## \& UNIT 1 <br> Number and Operations

Rational Numbers<br>+ Titers 7.2, 7.3.A, 7.3.B



Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters within found words to answer the riddle at the bottom of the page.

## 1. NIOARLTA MURNEB


2. TES
3. BTSEUS
4. PIRGENAET CMSEADIL

5. EADITIVD SENEIRV

6. TIIRANGTNEM SAELIDMC


1. Any number that can be written as a ratio of two integers. (Lesson 1-1)
2. A group of items. (Lesson 1-2)
3. A set that is contained within another set. (Lesson 1-2)
4. Decimals in which one or more digits repeat infinitely. (Lesson 1-1)
5. The opposite of any number. (Lesson 1-3)
6. Decimals that have a finite number of digits. (Lesson 1-1)

Q: Why were the two fractions able to settle their differences peacefully?
A: They were both $\qquad$ !

## Rational Numbers



## Are

Complete these exercises to review skills you will need for this chapter.

Multiply Fractions

EXAMPLE $\quad \frac{3}{8} \times \frac{4}{9} \quad \frac{3}{8} \times \frac{4}{9}=\frac{1}{8} \times \frac{1}{8} \times \frac{1}{9} \quad$ Divide by the common factors.

$$
=\frac{1}{6} \quad \text { Simplify } .
$$

Multiply. Write the product in simplest form.

1. $\frac{9}{14} \times \frac{7}{6}$
2. $\frac{3}{5} \times \frac{4}{7}$ $\qquad$ 3. $\frac{11}{8} \times \frac{10}{33}$
3. $\frac{4}{9} \times 3$
$\qquad$

## Operations with Fractions

EXAMPLE $\quad \frac{2}{5} \div \frac{7}{10}=\frac{2}{5} \times \frac{10}{7} \quad$ Multiply by the reciprocal of the $=\frac{2}{5} \times \frac{{ }^{2}}{7} \quad$ Divide by the common factors.
$=\frac{4}{7} \quad$ Simplify.

## Divide.

5. $\frac{1}{2} \div \frac{1}{4}$ $\qquad$ 6. $\frac{3}{8} \div \frac{13}{16}$ $\qquad$ 7. $\frac{2}{5} \div \frac{14}{15}$ $\qquad$ 8. $\frac{4}{9} \div \frac{16}{27}$ $\qquad$
6. $\frac{3}{5} \div \frac{5}{6}$ $\qquad$ 10. $\frac{1}{4} \div \frac{23}{24}$ $\qquad$ 11. $6 \div \frac{3}{5}$ $\qquad$ 12. $\frac{4}{5} \div 10$ $\qquad$

## Order of Operations

EXAMPLE $50-3(3+1)^{2} \quad$ To evaluate, first operate within parentheses.
$50-3(4)^{2} \quad$ Next simplify exponents.
$50-3(16) \quad$ Then multiply and divide from left to right.
50-48 Finally add and subtract from left to right.
2
Evaluate each expression.
13. $21-6 \div 3$ $\qquad$ 14. $18+(7-4) \times 3$ $\qquad$ 15. $5+(8-3)^{2}$ $\qquad$
16. $9+18 \div 3+10$ $\qquad$ 17. $60-(3-1)^{4} \times 3$ $\qquad$ 18. $10-16 \div 4 \times 2+6$ $\qquad$

## Reading Start-Up

## Visualize Vocabulary

## Use the $\checkmark$ words to complete the graphic. You can put more than one word in each section of the triangle.



## Understand Vocabulary

Complete the sentences using the preview words.

1. A group of items is a $\qquad$ . A set contained within another set is a $\qquad$ .
2. The $\qquad$ of a number is the same distance from 0 on a number line as the original number, but on the other side of 0 .
3. A $\qquad$ can be expressed as a ratio of two integers.

## Active Reading

Layered Book Before beginning the module, create a layered book to help you learn the concepts in this module. At the top of the first flap, write the title of the module, "Rational Numbers." Label the other flaps "Adding," "Subtracting,""Multiplying," and "Dividing." As you study each lesson, write important ideas, such as vocabulary and formulas, on the appropriate flap.


## MODULE 1

## Unpacking the Tfus

Understanding the TEKS and the vocabulary terms in the TEKS will help you know exactly what you are expected to learn in this module.

## teks 7.3.A

Add, subtract, multiply, and divide rational numbers fluently.

## Key Vocabulary

 rational number (número racional)Any number that can be expressed as a ratio of two integers.

## What It Means to You

You will add, subtract, multiply, and divide rational numbers.

## UNPACKING EXAMPLE 7.3.A

$$
\begin{array}{ll}
15 \cdot \frac{2}{3}-12 \div 1 \frac{1}{3} & \\
\frac{15}{1} \cdot \frac{2}{3}-\frac{12}{1} \div \frac{4}{3} & \begin{array}{l}
\text { Write the whole numbers and } \\
\frac{15}{1} \cdot \frac{2}{3}-\frac{12}{1} \cdot \frac{3}{4}
\end{array} \\
\frac{\text { mixed number as fractions. }}{} \quad \begin{array}{l}
\text { Convert division to multiplication } \\
\text { by the reciprocal. } \\
1 \cdot \frac{3}{1}
\end{array}-\frac{12^{3} \cdot 3}{1 \cdot 4} & \text { Simplify. } \\
\frac{10}{1}-\frac{9}{1}=10-9=1 & \begin{array}{l}
\text { Multiply numerators. Multiply } \\
\text { denominators. }
\end{array}
\end{array}
$$

Apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers.


## What It Means to You

You will solve real-world and mathematical problems involving the four operations with rational numbers.

## UNPACKING EXAMPLE 7.3.B

In 1954, the Sunshine Skyway Bridge toll for a car was $\$ 1.75$. In 2012, the toll was $\frac{5}{7}$ of the toll in 1954. What was the toll in 2012?

$$
\begin{aligned}
1.75 \cdot \frac{5}{7} & =1 \frac{3}{4} \cdot \frac{5}{7} & & \text { Write the decimal as a fraction. } \\
& =\frac{7}{4} \cdot \frac{5}{7} & & \text { Write the mixed number as an } \\
& =\frac{175 \cdot 5}{4 \cdot 7} & & \text { Simplify. } \\
& =\frac{5}{4}=1.25 & & \begin{array}{l}
\text { Convert the improper fraction. } \\
\text { a decimal. }
\end{array}
\end{aligned}
$$

The Sunshine Skyway Bridge toll for a car was \$1.25 in 2012.

