACTICE

32. Simplify the algebraic expression $(14n + m) + (n - 9m)$.

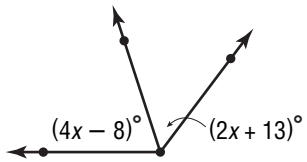
A $13n - 8m$

B $14n - 9m$

C $15n - 10m$

D $15n - 8m$

33. Which expression best represents the measure of angle ABC?



F $(2x + 13)°$

G $(4x - 5)°$

H $(6x + 5)°$

J $(6x + 21)°$

Spiral Review

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*. (Lesson 12-3)

34. $3t + 2s + s + 8t$

35. $4q^2 - q - 7 + 6q + 2$

36. **SKYDIVING** The distance d in feet a skydiver falls in t seconds is given by the function $d = 16t^2$. Graph this function and estimate how far a skydiver will fall in 5.5 seconds. (Lesson 12-2)

GET READY for the Next Lesson

PREREQUISITE SKILL **Rewrite each expression as an addition expression.** (Lessons 1-4 and 1-5)

37. $6 - 7$

38. $a^2 - 8$

39. $4x - 5y$

40. $(c + d) - 3c$

CHAPTER 12

Mid-Chapter Quiz

Lessons 12-1 through 12-4

Determine whether each equation or table represents a *linear* or *nonlinear* function. Explain. (Lesson 12-1)

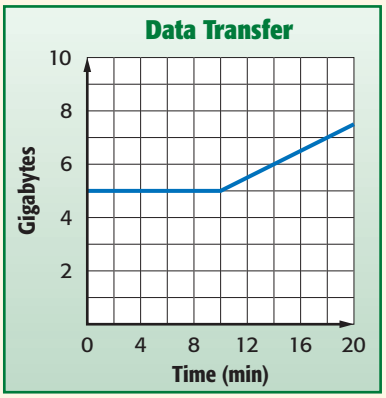
1. $3y = x$
2. $y = 5x^3 + 2$
3.

x	1	3	5	7
y	−5	−6	−7	−8

4.

x	−1	0	1	2
y	1	0	1	4

5. **LONG DISTANCE** The graph shows the amount of data transferred as a function of time. Is this a linear or nonlinear function? Explain your reasoning. (Lesson 12-1)



Graph each function. (Lesson 12-2)

6. $y = 2x^2$
7. $y = -x^2 + 3$
8. $y = 4x^2 - 1$
9. $y = -3x^2 + 1$

10. **AMUSEMENT PARK RIDES** Your height h feet above the ground t seconds after being released at the top of a free-fall ride is given by the function $h = -16t^2 + 200$. Graph this function. After about how many seconds will the ride be 60 feet above the ground? (Lesson 12-2)

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*. (Lesson 12-3)

11. $3x + 2 - 5x + 1$
12. $6a^2 + 5x - 2a^2$
13. $y^2 + 3y + 1 + 5y - 2y^2$
14. $3x^2 - 6x + 5x + 8$

15. **TEST PRACTICE** Simplify the algebraic expression $x^2 - 4x - 5x + 3 - 2x^2 + 9$. (Lesson 12-3)

A $3x^2 - 9x + 12$

B $2x^2$

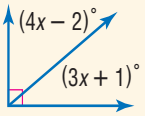
C $-2x^2 - 9x + 12$

D $-x^2 - 9x + 12$

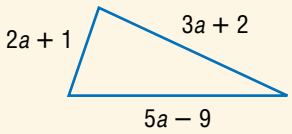
Add. (Lesson 12-4)

16. $(3a + 6) + (2a - 5)$
17. $(5y - 11) + (-3y - 3)$
18. $(3q^2 - 5) + (2q^2 - q)$
19. $(a^2 - 2a + 3) + (3a^2 - 5a + 6)$

20. **GEOMETRY** Find the measure of each angle in the figure. (Lesson 12-4)



21. **TEST PRACTICE** Which expression best represents the perimeter of $\triangle ABC$?



F $5a - 3$

G $6a + 9$

H $10a - 6$

J $11a + 12$

12-5 Subtracting Polynomials

Main IDEA

Subtract polynomials.

Preparation for TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions**, transform and solve equations, and factor **as necessary in problem situations. (B) Use the commutative, associative, and distributive properties to simplify algebraic expressions.**

REVIEW Vocabulary

additive inverse a number and its opposite (Lesson 1-4)

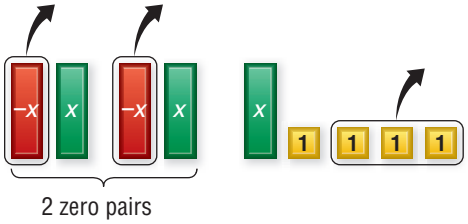
MINI Lab

You can use algebra tiles to find $(x + 4) - (-2x + 3)$.

STEP 1 Model the polynomial $x + 4$.

STEP 2 To subtract $-2x + 3$, you need to remove 2 negative x-tiles and 3 1-tiles.

STEP 3 Since there are no negative x-tiles to remove, add 2 zero pairs of x-tiles. Then remove 2 negative x-tiles and 3 1-tiles.



1. From the tiles that remain, find $(x + 4) - (-2x + 3)$.
2. Use algebra tiles to find $(2x^2 + 3x + 5) - (x^2 - x + 2)$.

To subtract two polynomials, subtract the like terms.

EXAMPLES Subtract Polynomials

Subtract.

1 $(7a + 5) - (3a + 4)$

$$\begin{array}{rl} 7a + 5 & \\ (-)\ 3a + 4 & \text{Align like terms.} \\ \hline 4a + 1 & \text{Subtract.} \end{array}$$

2 $(5x^2 + 3x + 4) - (3x^2 - 2)$

$$\begin{array}{rl} 5x^2 + 3x + 4 & \\ (-)\ 3x^2 \quad\quad - 2 & \text{Align like terms.} \\ \hline 2x^2 + 3x + 6 & \text{Subtract.} \end{array}$$

CHECK Your Progress

a. $(9y + 6) - (2y + 3)$

b. $(4n^2 + 5n + 6) - (4n - 1)$

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Recall that you can subtract a number by adding its *additive inverse*. You can also subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, find the opposite of each term.

Polynomial	Terms	Opposites	Additive Inverse
$x + 5$	$x, 5$	$-x, -5$	$-x - 5$
$-x^2 - 4x + 2$	$-x^2, -4x, 2$	$x^2, 4x, -2$	$x^2 + 4x - 2$

EXAMPLES Subtract Using the Additive Inverse

3 **Find $(4x + 9) - (7x - 2)$.**

The additive inverse of $7x - 2$ is $-7x + 2$.

$(4x + 9) - (7x - 2)$

$= (4x + 9) + (-7x + 2)$ To subtract $(7x - 2)$, add $(-7x + 2)$.

$= (4x - 7x) + (9 + 2)$ Group like terms.

$= -3x + 11$ Simplify by combining like terms.

4 **Find $(6y^2 - 5) - (-3y + 4)$.**

The additive inverse of $-3y + 4$ is $3y - 4$.

$$\begin{array}{rr} 6y^2 & -5 \\ (-) \quad -3y & +4 \end{array} \quad \rightarrow \quad \begin{array}{rr} 6y^2 & -5 \\ (+) \quad 3y & -4 \\ \hline 6y^2 + 3y & -9 \end{array}$$

Write the additive inverse.

Simplify.

CHECK Your Progress **Subtract.**

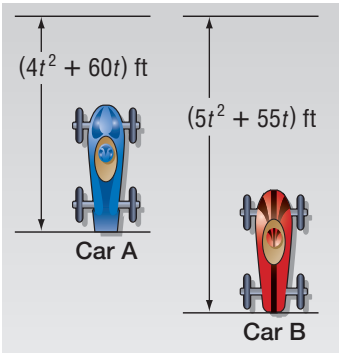
c. $(5p + 3) - (12p - 8)$

d. $(x^2 - 6x + 4) - (2x^2 - 7x - 1)$

Real-World EXAMPLE

5 **CARS** **Car A travels a distance of $4t^2 + 60t$ feet t seconds after the start of a soapbox derby. Car B travels $5t^2 + 55t$ feet. How far apart are the two cars 8 seconds after the start of the race?**

Write an expression for the difference of the distances traveled by each car.



$$\begin{array}{rr} 5t^2 & +55t \\ (-) \quad 4t^2 & +60t \end{array} \quad \rightarrow \quad \begin{array}{rr} 5t^2 & +55t \\ (+) \quad -4t^2 & -60t \\ \hline t^2 & -5t \end{array}$$

Distance of Car B

Distance of Car A

Now evaluate this expression for a time of 8 seconds.

$t^2 - 5t = (8)^2 - 5(8)$ Replace t with 8.

$= 64 - 40$ or 24 Simplify.

After 8 seconds, the cars are 24 feet apart.

CHECK Your Progress

e. **GEOMETRY** The length of a rectangle is $4x - 1$ centimeters, and the width is $2x + 3$ centimeters. How much longer is the length than the width if $x = 3$?

Tony Freeman/PhotoEdit

Real-World Career . . . .

How Does an Automotive Engineer Use Math?

An automotive engineer uses math to model a car's speed under different road conditions.

MathOnline

For more information, go to tx.msmath3.com.

CHECK Your Understanding

Examples 1, 2 (p. 651)

Subtract.

1. $5z + 2$
   $(-)\ 3z + 1$

2. $7c^2 + c + 5$
   $(-)\ 2c^2 \quad + 4$

3. $2m^2 + 6m + 8$
   $(-) \quad 3m - 1$

Examples 3, 4 (p. 652)

4. $(5n^2 - 2) - (2n - 1)$

5. $(r^2 + r - 1) - (2r^2 - r + 2)$

Example 5 (p. 652)

6. **SCHOOL** Mrs. Wilson gives extra points for every day a book report is turned in early and deducts the same number of points for every day it is late. Let d represent the points per day. On a 50-point book report, Ryan earned $45 + 2d$ points, and Amber earned $50 - 4d$ points. What is the difference in their scores if Mrs. Wilson adds or subtracts 5 points per day?

Exercises

HOMEWORK HELP

For Exercises	See Examples
7–12	1, 2
13–16	3, 4
17–18	5

Subtract.

7. $3x + 6$
   $(-)\ 2x + 5$

8. $9w + 15$
   $(-)\ 4w + 12$

9. $4u^2 + 3u + 2$
   $(-)\ 2u^2 \quad - 4$

10. $7y^2 + y + 6$
    $(-)\ 5y^2 \quad + 1$

11. $8g^2 + 8g + 5$
    $(-)\ 7g^2 + 5g + 1$

12. $10b^2 - 4b + 9$
    $(-)\ 5b^2 + b + 3$

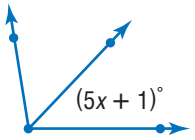
13. $(4m^2 - 8) - (-3m + 2)$

14. $(5k^2 - 7) - (9k + 13)$

15. $(c^2 - 2c + 1) - (c^2 + c - 5)$

16. $(3r^2 + r - 1) - (r^2 - r + 3)$

17. **GEOMETRY** The measure of $\angle ABC$ is $(12x - 8)°$. Write an expression that could be used to find the measure of $\angle ABD$. Then find the measure of $\angle ABD$ if $x = 15$.



18. **FAST FOOD** Khadijah ordered 3 burritos and 7 tacos from a fast-food drive through. She later discovered that she had been charged for 5 burritos and 5 tacos. If burritos cost b dollars and tacos cost t dollars, write an expression that could represent the amount Khadijah was overcharged. Then find the amount she was overcharged if tacos cost \$0.99 and burritos cost \$1.59.

Subtract. Then evaluate the difference if $x = -8$ *and* $y = 5$.

19. $(4x + 10) - (3x + 7)$

20. $(6y - 2) - (2y + 6)$

21. $(-3x - 8) - (y - 5)$

22. $(9x + 2y) - (8x - 4)$

23. $(x + 5y) - (-4x + 3y)$

24. $(-2x - y) - (-6x - 3y)$

EXTRA PRACTICE

See pages 726, 739.

Self-Check Quiz at tx.msmath3.com

25. **FUND-RAISER** Your club spends \$200 on a pizza fund-raiser kit. Each pizza costs you \$6.50 to make. You sell each pizza for \$10. Write an expression that you could use to model your profit from selling x pizzas. How much profit will you make if you sell 150 pizzas?

H.O.T. Problems

26. **CHALLENGE** Suppose A and B represent polynomials. If $A + B = 7x + 4$ and $A - B = 3x + 2$, find A and B.

27. **OPEN ENDED** Write two polynomials whose difference is $3x - 8$.

28. **FIND THE ERROR** Karen and Jacqui are finding $(3a^2 - 3a + 5) - (2a^2 + a - 1)$. Who is correct? Explain your reasoning.

Karen

$(3a^2 - 3a + 5) - (2a^2 + a - 1)$
$= (3a^2 - 3a + 5) + (-2a - a + 1)$
$= a^2 - 4a + 6$

$(3a^2 - 3a + 5) - (2a^2 + a - 1)$
$= (3a^2 - 3a + 5) + (-2a^2 + a - 1)$
$= a^2 - 2a + 4$

Jacqui

29. **WRITING IN MATH** Write a problem about a real-world situation in which you would subtract polynomials. Then solve the problem.

TEST PRACTICE

30. Simplify the algebraic expression $(6p + 2) - (p - 1)$.

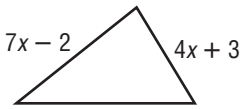
A $5p + 1$

B $5p - 1$

C $5p + 3$

D $5p - 3$

31. **GRIDDABLE** The perimeter of the triangle is $16x - 7$ units. Find the length of the missing side if $x = 2$.