# LEs5on Rational Numbers and Decimals 

## EXPLORE ACTIVITY

## Describing Decimal Forms of Rational Numbers

A rational number is a number that can be written as a ratio of two integers $a$ and $b$, where $b$ is not zero. For example, $\frac{4}{7}$ is a rational number, as is 0.37 because it can be written as the fraction $\frac{37}{100}$.

A Use a calculator to find the equivalent decimal form of each fraction.
Remember that numbers that repeat can be written as $0.333 \ldots$ or $0 . \overline{3}$.

| Fraction | $\frac{1}{4}$ | $\frac{5}{8}$ | $\frac{2}{3}$ | $\frac{2}{9}$ | $\frac{12}{5}$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Decimal <br> Equivalent |  |  |  |  |  | 0.2 | 0.875 |

B Now find the corresponding fraction of the decimal equivalents given in the last two columns in the table. Write the fractions in simplest form.

C Conjecture What do you notice about the digits after the decimal point in the decimal forms of the fractions? Compare notes with your neighbor and refine your conjecture if necessary.

## Reflect

1. Consider the decimal 0.101001000100001000001 .... Do you think this decimal represents a rational number? Why or why not?
2. Do you think a negative sign affects whether or not a number is a rational number? Use $-\frac{8}{5}$ as an example.
3. Do you think a mixed number is a rational number? Explain.
$\qquad$
$\qquad$
$\qquad$

## Writing Rational Numbers as Decimals

You can convert a rational number to a decimal using long division. Some decimals are terminating decimals because the decimals come to an end. Other decimals are repeating decimals because one or more digits repeat infinitely.

## EXAMPLE 1

## Write each rational number as a decimal.

(A) $\frac{5}{16}$

Divide 5 by 16 .
Add a zero after the decimal point. Subtract 48 from 50.
Use the grid to help you complete the long division.

Add zeros in the dividend and continue dividing until the remainder is 0 .

The decimal equivalent of $\frac{5}{16}$ is 0.3125 .
B $\frac{13}{33}$


Divide 13 by 33 .
Add a zero after the decimal point.
Subtract 99 from 130.
Use the grid to help you complete the long division.

You can stop dividing once you discover a repeating pattern in the quotient.

Write the quotient with its repeating pattern and indicate that the repeating numbers continue.

The decimal equivalent of $\frac{13}{33}$ is $0.3939 \ldots$,


Mathematical Processes
Do you think that decimals that have repeating patterns always have the same number of digits in their pattern? Explain.

Write each rational number as a decimal.
4. $\frac{4}{7}$ $\qquad$ 5. $\frac{1}{3}$
6. $\frac{9}{20}$

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## Writing Mixed Numbers as Decimals

You can convert a mixed number to a decimal by rewriting the fractional part of the number as a decimal.

## EXAMPLE 2 <br> 

Shawn rode his bike $6 \frac{3}{4}$ miles to the science museum. Write $6 \frac{3}{4}$ as a decimal.


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STEP 1 Rewrite the fractional part of the number as a decimal.

| 0.75 |  |
| ---: | :--- |
| $4 \longdiv { 3 . 0 0 }$ | Divide the numerator by |
| $\frac{-28}{20}$ |  |
| $\frac{-20}{0}$ |  |

STEP 2 Rewrite the mixed number as the sum of the whole part and the decimal part.

$6 \frac{3}{4}=6+\frac{3}{4}$

$$
=6+0.75
$$

$\div \quad=6.75$

## YOUR TURN

7. Yvonne made $2 \frac{3}{4}$ quarts of punch. Write $2 \frac{3}{4}$ as a decimal. $2 \frac{3}{4}=$ $\qquad$ Is the decimal equivalent a terminating or repeating decimal
8. Yvonne bought a watermelon that weighed $7 \frac{1}{3}$ pounds. Write $7 \frac{1}{3}$ as a decimal. $7 \frac{1}{3}=$ $\qquad$
Is the decimal equivalent a terminating or repeating decimal?

## Guided Practice

Write each rational number as a decimal. Then tell whether each decimal
is a terminating or a repeating decimal. (Explore Activity and Example 1)

1. $\frac{3}{5}=$
$\qquad$
2. $\frac{25}{99}=$ $\qquad$ 5. $\frac{7}{9}=$ $\qquad$ 6. $\frac{9}{25}=$ $\qquad$
3. $\frac{1}{25}=$ $\qquad$ 8. $\frac{25}{176}=$ $\qquad$ 9. $\frac{12}{1,000}=$ $\qquad$
$\qquad$

Write each mixed number as a decimal. (Example 2)
10. $11 \frac{1}{6}=$ $\qquad$
11. $2 \frac{9}{10}=$ $\qquad$
12. $8 \frac{23}{100}=$ $\qquad$
13. $7 \frac{3}{15}=$ $\qquad$ 14. $54 \frac{3}{11}=$ $\qquad$
$\qquad$
16. Maggie bought $3 \frac{2}{3} \mathrm{lb}$ of apples to make some apple pies. What is the weight of the apples written as a decimal? (Example 2)
$3 \frac{2}{3}=$ $\qquad$
17. Harry's dog weighs $12 \frac{7}{8}$ pounds. What is the weight of Harry's dog written as a decimal? (Example 2)
$12 \frac{7}{8}=$ $\qquad$

## ESSENTIAL QUESTION CHECK-IN

18. Tom is trying to write $\frac{3}{47}$ as a decimal. He used long division and divided until he got the quotient 0.0638297872 , at which point he stopped. Since the decimal doesn't seem to terminate or repeat, he concluded that $\frac{3}{47}$ is not rational. Do you agree or disagree? Why?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

### 1.1 Independent Practice



## Use the table for 19-23. Write each ratio in the form and then as a decimal. Tell whether each decimal is a terminating or a repeating decimal.