Spiral Review

Add. (Lesson 12-4)

32. $(7b + 2) + (-5b + 3)$

33. $(6v^2 - 4) + (v - 1)$

34. $(t^2 - 8t) + (t^2 + 5)$

35. $(d^2 + d) + (2d - 1)$

SCHOOL **For Exercises 36 and 37, use the following information.** The drama club is selling flowers. The sales for the first two weeks are shown in the table. (Lesson 12-3)

Number of Flowers Sold		
Week	Carnations	Roses
1	54	38
2	65	42

36. The selling prices of a carnation and a rose are C and R, respectively. Write a polynomial expression for the total sales.

37. If carnations cost \$2 each and roses cost \$5 each, what were the total sales?

GET READY for the Next Lesson

PREREQUISITE SKILL **Write each expression using exponents.** (Lesson 2-9)

38. $3 \cdot 3 \cdot 3 \cdot 3$

39. $5 \cdot 4 \cdot 5 \cdot 5 \cdot 4$

40. $7 \cdot (7 \cdot 7)$

41. $(2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$

CONTENTS

12-6 Multiplying and Dividing Monomials

Main IDEA

Multiply and divide monomials.

Preparation for TEKS A.3 The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. **(A) Use symbols to represent unknowns and variables. A.11** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. **(A) Use patterns to generate the laws of exponents and apply them in problem-solving situations.**

REVIEW Vocabulary

powers In the expression x^5, which is read *x to the fifth power*, x is the base and 5 is the exponent. (Lesson 2-9)

GET READY for the Lesson

SCIENCE The pH of a solution describes its acidity. Neutral water has a pH of 7. Lemon juice has a pH of 2. Each one-unit decrease in the pH means that the solution is 10 times more acidic. So a pH of 8 is 10 times more acidic than a pH of 9.

pH	Times More Acidic Than a pH of 9	Written Using Powers
8	10	10^1
7	$10 \times 10 = 100$	$10^1 \times 10^1 = 10^2$
6	$10 \times 10 \times 10 = 1{,}000$	$10^1 \times 10^2 = 10^3$
5	$10 \times 10 \times 10 \times 10 = 10{,}000$	$10^1 \times 10^3 = 10^4$
4	$10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$	$10^1 \times 10^4 = 10^5$

1. Examine the exponents of the factors and the exponents of the products in the last column. What do you observe?

Exponents are used to show repeated multiplication. You can use this fact to help find a rule for multiplying powers with the same base.

$$3^2 \cdot 3^4 = \underbrace{(3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3)}_{\text{6 factors}} \text{ or } 3^6$$

(2 factors, 4 factors)

Notice the sum of the original exponents and the exponent in the final product. This relationship is stated in the following rule.

STUDY TIP

Common Error When multiplying powers, do not multiply the bases. $4^5 \cdot 4^2 = 4^7$, not 16^7.

KEY CONCEPT Product of Powers

Words To multiply powers with the same base, add their exponents.

Examples

Numbers	Algebra
$2^4 \cdot 2^3 = 2^{4+3}$ or 2^7	$a^m \cdot a^n = a^{m+n}$

EXAMPLES Multiply Powers

1 **Find $5^2 \cdot 5$. Express using exponents.**

$5^2 \cdot 5 = 5^2 \cdot 5^1$ $\quad 5 = 5^1$

$= 5^{2+1}$ $\quad$ The common base is 5.

$= 5^3$ $\quad$ Add the exponents.

Check $5^2 \cdot 5 = (5 \cdot 5) \cdot 5$

$= 5 \cdot 5 \cdot 5$

$= 5^3$ ✓

2 **Find $-3x^2(4x^5)$. Express using exponents.**

$-3x^2(4x^5) = (-3 \cdot 4)(x^2 \cdot x^5)$ Commutative and Associative Properties

$= (-12)(x^{2+5})$ The common base is x.

$= -12x^7$ Add the exponents.

CHECK Your Progress **Multiply. Express using exponents.**

a. $9^3 \cdot 9^2$ b. $y^4 \cdot y^9$ c. $-2m(-8m^5)$

There is also a rule for dividing powers that have the same base.

Concepts in MOtion
BrainPOP® tx.msmath3.com

KEY CONCEPT Quotient of Powers

Words To divide powers with the same base, subtract their exponents.

Examples

Numbers	Algebra
$\frac{3^7}{3^3} = 3^{7-3}$ or 3^4	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

EXAMPLES Divide Powers

3 **Divide $\frac{4^8}{4^2}$.**

$\frac{4^8}{4^2} = 4^{8-2}$ The common base is 4.

$= 4^6$ Simplify.

4 **Divide $\frac{n^9}{n^4}$.**

$\frac{n^9}{n^4} = n^{9-4}$ The common base is n.

$= n^5$ Simplify.

CHECK Your Progress **Divide. Express using exponents.**

d. $\frac{5^7}{5^4}$ e. $\frac{x^{10}}{x^3}$ f. $\frac{12w^5}{2w}$

Real-World EXAMPLE

5 **SOUND The loudness of a conversation is 10^6 times as intense as the loudness of a pin dropping, while the loudness of a jet engine is 10^{12} times as intense. How many times more intense is the loudness of a jet engine than the loudness of a conversation?**

To find how many times more intense, divide 10^{12} by 10^6.

$\frac{10^{12}}{10^6} = 10^{12-6}$ or 10^6 Quotient of Powers

The loudness of a jet engine is 10^6 or 1,000,000 times as intense.

CHECK Your Progress

g. **RIVERS** The Guadalupe River is 2^8 miles long. The Amazon River is almost 2^4 times as long. Find the length of the Amazon River.

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Gabe Palmer/CORBIS

Real-World Link
The decibel measure of the loudness of a sound is the exponent of its relative intensity multiplied by 10. A jet engine has a loudness of 120 decibels.

CHECK Your Understanding

Examples 1–4 (pp. 655–656)

Multiply or divide. Express using exponents.

1. $4^5 \cdot 4^3$
2. $n^2 \cdot n^9$
3. $-2a(3a^4)$
4. $\frac{7^6}{7}$
5. $\frac{y^8}{y^5}$
6. $\frac{9c^7}{3c^2}$

Example 5 (p. 656)

7. **ASTRONOMY** Venus is approximately 10^8 kilometers from the Sun. Saturn is more than 10^9 kilometers from the Sun. About how many times farther away from the Sun is Saturn than Venus?

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–17	1, 2
18–23	3, 4
24, 25	5

Multiply or divide. Express using exponents.

8. $6^8 \cdot 6^5$
9. $2^9 \cdot 2$
10. $n \cdot n^7$
11. $b^{13} \cdot b$
12. $2g \cdot 7g^6$
13. $(3x^8)(5x)$
14. $-4a^5(6a^5)$
15. $(8w^4)(-w^7)$
16. $(-p)(-9p^2)$
17. $-5y^3(-8y^6)$
18. $\frac{3^9}{3^2}$
19. $\frac{8^4}{8}$
20. $\frac{r^7}{r^2}$
21. $\frac{x^{14}}{x^8}$
22. $\frac{14n^6}{7n}$
23. $\frac{24k^3}{8k^2}$

24. **LIFE SCIENCE** A cell culture contains 2^6 cells. By the end of the day, there are 2^{10} times as many cells in the culture. How many cells are there in the culture by the end of the day?

25. **POPULATION** The continent of North America contains approximately 10^7 square miles of land. If the population doubles, there will be about 10^9 people on the continent. At that point, on average, how many people will occupy each square mile of land?

Multiply or divide. Express using exponents.

26. $xy^2(x^3y)$
27. $\frac{20a^5b}{4ab}$
28. $2^6 \cdot 2 \cdot 2^3$
29. $4a^2b^3(7ab^2)$
30. $\frac{16x^3y^2}{2x^2y}$
31. $x^3 \cdot x^9 \div x^5$
32. $(18m^7n^5) \div (9m^6n^2)$

ANALYZE TABLES **For Exercises 33 and 34, use information below and in the table.**
For each increase of one on the Richter scale, an earthquake's vibrations, or *seismic waves*, are 10 times greater.

Earthquake	Richter Scale Magnitude
San Francisco, 1906	8.3
Adana, Turkey, 1998	6.3

EXTRA PRACTICE
See pages 726, 739.
Self-Check Quiz at tx.msmath3.com

33. How many times greater are the seismic waves of an earthquake with a magnitude of 6 than an aftershock with a magnitude of 3?
34. How many times greater were the seismic waves of the 1906 San Francisco earthquake than the 1998 Adana earthquake?

H.O.T. Problems

35. **CHALLENGE** What is half of 2^{30}? Write using exponents.

36. **OPEN ENDED** Write a multiplication expression whose product is 4^{15} and a division expression whose quotient is 4^{15}.

37. **NUMBER SENSE** Is $\frac{2^{100}}{2^{99}}$ greater than, less than, or equal to 2? Explain your reasoning.

38. **WRITING IN MATH** Determine whether the following statement is *true* or *false*. Explain your reasoning or give a counterexample.

If you change the order in which you multiply two monomials, the product will be different.

TEST PRACTICE

39. Which expression is equivalent to $\frac{(8y^2)(3y^4)}{6y^5}$?

A $4y$

B $4y^3$

C $5y^6$

D $6y^4$

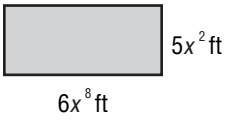
40. Which expression describes the area in square units of the rectangle below?

F $11x^{10}$

G $30x^{10}$

H $11x^{16}$

J $30x^{16}$



Spiral Review

Subtract. (Lesson 12-5)

41. $(3x + 8) - (5x + 1)$

42. $(5a - 2) - (3a - 4)$

43. $(6y^2 + 3y + 9) - (2y^2 + 8y + 1)$

SCHOOL For Exercises 44 and 45, use the following information and the table at the right.
Suppose your total number grade points for the first semester was $2A + 2B + C$ and your total for the second semester was $A + 3B + D$. (Lesson 12-4)

Grade	Grade Points
A	4
B	3
C	2
D	1
F	0

44. Add the polynomials to find your total grade points for the year.

45. Evaluate the sum by substituting each grade point value.

46. **FITNESS** The following are the number of sit-ups Carlos did every day for one week: 52, 57, 52, 33, 39, 43, 53. Find the mean, median, and mode of the data. (Lesson 9-4)

GET READY for the Next Lesson

47. **PREREQUISITE SKILL** A section of a theater is arranged so that each row has the same number of seats. You are seated in the 5th row from the front and the 3rd row from the back. If your seat is 6th from the left and 2nd from the right, how many seats are in this section of the theater? Use the *draw a diagram* strategy. (Lesson 4-4)

12-7 Problem-Solving Investigation

MAIN IDEA: Solve problems by making a model.

Targeted TEKS 8.14 The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. **(C) Select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture…to solve a problem.**

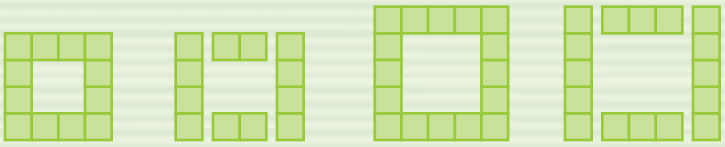
P.S.I. TEAM +

e-Mail: MAKE A MODEL

YOUR MISSION: Make a model to solve the problem.

THE PROBLEM: Determine if there are enough tables to make a 10 by 10 square arrangement.

Tonya: We have 35 square tables. We need to arrange them into a square that is open in the middle and has 10 tables on each side.

EXPLORE	You know Tonya has 35 square tables.
PLAN	Start by making models of a 4-by-4 square and of a 5-by-5 square. Then look for a pattern.
SOLVE	 4-by-4 square; 2 groups of 4 and 2 groups of 2; 5-by-5 square; 2 groups of 5 and 2 groups of 3 For a 10-by-10 square, Tonya needs $2 \cdot 10 + 2 \cdot 8$ or 36 tables. She has 35 tables, so she needs one more.
CHECK	You can estimate that Tonya needs 4×10 or 40 tables. But each of the corner tables is counted twice. So, she needs $40 - 4$ or 36 tables.

Analyze The Strategy

1. Draw a diagram showing another way the students could have grouped the tiles to solve this problem. Use a 4-by-4 square.
2. WRITING IN MATH **Write** a problem that can be solved by making a model. Describe the model. Then solve the problem.

Mixed Problem Solving

For Exercises 3–5, solve by making a model.

3. **STICKERS** In how many different ways can three rectangular stickers be torn from a sheet of 3 × 3 stickers so that all three stickers are still attached? Draw each arrangement.

4. **GEOMETRY** A 10-inch by 12-inch piece of cardboard has a 2-inch square cut out of each corner. Then the sides are folded up and taped together to make an open box. Find the volume of the box.

5. **GAMES** A computer game requires players to stack arrangements of five squares arranged to form a single shape. One arrangement is shown at the right. How many different arrangements are there if touching squares must border on a full side?

Use any strategy to solve Exercises 6–11. Some strategies are shown below.

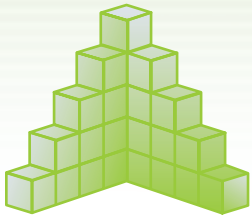
PROBLEM-SOLVING STRATEGIES
- Use the four-step plan.
- Draw a diagram.
- Act it out.
- Make a model.

6. **CAMP** The camp counselor lists 21 chores on separate pieces of paper and places them in a basket. The counselor takes one piece of paper, and each camper takes one as the basket is passed around the circle. There is one piece of paper left when the basket returns to the counselor. How many people could be in the circle if the basket goes around the circle more than once?

7. **PARKING** Parking space numbers consist of 3 digits. They are typed on a slip of paper and given to students at orientation. Tara accidentally read her number upside-down. The number she read was 795 more than her actual parking space number. What is Tara's parking space number?

8. **PETS** Mrs. Harper owns both cats and canaries. Altogether her pets have thirty heads and eighty legs. How many cats does she have?

TOWERS For Exercises 9 and 10, use the figure at the right.