19. basketball players to football players
20. hockey players to lacrosse players
21. polo players to football players

| Team Sports |  |
| :--- | :---: |
| Sport | Number of <br> Players |
| Baseball | 9 |
| Basketball | 5 |
| Football | 11 |
| Hockey | 6 |
| Lacrosse | 10 |
| Polo | 4 |
| Rugby | 15 |
| Soccer | 11 |

22. lacrosse players to rugby players
$\qquad$
23. football players to soccer players
$\qquad$
24. Look for a Pattern Beth said that the ratio of the number of players
 in any sport to the number of players on a lacrosse team must always be a terminating decimal. Do you agree or disagree? Why?
$\qquad$
$\qquad$
$\qquad$
25. Yvonne bought $4 \frac{7}{8}$ yards of material to make a dress.
a. What is $4 \frac{7}{8}$ written as an improper fraction? $\qquad$
b. What is $4 \frac{7}{8}$ written as a decimal? $\qquad$
c. Communicate Mathematical Ideas If Yvonne wanted to make 3 dresses that use $4 \frac{7}{8}$ yd of fabric each, explain how she could use estimation to make sure she has enough fabric for all of them.
$\qquad$
$\qquad$
$\qquad$
26. Vocabulary A rational number can be written as the ratio of one
$\qquad$ to another and can be represented by a repeating or $\qquad$ decimal.
27. Problem Solving Marcus is $5 \frac{7}{24}$ feet tall. Ben is $5 \frac{5}{16}$ feet tall. Which of the two boys is taller? Justify your answer.
$\qquad$
28. Represent Real-World Problems If one store is selling $\frac{3}{4}$ of a bushel of apples for $\$ 9$, and another store is selling $\frac{2}{3}$ of a bushel of apples for $\$ 9$, which store has the better deal? Explain your answer.
$\qquad$
$\qquad$
$\qquad$

## focus on hicher order thinking

29. Analyze Relationships You are given a fraction in simplest form. The numerator is not zero. When you write the fraction as a decimal, it is a repeating decimal. Which numbers from 1 to 10 could be the denominator?
$\qquad$
$\qquad$
30. Communicate Mathematical Ideas Julie got 21 of the 23 questions on her math test correct. She got 29 of the 32 questions on her science test correct. On which test did she get a higher score? Can you compare the fractions $\frac{21}{23}$ and $\frac{29}{32}$ by comparing 29 and 21? Explain. How can Julie compare her scores?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
31. Look for a Pattern Look at the decimal $0.121122111222 . .$. . If the pattern continues, is this a repeating decimal? Explain.

## LESSON <br> 1.2Relationships Between Sets of Rational Numbers

## ESSENTIAL QUESTION

How can you describe relationships between sets of rational numbers?

## Classifying Rational Numbers

A group of items is called a set. A Venn diagram uses intersecting circles to show relationships among sets of numbers or things. The Venn diagram below shows how the sets of whole numbers, integers, and rational numbers are related to each other.

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When a set is contained within a larger set in a Venn diagram, the numbers in the smaller set are also members of the larger set.

## EXAMPLE 1

## TEKS 7.2

Classify each number by naming the set or sets to which it belongs.
(A) 37
whole, integer, rational
(B) -56.12 rational

C -98
integer, rational
D $\frac{7}{8}$ rational

37 is a whole number. All whole numbers are integers. All integers are rational numbers.
-56.12 is rational number. It is not a whole number because it is negative. It is not an integer because there are non-zero digits after the decimal point.
-98 is an integer. All integers are rational numbers.
$\frac{7}{8}$ is a rational number. It is not a whole number because it is a fraction of a whole number. It is not an integer because it is not a whole number or the opposite of a whole number.

When you classify a number, you can use the Venn diagram to help figure out which other sets, if any, it belongs to.

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2. $-102.55 \ldots$
$\qquad$
$\qquad$
4. 3
$\qquad$
$\qquad$

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My Notes

Classify each number by naming the set or sets to which it belongs.

1. -8
$\qquad$
$\qquad$
2. $\frac{9}{2}$
$\qquad$
$\qquad$

## Understanding Sets and Subsets of Rational Numbers

When one set is entirely contained in another set, we say the first set is a subset of the second set. You can use the Venn diagram to decide whether or not a given set of numbers is a subset of another set.

## EXAMPLE 2

Tell whether the given statement is true or false. Explain your choice.
A All integers are rational numbers.
True. Every integer is included in the set of rational numbers.
Integers are a subset of the set of rational numbers.
B All integers are whole numbers.
False. Every whole number is an integer, but it is not true that every integer is a whole number.

Integers such as -1 and -6 are not whole numbers.

## Reflect

5. Make a Conjecture Jared said that every prime number is an integer. Do you agree or disagree? Explain.

## YOUR TURN

6. Tell whether the statement "Some rational numbers are integers" is

Personal true or false. Explain your choice.
7. Describe a real-world situation that is best described by the set of rational numbers.

Are there any integers that are not rational numbers? Explain.

## Identifying Sets for Real-World Situations

Numbers in real-world situations can be whole numbers, integers, rational numbers, or some combination of the three sets. When you have a choice of sets to describe a number in a real-world situation, you may be able to choose
 one that gives a more precise description than the others.

## EXAMPLE 3

Identify the set of numbers that best describes each situation. Explain your choice.

A the number of bills in a person's wallet
The set of whole numbers best describes the situation.
The wallet may contain no bills or any counting number of bills.
The possible numbers of bills $0,1,2,3, \ldots$ are whole numbers.
Whole numbers are also integers and rational numbers. But since there cannot be a negative or a fractional number of bills, the set of whole numbers is the most precise description.

B golf scores shown on a golf leaderboard
The set of integers best describes the situation. The scores are also rational numbers. But since there cannot be fractional scores, the set of integers is the more precise description.

| Leaderboard |  |  |  |
| :---: | :--- | :---: | :---: |
| Position | Player | Score | Hole |
| 1 | Chin | -5 | 16 |
| 2 | Smith | -2 | 14 |
| 3 | Mehta | -1 | 12 |
| 4 | Adams | 0 | 15 |
| 5 | Ramirez | +1 | 13 |

## Guided Practice

Classify each number by naming the set or sets to which it belongs. (Example 1)

1. 5
whole number, $\qquad$
$\qquad$
2. -23
integer, $\qquad$
$\qquad$
3. $-\frac{3}{14}$
4. 4.5
$\qquad$

Tell whether the given statement is true or false. Explain your choice.
(Example 2)
5. All rational numbers are integers. True / False
$\qquad$
$\qquad$
6. Some integers are whole numbers. True / False
$\qquad$
$\qquad$

Identify the set of numbers that best describes each situation. Explain your choice. (Example 3)
7. the number of students in a school

The set of $\qquad$ best describes the situation because
$\qquad$
8. possible points in a certain board game (... $-3,-2,-1,0,1,2,3, \ldots)$

The set of $\qquad$ best describes the situation because

## ESSENTIAL QUESTION CHECK-IN

9. How can you represent how the sets of whole numbers, integers, and rational numbers are related to each other?

### 1.2 Independent Practice

Classify each number by naming the set or sets to which it belongs.
10. -9
$\qquad$
12. 789
$\qquad$
$\qquad$
11. 7.5
13. $5 \frac{3}{4}$
$\qquad$
$\qquad$

Fill in each Venn diagram with the whole numbers from 1 to 15 . Remember that a composite number is a whole number greater than 1 that is not a prime number.