9. How many cubes would it take to build this tower?

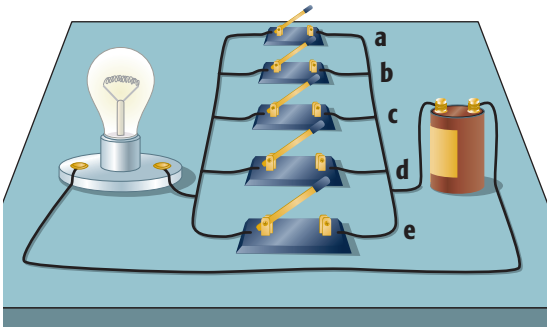
10. How many cubes would it take to build a similar tower that is 12 cubes high?

11. **CARS** Yesterday you noted that the mileage on the family car read 60,094.8 miles. Today it reads 60,099.1 miles. Was the car driven about 4 or 40 miles?

Select the Operation

For Exercises 12 and 13, select the appropriate operation(s) to solve the problem. Justify your selections(s) and solve the problem.

12. **SCIENCE** The light in the circuit will turn on if one or more switches are closed. How many combinations of open and closed switches will result in the light being on?



13. **HOBBIES** Lorena says to Angela, "If you give me one of your baseball cards, I will have twice as many baseball cards as you have." Angela answers, "If you give me one of your cards, we will have the same number of cards." How many cards does each girl have?

12-8 Multiplying Monomials and Polynomials

Main IDEA

Multiply monomials and polynomials.

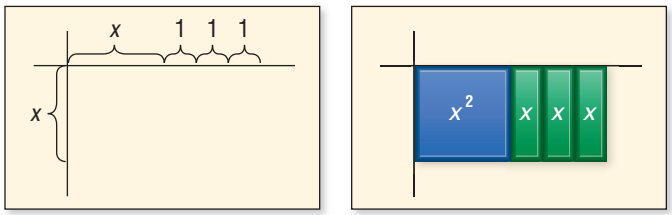
Preparation for TEKS A.3 The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. **(A) Use symbols to represent unknowns and variables. A.11** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. **(A) Use patterns to generate the laws of exponents and apply them in problem-solving situations.**

MINI Lab

Algebra tiles can be used to form a rectangle whose length and width each represent a polynomial. The area of the rectangle is the product of the polynomials. Use algebra tiles to find $x(x + 3)$.

STEP 1 Use algebra tiles to mark off a rectangle with a width of x and a length of $x + 3$ on a product mat.

STEP 2 Using the marks as a guide, fill in the rectangles with algebra tiles.



1. What is $x(x + 3)$ in simplest form?

Use algebra tiles to find each product.

2. $x(x + 4)$
3. $x(3x + 1)$
4. $2x(x + 3)$

In Lesson 10–1, you learned how to rewrite an expression like $4(x + 3)$ using the Distributive Property. This is an example of multiplying a polynomial by a monomial.

polynomial: $4(x + 3)$ — monomial: 4

$4(x + 3) = 4(x) + 4(3)$ Distributive Property

$= 4x + 12$ Simplify.

STUDY TIP

Check for Reasonableness You can use algebra tiles to check the product in Example 1.

Often, the Distributive Property and the definition of exponents are needed to simplify the product of a monomial and a polynomial.

EXAMPLES Use the Distributive Property

1 **Find $x(x + 2)$.**

$x(x + 2) = x(x) + x(2)$ Distributive Property

$= x^2 + 2x$ $x \cdot x = x^2$

2 **Find $-5y(y + 8)$.**

$-5y(y + 8) = -5y(y) + (-5y)(8)$ Distributive Property

$= -5y^2 + (-40y)$ $-5 \cdot y \cdot y = -5y^2$

$= -5y^2 - 40y$ Definition of subtraction

CHECK Your Progress **Multiply.**

a. $n(n - 9)$ b. $(10 + 2p)4p$ c. $-3x(6x - 4)$

EXAMPLES Use the Product of Powers Rule

3 **Find $3n(n^2 - 7)$.**

$3n(n^2 - 7) = 3n[n^2 + (-7)]$ Rewrite $n^2 - 7$ as $n^2 + (-7)$.

$= 3n(n^2) + 3n(-7)$ Distributive Property

$= 3n^3 + (-21n)$ $3n(n^2) = 3n^{1+2}$ or $3n^3$

$= 3n^3 - 21n$ Definition of subtraction

4 **Find $2x(x^2 + 3x - 5)$.**

$2x(x^2 + 3x - 5)$

$= 2x[x^2 + 3x + (-5)]$ Rewrite $x^2 + 3x - 5$ as $x^2 + 3x + (-5)$.

$= 2x(x^2) + 2x(3x) + 2x(-5)$ Distributive Property

$= 2x^3 + 6x^2 + (-10x)$ Simplify.

$= 2x^3 + 6x^2 - 10x$ Definition of subtraction

CHECK Your Progress **Multiply.**

d. $5y(4y^2 - 2y)$ e. $a(a^2 - 4a + 6)$ f. $-4p(2p^2 - p + 3)$

Real-World EXAMPLE

5 **TENNIS** **The length ℓ of a tennis doubles court is 6 feet longer than twice the width w. Write two expressions for the area of the court.**

First, write an expression for the length of the court. The length of the court is $2w + 6$. Next, write an expression for the court's area.

$A = (2w + 6)w$ $A = \ell \cdot w$; Replace ℓ with $2w + 6$

$A = (2w)w + (6)w$ Distributive Property

$A = 2w^2 + 6w$ Simplify.

Two expressions for the area of the court are $(2w + 6)w$ and $2w^2 + 6w$.

CHECK Your Progress

g. **CONSTRUCTION** When the Malones built their home, the width w of their driveway was one foot more than half the length ℓ. Write two expressions for the area of the Malone's driveway.

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Real-World Link
In an early version of modern tennis from the 19th century, the court was shaped like an hourglass.
Source: sportsknowhow.com

Mary Evans Picture Library

CHECK Your Understanding

Examples 1–4 (pp. 661–662)

Multiply.

1. $m(m + 5)$
2. $(2w - 1)(3w)$
3. $-4x(x + 1)$
4. $k(k^2 - 7)$
5. $g(2g^2 - 5g + 9)$
6. $3z(4z^2 - 6z - 10)$

Example 5 (p. 662)

7. **ART** The length ℓ of Leonardo da Vinci's *Mona Lisa* is 29 centimeters less than twice the width w. Write two expressions for the area of the *Mona Lisa*.

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–13	1, 2
14–19	3, 4
20, 21	5

Multiply.

8. $r(r + 9)$
9. $t(t - 4)$
10. $(3b - 2)(3b)$
11. $(5x + 1)(2x)$
12. $-6d(d + 5)$
13. $8w(1 - 7w)$
14. $4y(y^2 - 9)$
15. $-6g(2g^2 + 1)$
16. $t(t^2 + 5t + 9)$
17. $-n(3n^2 - 4n + 13)$
18. $-2r(4r^2 - r - 8)$
19. $11c(6c^2 - 8c + 1)$

20. **GARDENING** A square garden plot measures x feet on each side. Suppose you double the length of the plot and increase the width by 4 feet. Write two expressions for the area of the new plot.