Tell whether the given statement is true or false. Explain your choice.
16. All rational numbers are whole numbers.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
18. Some whole numbers are negative.
$\qquad$
$\qquad$
17. All whole numbers are integers.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
19. No positive numbers are integers.
$\qquad$
$\qquad$

## Identify the set of numbers that best describes each situation. Explain your choice.

20. possible number of miles you can walk in 1 hour
$\qquad$
$\qquad$
$\qquad$
21. possible number of marbles in a jar
$\qquad$
$\qquad$
22. Represent Real-World Problems Using what you know of rational numbers, describe a real-world situation where a doctor might use the set of rational numbers on a daily basis.
$\qquad$
$\qquad$
23. Communicate Mathematical Ideas The letters in the Venn diagram represent whole numbers. Describe the numbers you would find in Section $c$, Section $d$, and Section $e$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
24. Analyze Relationships Explain how the set of integers differs from the set of whole numbers.
$\qquad$
$\qquad$
25. Justify Reasoning Explain why a mixed number is not in the set of integers or whole numbers.

Divisible by 2 Divisible by 3


Divisible by 5
$\qquad$
$\qquad$
$\qquad$

## LEsson Adding Rational Numbers

## ESSENTIAL QUESTION

## Adding Rational Numbers with the Same Sign

To add rational numbers with the same sign, apply the rules for adding integers. The sum has the same sign as the sign of the rational numbers.

## EXAMPLE 1

## TEKS 7.3.A

A Malachi hikes for 2.5 miles and stops for lunch. Then he hikes for $\mathbf{1 . 5}$ more miles. How many miles did he hike altogether?

STEP 1 Use positive numbers to represent the distance Malachi hiked.
STEP 2 Find $2.5+1.5$.

STEP 3 Start at 2.5.


STEP 4 Move 1.5 units to the right because the second addend is positive.

The result is 4 .

- Malachi hiked 4 miles.

B Kyle pours out $\frac{3}{4}$ liter of liquid from a beaker. Then he pours out another $\frac{1}{2}$ liter of liquid. What is the overall change in the amount of liquid in the beaker?

STEP 1 Use negative numbers to represent the amount of change each time Kyle pours liquid from the beaker.

Find $-\frac{3}{4}+\left(-\frac{1}{2}\right)$.
STEP 3 Start at $-\frac{3}{4}$.


STEP 4 Move $\left|-\frac{1}{2}\right|=\frac{1}{2}$ unit to the left because the second addend is negative.

The result is $-1 \frac{1}{4}$.
The amount of liquid in the beaker has decreased by $1 \frac{1}{4}$ liters.

## Reflect

1. Explain how to determine whether to move right or left on the number line when adding rational numbers.
$\qquad$
$\qquad$

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## YOUR TURN

## Use a number line to find each sum.

2. $3+1 \frac{1}{2}=$ $\qquad$

3. $-2.5+(-4.5)=$ $\qquad$


## Adding Rational Numbers with Different Signs

To add rational numbers with different signs, find the difference of their absolute values. Then use the sign of the rational number with the greater absolute value.

## EXAMPLE 2

A During the day, the temperature increases by 4.5 degrees. At night, the temperature decreases by $\mathbf{7 . 5}$ degrees. What is the overall change in temperature?

STEP 1 Use a positive number to represent the increase in temperature and a negative number to represent a decrease in temperature.

STEP 2 Find $4.5+(-7.5)$.

STEP 3 Start at 4.5.


STEP 4 Move $|-7.5|=7.5$ units to the left because the second addend is negative.

The result is -3 .

- The temperature decreased by 3 degrees overall.

B Ernesto writes a check for $\mathbf{\$ 2 . 5 0}$. Then he deposits $\mathbf{\$ 6}$ in his checking account. What is the overall increase or decrease in the account balance?

STEP 1 Use a positive number to represent a deposit and a negative number to represent a withdrawal or a check.

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STEP 2 Find $-2.5+6$.

STEP 3 Start at -2.5.


STEP 4 Move $|6|=6$ units to the right because the second addend is positive.

The result is 3.5 .

- The account balance will increase by $\$ 3.50$.


## Reflect

4. Do $-3+2$ and $2+(-3)$ have the same sum? Does it matter if the negative number is the first addend or the second addend?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
5. Make a Conjecture Do you think the sum of a negative number and a positive number will always be negative? Explain your reasoning.
$\qquad$
$\qquad$

## YOUR TURN

Use a number line to find each sum.
6. $-8+5=$ $\qquad$

7. $\frac{1}{2}+\left(-\frac{3}{4}\right)=$ $\qquad$

8. $-1+7=$ $\qquad$


## Finding the Additive Inverse

The opposite, or additive inverse, of a number is the same distance from 0 on a number line as the original number, but on the other side of 0 . The sum of a number and its additive inverse is 0 . Zero is its own additive inverse.

## EXAMPLE 3

## Math Talk <br> Mathematical Processes

Explain how to use a number line to find the additive inverse, or opposite, of -3.5 .

My Notes


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A A football team loses 3.5 yards on their first play. On the next play, they gain 3.5 yards. What is the overall increase or decrease in yards?

STEP 1 Use a positive number to represent the gain in yards and a negative number to represent the loss in yards.

STEP 2 Find $-3.5+3.5$.

STEP 3 Start at -3.5 .


STEP 4 Move $|3.5|=3.5$ units to the right, because the second addend is positive.
$\therefore \quad$ The result is 0 . This means the overall change is 0 yards.
B Kendrick adds $\frac{3}{4}$ cup of chicken stock to a pot. Then he takes $\frac{3}{4}$ cup of stock out of the pot. What is the overall increase or decrease in the amount of chicken stock in the pot?

STEP 1 Use a positive number to represent chicken stock added to the pot and a negative number to represent chicken stock taken out of the pot.

STEP 2 Find $\frac{3}{4}+\left(-\frac{3}{4}\right)$.
STEP 3 Start at $\frac{3}{4}$.


STEP 4 Move $\left|-\frac{3}{4}\right|=\frac{3}{4}$ units to the left because the second addend is negative.
$\therefore \quad$ The result is 0 . This means the overall change is 0 cups.

## YOUR TURN

Use a number line to find each sum.
9. $2 \frac{1}{2}+\left(-2 \frac{1}{2}\right)=$ $\qquad$ 10. $-4.5+4.5=$ $\qquad$


## Adding Three or More Rational Numbers

Recall that the Associative Property of Addition states that when you are adding more than two numbers, you can group any of the numbers together. This property can help you add numbers with different signs.

## EXAMPLE 4



Tina spent $\mathbf{\$ 5 . 2 5}$ on craft supplies to make friendship bracelets. She made $\mathbf{\$ 3 . 7 5}$ on Monday. On Tuesday, she sold an additional \$4.50 worth of bracelets. What was Tina's overall profit or loss?

STEP 1 Use negative numbers to represent the amount Tina spent and positive numbers to represent the money Tina earned.

STEP 2 Find $-5.25+3.75+4.50$.

STEP 3 Group numbers with the same sign.

$$
-5.25+(3.75+4.50) \quad \text { Associative Property }
$$

STEP $4-5.25+8.25$ Add the numbers inside the parentheses.

3

Profit means the difference between income and costs is positive.

Find the difference of the absolute values: $8.25-5.25$.

Use the sign of the number with the greater absolute value. The sum is positive.
$\div \quad$ Tina earned a profit of $\$ 3.00$.

## YOUR TURN

## Find each sum.

11. $-1.5+3.5+2=$ $\qquad$
12. $3 \frac{1}{4}+(-2)+\left(-2 \frac{1}{4}\right)=$ $\qquad$
13. $-2.75+(-3.25)+5=$ $\qquad$
14. $15+8+(-3)=$ $\qquad$
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## Guided Practice

Use a number line to find each sum. (Example 1 and Example 2)

1. $-3+(-1.5)=$ $\qquad$

2. $\frac{1}{4}+\frac{1}{2}=$ $\qquad$

3. $3+(-5)=$

4. $1.5+3.5=$ $\qquad$

5. $-1 \frac{1}{2}+\left(-1 \frac{1}{2}\right)=$ $\qquad$

6. $-1.5+4=$ $\qquad$

7. Victor borrowed $\$ 21.50$ from his mother to go to the theater. A week later,
he paid her $\$ 21.50$ back. How much does he still owe her? (Example 3)
$\qquad$
8. Sandra used her debit card to buy lunch for $\$ 8.74$ on Monday. On

Tuesday, she deposited $\$ 8.74$ back into her account. What is the overall increase or decrease in her bank account? (Example 3)
$\qquad$
Find each sum without using a number line. (Example 4)
9. $2.75+(-2)+(-5.25)=$ $\qquad$
10. $-3+\left(1 \frac{1}{2}\right)+\left(2 \frac{1}{2}\right)=$ $\qquad$
11. $-12.4+9.2+1=$ $\qquad$ 12. $-12+8+13=$ $\qquad$
13. $4.5+(-12)+(-4.5)=$ $\qquad$ 14. $\frac{1}{4}+\left(-\frac{3}{4}\right)=$
16. $-8+\left(-1 \frac{1}{8}\right)=$
$\qquad$
15. $-4 \frac{1}{2}+2=$ $\qquad$
$\qquad$

ESSENTIAL QUESTION CHECK-IN
17. How can you use a number line to find the sum of -4 and 6 ?
$\qquad$
$\qquad$

### 1.3 Independent Practice

TEKS 7.3.A, 7.3.B

18. Samuel walks forward 19 steps. He represents this movement with a positive 19. How would he represent the opposite of this number?
19. Julia spends $\$ 2.25$ on gas for her lawn mower. She earns $\$ 15.00$ mowing her neighbor's yard. What is Julia's profit?
20. A submarine submerged at a depth of -35.25 meters dives an additional 8.5 meters. What is the new depth of the submarine?
21. Renee hiked for $4 \frac{3}{4}$ miles. After resting, Renee hiked back along the same route for $3 \frac{1}{4}$ miles. How many more miles does Renee need to hike to return to the place where she started?
22. Geography The average elevation of the city of New Orleans, Louisiana, is 0.5 m below sea level. The highest point in Louisiana is Driskill Mountain at about 163.5 m higher than New Orleans. How high is Driskill Mountain?
23. Problem Solving A contestant on a game show has 30 points. She answers a question correctly to win 15 points. Then she answers a question incorrectly and loses 25 points. What is the contestant's final score?

## Financial Literacy Use the table for 24-26. Kameh owns

 a bakery. He recorded the bakery income and expenses in a table.24. In which months were the expenses greater than the income? Name the month and find how much money was lost.
25. In which months was the income greater than the

| Month | Income $\mathbf{( \$ )}$ | Expenses $(\mathbf{\$})$ |
| :---: | :---: | :---: |
| January | 1,205 | $1,290.60$ |
| February | 1,183 | $1,345.44$ |
| March | 1,664 | $1,664.00$ |
| June | 2,413 | $2,106.23$ |
| July | 2,260 | $1,958.50$ |
| August | 2,183 | $1,845.12$ | expenses? Name the months and find how much money was gained each of those months.

26. Communicate Mathematical Ideas If the bakery started with an extra $\$ 250$ from the profits in December, describe how to use the information in the table to figure out the profit or loss of money at the bakery by the end of August. Then calculate the profit or loss.
$\qquad$
$\qquad$
$\qquad$
27. Vocabulary -2 is the $\qquad$ of 2.
28. The basketball coach made up a game to play where each player takes 10 shots at the basket. For every basket made, the player gains 10 points. For every basket missed, the player loses 15 points.
a. The player with the highest score sank 7 baskets and missed 3 . What was the highest score?
b. The player with the lowest score sank 2 baskets and missed 8 . What was the lowest score?
$\qquad$
c. Write an expression using addition to find out what the score would be if a player sank 5 baskets and missed 5 baskets.