21. **FUND-RAISER** Jim is selling T-shirts for a fund-raiser. He makes a profit of $2x + 5$ dollars for each T-shirt he sells. Write two expressions for the total profit Jim makes from selling x T-shirts.

Multiply.

22. $-a(7a - 8)$
23. $-5w(3w^2 - w - 4)$
24. $y(2y^3 - 6y)$
25. $a^2(a + 4)$
26. $-6n(n^3 - 7)$
27. $-t(-2t^3 + 5t^2 - 1)$

28. **FLAGS** The large flag that flew over Fort McHenry in Maryland during the War of 1812 inspired Francis Scott Key to write the national anthem. The length ℓ of this flag is 12 feet more than the width w. Write two expressions for the area of the flag. Find the area of the flag if it is 30 feet wide.

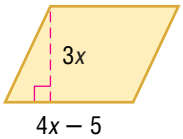
EXTRA PRACTICE

See pages 727, 739.

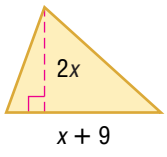
Self-Check Quiz at tx.msmath3.com

GEOMETRY Write an expression in simplest form for the area of each figure.

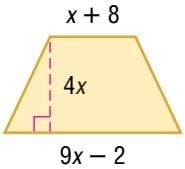
29.



30.



31.



H.O.T. Problems

32. **CHALLENGE** Use algebra tiles to find the following product of two *binomials*, or polynomials with two terms: $(x + 2)(x + 3)$.

33. **OPEN ENDED** Write a polynomial with three terms and a monomial that contains a variable with a power of 1. Then find their product.

34. **FIND THE ERROR** Christopher and Daniela are finding the product of $3x$ and $2x^2 - 3x + 8$. Who is correct? Explain your reasoning.

Christopher: $3x(2x^2 - 3x + 8) = 6x^2 - 9x + 24$

Daniela: $3x(2x^2 - 3x + 8) = 6x^3 - 9x^2 + 24x$

35. **WRITING IN MATH** Write a problem about a real-world situation in which you would multiply a monomial and a polynomial. Then solve the problem.

TEST PRACTICE

36. What is the product of $4x^2$ and $x^2 + 2x - 3$?

A $4x^2 + 8x - 12$

B $4x^4 + 8x^2 - 12x$

C $4x^4 + 8x^3 - 12x^2$

D $5x^2 + 6x + 1$

37. Which expression describes the area in square units of a rectangle that has a width of $6h$ and a length of $(4 - 3h)$?

F $24 - 18h$

G $24 + 18h$

H $24h - 18h^2$

J $24h + 18h^2$

Spiral Review

38. **DESKS** Mrs. Garcia is rearranging the 31 desks in her classroom. She wants to place the desks into groups of four. How many extra desks will she have? Use the *make a model* strategy. (Lesson 12-7)

Multiply or divide. Express using exponents. (Lesson 12-6)

39. $5^2 \cdot 5$

40. $\frac{11^8}{11^5}$

41. $3x^3 \cdot 9x^3$

42. $\frac{21a^5}{3a^4}$

43. **BUSINESS** Allison's income from selling x beaded bracelets is $6.50x$. Her expenses are $4x + 35$. Write an expression for her profit from selling x bracelets. (Lesson 12-5)

Cross-Curricular Project

Math and Economics

Getting Down to Business It's time to complete your project. Use the information and data you have gathered about the cost of materials and the feedback from your peers to prepare a video or brochure. Be sure to include a scatter plot with your project.

MathOnline **Cross-Curricular Project at** tx.msmath3.com

CONTENTS

CHAPTER 12

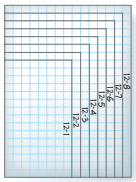
Study Guide and Review

STUDY TO GO Download Vocabulary Review from tx.msmath3.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Functions (Lessons 12-1 and 12-2)

- Linear functions have constant rates of change.
- Nonlinear functions do not have constant rates of change.
- Quadratic functions are functions in which the greatest power of the variable is 2.

Monomials (Lessons 12-3 and 12-6)

- A monomial is a number, a variable, or a product of numbers and/or variables.
- Product of Powers states when multiplying powers with the same base, add their exponents.
- Quotient of Powers states when dividing powers with the same base, subtract their exponents.

Polynomials (Lessons 12-3 to 12-5)

- A polynomial is an algebraic expression that is the sum or difference of one or more monomials.
- Add polynomials by combining like terms.
- Subtract polynomials by subtracting like terms.

Multiplying Monomials and Polynomials (Lesson 12-8)

- Use the Distributive Property to simplify the product of a monomial and a polynomial.

Key Vocabulary

monomial (p. 642)
polynomial (p. 642)
nonlinear function (p. 630)
quadratic function (p. 637)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. A polynomial is the sum or <u>difference</u> of 1 or more monomials.
2. The expression $x^2 - 3x$ is an example of a <u>monomial</u>.
3. A <u>nonlinear</u> function has a constant rate of change.
4. A quadratic function is a function whose greatest power is <u>2</u>.
5. The <u>additive</u> inverse of $9y^2 - 5y + 2$ *is* $-9y^2 + 5y - 2$.
6. To <u>multiply</u> two polynomials, you combine like terms.
7. The product of $3x$ and $x^2 + 3x$ will have <u>three</u> terms.
8. A quadratic function is a <u>nonlinear</u> function.
9. The graph of a linear function is a <u>curve</u>.
10. To divide powers with the same base, <u>subtract</u> the exponents.
11. Use the <u>Associative Property</u> to simplify the product of a monomial and a polynomial.
12. The Quotient of Powers states when dividing powers with the same base, <u>subtract</u> their exponents.

Lesson-by-Lesson Review

12-1 Linear and Nonlinear Functions (pp. 630–635)

Determine whether each equation or table represents a *linear* or *nonlinear* function. Explain.

13. $y - 4x = 1$

14. $y = x^2 + 3$

15. **READING** Use the table to determine whether the number of pages read is a linear function of the time spent reading. Explain.

Time (h)	2	3	4	5
Pages Read	98	147	199	248

Example 1 **Determine whether the table represents a *linear* or *nonlinear* function.**

x	y
−2	−3
−1	−1
0	1
1	3

As x increases by 1, y increases by 2. The rate of change is constant, so this function is linear.

12-2 Graphing Quadratic Functions (pp. 637–641)

Graph each function.

16. $-4x^2$

17. $y = x^2 + 4$

18. $y = -2x^2 + 1$

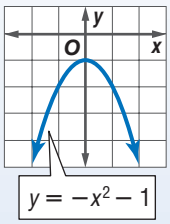
19. $y = 3x^2 - 1$

20. **SCIENCE** A ball is dropped from the top of a 36-foot tall building. The quadratic equation $d = -16t^2 + 36$ models the distance d in feet the ball is from the ground at time t seconds. Graph the function. Then use your graph to find how long it takes for the ball to reach the ground.