## M.0.7. <br> focus on hicher order thinking

29. Communicate Mathematical Ideas Explain the different ways it is possible to add two rational numbers and get a negative number.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
30. Explain the Error A student evaluated $-4+x$ for $x=-9 \frac{1}{2}$ and got the answer of $5 \frac{1}{2}$. What might the student have done wrong?
$\qquad$
$\qquad$
31. Draw Conclusions Can you find the sum $[5.5+(-2.3)]+(-5.5+2.3)$ without performing any additions?

## ESSENTIAL QUESTION

## How do you subtract rational numbers?

## Subtracting Positive Rational Numbers

To subtract rational numbers, you can apply the same rules you use to subtract integers.

## EXAMPLE 1

## Reव

## teks 7.3.A



The temperature on an outdoor thermometer on Monday was $5.5^{\circ} \mathrm{C}$. The temperature on Thursday was $\mathbf{7 . 2 5}$ degrees less than the temperature on Monday. What was the temperature on Thursday?

Subtract to find the temperature on Thursday.

STEP 1 Find 5.5-7.25.

STEP 2 Start at 5.5.


STEP 3 Move $|7.25|=7.25$ units to the left because you are subtracting a positive number.

The result is -1.75 .

- $\quad$ The temperature on Thursday was $-1.75^{\circ} \mathrm{C}$.


## YOUR TURN

Use a number line to find each difference.

1. $-6.5-2=$ $\qquad$

2. $1 \frac{1}{2}-2=$ $\qquad$

3. $-2.25-5.5=$ $\qquad$


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## Subtracting Negative Rational Numbers

To subtract negative rational numbers, move in the opposite direction on the number line.

## EXAMPLE 2 (Reald

During the hottest week of the summer, the water level of the Muskrat River was $\frac{5}{6}$ foot below normal. The following week, the level was $\frac{1}{3}$ foot below normal. What is the overall change in the water level?

Subtract to find the difference in water levels.
STEP 1 Find $-\frac{1}{3}-\left(-\frac{5}{6}\right)$.
STEP 2 Start at $-\frac{1}{3}$.


STEP 3 Move $\left|-\frac{5}{6}\right|=\frac{5}{6}$ to the right because you are subtracting a negative number.

The result is $\frac{1}{2}$.

- So, the water level increased $\frac{1}{2}$ foot.


## Reflect

4. Work with other students to compare addition of negative numbers on a number line to subtraction of negative numbers on a number line.
$\qquad$
$\qquad$
5. Compare the methods used to solve Example 1 and Example 2.
$\qquad$
$\qquad$
$\qquad$

## YOUR TURN

Use a number line to find each difference.
6. $0.25-(-1.50)=$

7. $-\frac{1}{2}-\left(-\frac{3}{4}\right)=$ $\qquad$


## EXPLORE ACTIVITY 1 <br> world <br> worl <br> Adding the Opposite

Joe is diving $2 \frac{1}{2}$ feet below sea level. He decides to descend $7 \frac{1}{2}$ more feet. How many feet below sea level is he?

STEP 1 Use negative numbers to represent the number of feet below sea level.

STEP 2 Find $-2 \frac{1}{2}-7 \frac{1}{2}$.
STEP 3 Start at $-2 \frac{1}{2}$.


STEP 4 Move $\left|7 \frac{1}{2}\right|=7 \frac{1}{2}$ units to the $\qquad$
because you are subtracting a $\qquad$ number.

The result is -10 .
$\therefore$ Joe is $\qquad$ sea level.

## Reflect

You move left on a number line to add a negative number. You move the same direction to subtract a positive number.
8. Use a number line to find each difference or sum.
a. $-3-3=$ $\qquad$

b. $-3+(-3)=$ $\qquad$

9. Make a Conjecture Work with other students to make a conjecture about how to change a subtraction problem into an addition problem.

## Adding the Opposite

To subtract a number, add its opposite. This can also be written as $p-q=p+(-q)$.

## EXPLORE ACTIVITY 2 Rerld

## Finding the Distance between Two Numbers

A cave explorer climbed from an elevation of $\mathbf{- 1 1}$ meters to an elevation of -5 meters. What vertical distance did the explorer climb?
There are two ways to find the vertical distance.
(A) Start at $\qquad$ .

Count the number of units on the vertical number line up to -5 .

The explorer climbed $\qquad$ meters.

This means that the vertical distance between
-11 meters and -5 meters is $\qquad$ meters.

B Find the difference between the two elevations and use absolute value to find the distance.
$-11-(-5)=$ $\qquad$


Take the absolute value of the difference because
 distance traveled is always a nonnegative number.
$|-11-(-5)|=$ $\qquad$
The vertical distance is $\qquad$ meters.

## Reflect

10. Does it matter which way you subtract the values when finding distance? Explain.
$\qquad$
$\qquad$
11. Would the same methods work if both the numbers were positive? What if one of the numbers were positive and the other negative?

## Distance Between Two Numbers

The distance between two values $a$ and $b$ on a number line is represented by the absolute value of the difference of $a$ and $b$. Distance between $a$ and $b=|a-b|$ or $|b-a|$.

## Guided Practice

## Use a number line to find each difference. (Example 1, Example 2 and Explore Activity 1)

1. $5-(-8)=$ $\qquad$

2. $-3 \frac{1}{2}-4 \frac{1}{2}=$ $\qquad$

3. $-7-4=$ $\qquad$

4. $-0.5-3.5=$ $\qquad$


Find each difference. (Explore Activity 1)
5. $-14-22=$ $\qquad$ 6. $-12.5-(-4.8)=$ $\qquad$ 7. $\frac{1}{3}-\left(-\frac{2}{3}\right)=$ $\qquad$
8. $65-(-14)=$ $\qquad$ 9. $-\frac{2}{9}-(-3)=$ $\qquad$ 10. $24 \frac{3}{8}-\left(-54 \frac{1}{8}\right)=$ $\qquad$
11. A girl is snorkeling 1 meter below sea level and then dives down another 0.5 meter. How far below sea level is the girl? (Explore Activity 1)
12. The first play of a football game resulted in a loss of $12 \frac{1}{2}$ yards. Then a penalty resulted in another loss of 5 yards. What is the total loss or gain? (Explore Activity 1)
13. A climber starts descending from 533 feet above sea level and keeps going until she reaches 10 feet below sea level. How many feet did she descend? (Explore Activity 2)
14. Eleni withdrew $\$ 45.00$ from her savings account. She then used her debit card to buy groceries for $\$ 30.15$. What was the total amount Eleni took out of her account? (Explore Activity 1)

## ESSENTIAL QUESTION CHECK-IN

15. Mandy is trying to subtract $4-12$, and she has asked you for help. How would you explain the process of solving the problem to Mandy, using a number line?