Example 2 **Graph $y = -x^2 - 1$.**

Make a table of values. Then plot and connect the ordered pairs with a smooth curve.

x	$y = -x^2 - 1$	(x, y)
−2	$-(-2)^2 - 1$	(−2, −5)
−1	$-(-1)^2 - 1$	(−1, −2)
0	$-(0)^2 - 1$	(0, −1)
1	$-(1)^2 - 1$	(1, −2)
2	$-(2)^2 - 1$	(2, −5)



12-3 Simplifying Polynomials (pp. 642–645)

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.

21. $3a - b - 7a + 2 + 4b$

22. $8x - y + 1$

23. $3n^2 + 7n - 4n^2 + n$

24. $-5y^2 + 4y - y + 8$

Example 3

Simplify $8a^2 - 5a + 6 - 9a^2 - 6$.

$$\begin{aligned} &8a^2 - 5a + 6 - 9a^2 - 6 \\ &= 8a^2 + (-5a) + 6 + (-9a^2) + (-6) \\ &= [8a^2 + (-9a^2)] + (-5a) + [6 + (-6)] \\ &= -1a^2 + (-5a) + 0 \\ &= -a^2 - 5a \end{aligned}$$

Mixed Problem Solving
For mixed problem-solving practice, see page 739.

12-4 Adding Polynomials (pp. 646–649)

Add.

25. $(4d + 1) + (8d + 6)$
26. $(3a^2 + 6a) + (2a^2 - 5a)$
27. $(b^2 - 2b + 4) + (2b^2 + b - 8)$
28. **WINDOWS** A window has a length of $3x - 5$ inches and a width of $x + 7$ inches. Write an expression to represent the perimeter of the window.

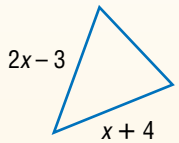
Example 4 **Find $(3x^2 - 2) + (2x^2 + 5)$.**

$$(3x^2 - 2) + (2x^2 + 5)$$
$$= (3x^2 + 2x^2) + (-2 + 5)$$
$$= 5x^2 + 3$$

12-5 Subtracting Polynomials (pp. 651–654)

Subtract.

29. $(7g + 2) - (5g + 1)$
30. $(3c - 7) - (-3c + 4)$
31. $(7p^2 + 2p - 5) - (4p^2 + 6p - 2)$
32. $(6k^2 - 3) - (k^2 - 5k - 2)$
33. **GEOMETRY** The perimeter of the triangle at the right is $4x$. Write an expression for its missing length.



Example 5 **Find $(5x - 1) - (6x + 4)$.**

To subtract $6x + 4$, add $-6x - 4$.

$$(5x - 1) - (6x + 4) = (5x - 1) + (-6x - 4)$$
$$= [5x + (-6x)] + [-1 + (-4)]$$
$$= -1x + (-5)$$
$$= -x - 5$$

Example 6 **Find $(-3x + 7) - (-5x - 11)$.**

To subtract $-5x - 11$, add $5x + 11$.

$$(-3x + 7) - (-5x - 11)$$
$$= (-3x + 7) + (5x + 11)$$
$$= [-3x + (5x)] + [7 + 11]$$
$$= 2x + 18$$

12-6 Multiplying and Dividing Monomials (pp. 655–658)

Multiply or divide. Express using exponents.

34. $4 \cdot 4^5$
35. $x^6 \cdot x^2$
36. $-9y^2(-4y^9)$
37. $\frac{5^9}{5^2}$
38. $\frac{n^5}{n}$
39. $\frac{21c^{11}}{-7c^8}$
40. **AREA** The area of the family room is 3^4 square feet. The area of the kitchen is 4^3 square feet. What is the difference in area between the two rooms?

Example 7 **Find $3a^3 \cdot 4a^7$.**

$$3a^3 \cdot 4a^7 = (3 \cdot 4)a^{3+7}$$
$$= 12a^{10}$$

Example 8 **Find $\frac{6^8}{6^3}$.**

$$\frac{6^8}{6^3} = 6^{8-3}$$
$$= 6^5$$

CHAPTER 12

Study Guide and Review

12-7 PSI: Make a Model (pp. 659–660)

Solve the problem by using the *make a model* strategy.

41. **POSTCARD** Sydney has a postcard that measures 5 inches by 3 inches. She decides to frame it, using a frame that is $1\frac{3}{4}$ inches wide. What is the perimeter of the framed postcard?

42. **MAGAZINES** A book store arranges its best-seller magazines in the front window. In how many different ways can 5 best-seller magazines be arranged in a row?

43. **BLUEPRINTS** Mr. Franz is looking at the blueprints for his new garage. On the blueprints, 1 foot $= \frac{3}{4}$ inch. If the garage measures 24 feet by 18 feet, what are the dimensions of the garage on the blueprints?

Example 9 **Cans of motor oil are being displayed in the shape of a pyramid. The top layer has 2 cans in it. If one more can is added to each layer and there are 6 layers in the pyramid, how many cans are there in the display? Solve by making a model.**

Make a model to solve the problem.

So, based on the model there are 27 cans.

12-8 Multiplying Monomials and Polynomials (pp. 661–664)

Multiply.

44. $a(a - 7)$
45. $(3y + 4)(3y)$
46. $-4n(n - 2)$
47. $p(p^2 - 6)$
48. $x(2x^2 + x - 5)$
49. $-2k(5k^2 - 3k + 8)$
50. $(5z - 4)(7z)$
51. $y(5y + 9)$

52. **SPORTS** The length of a football field is 120 feet less than 3 times the width. Write two expressions for the area of the football field.

53. **PROFIT** A cell phone store is having a sale on accessories. The store makes a profit of $3x + 2$ dollars for every accessory they sell. Write two expressions for the total profit the store makes from selling x accessories.

Example 10 **Find $-2x(5x + 3)$.**

$$\begin{aligned} -2x(5x + 3) &= -2x(5x) + (-2x)(3) \\ &= -10x^2 + (-6x) \\ &= -10x^2 - 6x \end{aligned}$$

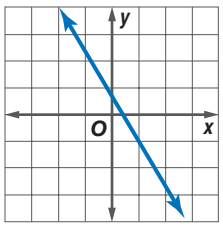
Example 11 **Find $4x(8x - 5)$.**

$$\begin{aligned} 4x(8x - 5) &= 4x(8x) + (4x)(-5) \\ &= 32x^2 + (-20x) \\ &= 32x^2 - 20x \end{aligned}$$

CHAPTER 12

Practice Test

Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

1. 

2. $2x = y$

3.

x	−3	−1	1	3
y	2	10	18	26

Graph each function.

4. $y = \frac{1}{2}x^2$

5. $y = -2x^2 + 3$

6. **BUSINESS** The function $p = 60 + 2d^2$ models the profit made by a manufacturer of digital audio players. Graph this function. Then use your graph to estimate the profit earned after making 20 players.

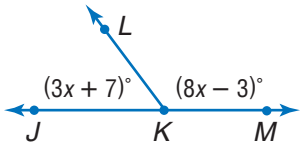
Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.

7. $-6x + 4y - 8 + y - 1$
8. $2a^2 + 4a + 3a^2 + 5a$
9. $7p^2 + 10p + 1$
10. **TEST PRACTICE** Simplify the algebraic expression $(3x + 5) - (6x - 4)$.

A $9x + 9$
B $9x - 1$
C $-3x + 9$
D $-3x - 1$

Add or subtract.

11. $(4c^2 + 2) + (-4c^2 + 1)$
12. $(-x^2 + 2x - 5) + (4x^2 - 6x)$
13. $(9z^2 - 3z) - (5z^2 + 8z)$
14. $(5n^2 - 4n + 1) - (4n - 5)$
15. **GEOMETRY** Write an expression for the measure of $\angle JKM$. Then find the value of x.



16. **CRAFTS** Martina is making cube-shaped gift boxes from decorative cardboard. Each side of the cube is to be 6 inches long, and there is a $\frac{1}{2}$-inch overlap on each side. How much cardboard does Martina need to make each box?

Multiply or divide. Express using exponents.

17. $15^3 \cdot 15^5$
18. $-5m^6(-9m^8)$
19. $\frac{3^{15}}{3^7}$
20. $\frac{-40w^8}{8w}$

Multiply.

21. $8n(n + 3)$
22. $g^3(6g - 5)$
23. $-4x(3x^2 - 6x + 8)$
24. **TEST PRACTICE** Which expression is equivalent to $\frac{(12x^4)(4x^3)}{8x^5}$?

F $12x^7$
G $12x^2$
H $6x^4$
J $6x^2$

25. **LANDSCAPING** The length of a flowerbed is 2 meters less than three times its width w. Write two expressions for the area of the flowerbed.

CHAPTER 12

Texas Test Practice
Cumulative, Chapters 1–12

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. The expression $n(n + 1) - 2$ describes a pattern of numbers. If n represents a number's position in the sequence, which pattern of numbers is generated by the expression?

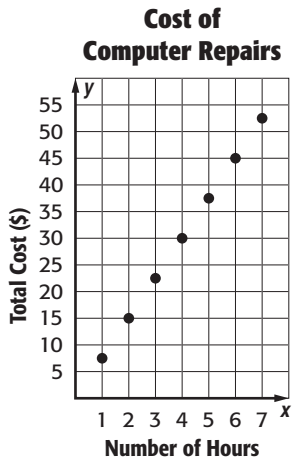
   A 0, 4, 8, 12, 16, …

   B 1, 3, 8, 14, 22, …

   C 2, 5, 12, 21, 35, …

   D 0, 4, 10, 18, 28, …

2. The scatter plot below shows the cost of computer repairs in relation to the number of hours the repair takes. Based on the information in the scatter plot, which statement is a valid conclusion?



   F As the length of time increases, the cost of the repair increases.

   G As the length of time increases, the cost of the repair stays the same.

   H As the length of time decreases, the cost of the repair increases.

   J As the length of time increases, the cost of the repair decreases.

3. The equation $c = 0.8t$ represents c, the cost of t tickets on a ferry. Which table contains values that satisfy this equation?

   A

Cost of Ferry Tickets				
t	1	2	3	4
c	\$0.80	\$1.00	\$1.20	\$1.40

   B

Cost of Ferry Tickets				
t	1	2	3	4
c	\$0.80	\$1.60	\$2.40	\$3.20

   C

Cost of Ferry Tickets				
t	1	2	3	4
c	\$0.75	\$1.50	\$2.25	\$3.00

   D

Cost of Ferry Tickets				
t	1	2	3	4
c	\$1.80	\$2.60	\$3.40	\$4.20

4. Shanelle purchased a new computer for \$1,099 and a computer desk for \$699 including tax. She plans to pay the total amount in 24 equal monthly payments. What is a reasonable amount for each monthly payment?

   F \$50 **H** \$150

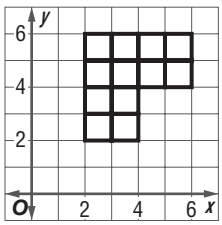
   G \$75 **J** \$1,800

TEST-TAKING TIP

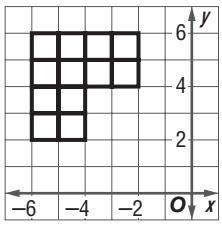
Question 4 You can often use estimation to eliminate incorrect answers. In this question, Shanelle's total spent can be estimated by adding \$1,100 and \$700, then dividing by 24. The sum of \$1,100 and \$700 is \$1,800 before dividing by 24, so choice J can be eliminated.

5. **GRIDDABLE** A car used 4.2 gallons of gasoline to travel 126 miles. How many gallons of gasoline would be needed to travel 195 miles?

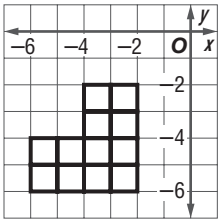
6. Which coordinate grid shows the reflection of the figure across the y-axis?



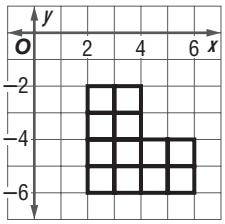
A



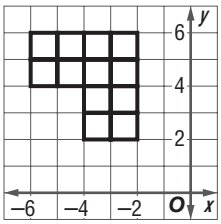
C



B



D



7. The diameter of a Jupiter is about 89,000 miles. Which expression represents this number in scientific notation?

F 89×10^3

G 8.9×10^{-4}

H 8.9×10^4

J 0.89×10^5

8. The area of a rectangle is $30m^{11}$ square feet. If the length of the rectangle is $6m^4$ feet, what is the width of the rectangle?

A $5m^7$ ft

B $24m^7$ ft

C $36m^{15}$ ft

D $180m^{15}$ ft

9. Kim has quiz scores of 90, 80, 85, 95, 90, 95, 95, and 75 this quarter. Which measure of data would give Kim the greatest quiz score?

F mean

G median

H mode

J range

10. Which function includes the data set $\{(-2, 4), (1, 7), (3, -1)\}$?

A $y = 2x^2$

B $y = \frac{x^2}{2}$

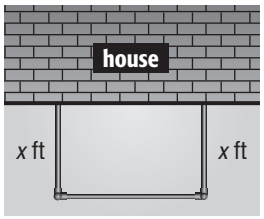
C $y = x^2 + 2$

D $-x^2 + 8$

Pre-AP

Record your answers on a sheet of paper. Show your work.

11. You have 40 feet of fencing to make a rectangular kennel for your dog. You will use your house as one side.



a. Write an algebraic expression for the kennel's length.

b. Write an algebraic expression in simplest form for the area of the kennel.

c. Write the area A of the kennel as a function of its width x.

d. Make a table of values and graph the function you wrote in part c.

e. Use your graph to determine the width that produces a kennel with the greatest area.

NEED EXTRA HELP?											
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson...	10-4	11-8	11-1	5-5	4-3	6-6	2-10	12-6	9-4	12-8	12-8
For Help with Test Objective...	2	5	2	1	5	4	1	3	5	2	2