### 1.4 Independent Practice

TEKS 7.3.A, 7.3.B
16. Science At the beginning of a laboratory experiment, the temperature of a substance is $-12.6^{\circ} \mathrm{C}$. During the experiment, the temperature of the substance decreases $7.5^{\circ} \mathrm{C}$. What is the final temperature of the substance?
17. A diver went 25.65 feet below the surface of the ocean, and then 16.5 feet further down, he then rose 12.45 feet. Write and solve an expression to find the diver's new depth.
$\qquad$
$\qquad$
18. A city known for its temperature extremes started the day at -5 degrees Fahrenheit. The temperature increased by 78 degrees Fahrenheit by midday, and then dropped 32 degrees by nightfall.
a. What expression can you write to find the temperature at nightfall?
b. What expression can you write to describe the overall change in temperature? Hint: Do not include the temperature at the beginning of the day since you only want to know about how much the temperature changed.
c. What is the final temperature at nightfall? What is the overall change in temperature?
19. Financial Literacy On Monday, your bank account balance was $-\$ 12.58$. Because you didn't realize this, you wrote a check for $\$ 30.72$ for groceries.
a. What is the new balance in your checking account?
b. The bank charges a $\$ 25$ fee for paying a check on a negative balance. What is the balance in your checking account after this fee?
c. How much money do you need to deposit to bring your account balance back up to $\$ 0$ after the fee?

## Astronomy Use the table for problems 20-21.

20. How much deeper is the deepest canyon on Mars than the deepest canyon on Venus?

| Elevations on Planets |  |  |
| :---: | :---: | :---: |
|  | Lowest (ft) | Highest (ft) |
| Earth | $-36,198$ | 29,035 |
| Mars | $-26,000$ | 70,000 |
| Venus | $-9,500$ | 35,000 |

21. Persevere in Problem Solving What is the difference between Earth's highest mountain and its deepest ocean canyon? What is the difference between Mars' highest mountain and its deepest canyon? Which difference is greater? How much greater is it?
$\qquad$
$\qquad$
$\qquad$
22. Pamela wants to make some friendship bracelets for her friends. Each friendship bracelet needs 5.2 inches of string.
a. If Pamela has 20 inches of string, does she have enough to make bracelets for 4 of her friends?
$\qquad$
$\qquad$
b. If so, how much string would she have left over? If not, how much more string would she need?
23. Jeremy is practicing some tricks on his skateboard. One trick takes him forward 5 feet, then he flips around and moves backwards 7.2 feet, then he moves forward again for 2.2 feet.
a. What expression could be used to find how far Jeremy is from his starting position when he finishes the trick?
b. How far from his starting point is he when he finishes the trick? Explain
24. Esteban has $\$ 20$ from his allowance. There is a comic book he wishes to buy that costs $\$ 4.25$, a cereal bar that costs $\$ 0.89$, and a small remote control car that costs $\$ 10.99$.
a. Does Esteban have enough to buy everything?
b. If so, how much will he have left over? If not, how much does he still need?
focus on hicher order thinking
25. Look for a Pattern Show how you could use the Commutative Property to simplify the evaluation of the expression $-\frac{7}{16}-\frac{1}{4}-\frac{5}{16}$.
$\qquad$
$\qquad$
$\qquad$
26. Problem Solving The temperatures for five days in Kaktovik, Alaska, are given below.
$-19.6^{\circ} \mathrm{F},-22.5^{\circ} \mathrm{F},-20.9^{\circ} \mathrm{F},-19.5^{\circ} \mathrm{F},-22.4^{\circ} \mathrm{F}$
Temperatures over the same 5-day period last year were 12 degrees lower. What were the highest and lowest temperatures over this period last year?
$\qquad$
$\qquad$
27. Make a Conjecture Must the difference between two rational numbers be a rational number? Explain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
28. Look for a Pattern Evan said that the difference between two negative numbers must be negative. Was he right? Use examples to illustrate your answer.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## LESSON <br> 1.5Multiplying Rational Numbers

## Multiplying Rational Numbers with Different Signs

## Products of Rational Numbers

| Sign of Factor $\boldsymbol{p}$ | Sign of Factor $\boldsymbol{q}$ | Sign of Product $\mathbf{p} \boldsymbol{q}$ |
| :---: | :---: | :---: |
| + | - | - |
| - | + | - |

You can also use the fact that multiplication is repeated addition.

## EXAMPLE 1

Gina hiked down a canyon and stopped each time she descended $\frac{1}{2}$ mile to rest. She hiked a total of 4 sections. What is her overall change in elevation?

STEP 1 Use a negative number to represent the change in elevation.
STEP 2 Find $4\left(-\frac{1}{2}\right)$.
STEP 3 Start at 0 . Move $\frac{1}{2}$ unit to the left 4 times.
The result is -2 .
○ The overall change is -2 miles. $-\mathbf{3}$
Check: Use the rules for multiplying rational numbers.

$$
\begin{aligned}
4\left(-\frac{1}{2}\right) & =\left(-\frac{4}{2}\right) & & \text { A negative times a positive equals a negative. } \\
& =-2 \checkmark & & \text { Simplify. }
\end{aligned}
$$



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## Multiplying Rational Numbers with the Same Sign

The rules for the signs of products with the same signs are summarized below.

| Products of Rational Numbers |  |  |
| :---: | :---: | :---: |
| Sign of Factor $\boldsymbol{p}$ | Sign of Factor $\boldsymbol{q}$ | Sign of Product $\boldsymbol{p} \boldsymbol{q}$ |
| + | + | + |
| - | - | + |

You can also use a number line to find the product of rational numbers with the same signs.

## EXAMPLE 2

Multiply - 2(-3.5).
STEP 1 First, find the product 2(-3.5).


STEP 2 Start at 0 . Move 3.5 units to the left two times.
STEP 3 The result is -7.
STEP 4 This shows that 2 groups of -3.5 equals -7 .
So, -2 groups of -3.5 must equal the opposite of -7 .


STEP $5-2(-3.5)=7$

Check: Use the rules for multiplying rational numbers.
$-2(-3.5)=7 \quad$ A negative times a negative equals a positive.

## YOUR TURN

2. Find $-3(-1.25)$.


## Multiplying More Than <br> Two Rational Numbers

If you multiply three or more rational numbers, you can use a pattern to find the sign of the product.

## EXAMPLE 3

## TIEKS 7.3.B

Multiply $\left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{5}\right)$.
STEP 1 First, find the product of the first two factors. Both factors are negative, so their product will be positive.

STEP $2\left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right)=+\left(\frac{x}{3} \cdot \frac{1}{2}\right)$

$$
=\frac{1}{3}
$$

STEP 3 Now, multiply the result, which is positive, by the third factor, which is negative. The product will be negative.

STEP $4 \frac{1}{3}\left(-\frac{3}{5}\right)=\frac{1}{3}\left(-\frac{3}{5}\right)$
-

Math Talk
Mathematical Processes
Suppose you find the product of several rational numbers, one of which is zero. What can you say about the product?

## Reflect

3. Look for a Pattern You know that the product of two negative numbers is positive, and the product of three negative numbers is negative. Write a rule for finding the sign of the product of $n$ negative numbers.

## YOUR TURN

Find each product.
4. $\left(-\frac{3}{4}\right)\left(-\frac{4}{7}\right)\left(-\frac{2}{3}\right)$
5. $\left(-\frac{2}{3}\right)\left(-\frac{3}{4}\right)\left(\frac{4}{5}\right)$ $\qquad$
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6. $\left(\frac{2}{3}\right)\left(-\frac{9}{10}\right)\left(\frac{5}{6}\right)$

## Guided Practice

Use a number line to find each product. (Example 1 and Example 2)

1. $5\left(-\frac{2}{3}\right)=$ $\qquad$

2. $-3\left(-\frac{4}{7}\right)=$ $\qquad$ 4. $-\frac{3}{4}(-4)=$ $\qquad$

3. $3\left(-\frac{1}{4}\right)=$ $\qquad$

4. $4(-3)=$
5. $-1.8(5)=$ $\qquad$ 7. $-2(-3.4)=$ $\qquad$
6. $0.54(8)=$ $\qquad$ 9. $-5(-1.2)=$ $\qquad$ 10. $-2.4(3)=$ $\qquad$

Multiply. (Example 3)
11. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}=\square \times \frac{3}{4}=$ $\qquad$ 12. $-\frac{4}{7}\left(-\frac{3}{5}\right)\left(-\frac{7}{3}\right)=(\square) \times\left(-\frac{7}{3}\right)=$ $\qquad$
13. $-\frac{1}{8} \times 5 \times \frac{2}{3}=$ $\qquad$ 14. $-\frac{2}{3}\left(\frac{1}{2}\right)\left(-\frac{6}{7}\right)=$ $\qquad$
15. The price of one share of Acme Company declined $\$ 3.50$ per day for 4 days in a row. What is the overall change in price of one share?
(Example 1)
16. In one day, 18 people each withdrew $\$ 100$ from an ATM machine. What is the overall change in the amount of money in the ATM machine?
(Example 1)
$\qquad$

## ESSENTIAL QUESTION CHECK-IN

17. Explain how you can find the sign of the product of two or more rational numbers.
$\qquad$
$\qquad$
$\qquad$

### 1.5 Independent Practice

## TEKS 7.3.A, 7.3.B

18. Financial Literacy Sandy has $\$ 200$ in her bank account.
a. If she writes 6 checks for exactly $\$ 19.98$, what expression describes the change in her bank account?
b. What is her account balance after the checks are cashed?
$\qquad$
19. Communicating Mathematical Ideas Explain, in words, how to find the product of $-4(-1.5)$ using a number line. Where do you end up?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
20. Greg sets his watch for the correct time on Wednesday. Exactly one week later, he finds that his watch has lost $3 \frac{1}{4}$ minutes. What is the overall change in time after 8 weeks?
$\qquad$
21. A submarine dives below the surface, heading downward in three moves. If each move downward was 325 feet, where is the submarine after it is finished diving?
$\qquad$
$\qquad$
$\qquad$ stops to buy gas, she loses 15 minutes of travel time. If she has to stop 5 times, how late will she be getting to her destination?
$\qquad$
$\qquad$
22. The table shows the scoring system for quarterbacks in Jeremy's fantasy football league. In one game, Jeremy's quarterback had 2 touchdown passes, 16 complete passes, 7 incomplete passes, and 2 interceptions. How many total points did Jeremy's quarterback score?

| Quarterback Scoring |  |
| :--- | :---: |
| Action | Points |
| Touchdown pass | 6 |
| Complete pass | 0.5 |
| Incomplete pass | -0.5 |
| Interception | -1.5 |

## \%.....

focus on hicher order thinking
26. Represent Real-World Problems The ground temperature at Brigham Airport is $12^{\circ} \mathrm{C}$. The temperature decreases by $6.8^{\circ} \mathrm{C}$ for every increase of 1 kilometer above the ground. What is the temperature outside a plane flying at an altitude of 5 kilometers?
27. Identify Patterns The product of four numbers, $a, b, c$, and $d$, is a negative number. The table shows one combination of positive and negative signs of the four numbers that could produce a negative product. Complete the table to show the seven other possible combinations.

| $\boldsymbol{a}$ | $\boldsymbol{b}$ | $\boldsymbol{c}$ | $\boldsymbol{d}$ |
| :---: | :---: | :---: | :---: |
| + | + | + | - |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

28. Reason Abstractly Find two integers whose sum is -7 and whose product is 12. Explain how you found the numbers.

## EXPLORE ACTIVITY 1

## 2ad

## Dividing Rational Numbers

A diver needs to descend to a depth of 100 feet below sea level. She wants to do it in $\mathbf{5}$ equal descents. How far should she travel in each descent?

A To solve this problem, you can set up a division problem:


B Rewrite the division problem as a multiplication problem. Think: Some number multiplied by 5 equals -100 .
$\qquad$ $\times ?=-100$

C Remember the rules for integer multiplication. If the product is negative, one of the factors must be negative. Since is positive, the unknown factor must be positive / negative.
D You know that $5 \times$ $\qquad$ $=100$. So, using the rules for integer multiplication you can say that $5 \times$ $\qquad$ $=-100$.

The diver should descend $\qquad$ feet in each descent.

## Reflect

1. What do you notice about the quotient of two rational numbers with different signs?
$\qquad$
$\qquad$
2. What do you notice about the quotient of two rational numbers with the same sign? Does it matter if both signs are positive or both are negative?

Let $p$ and $q$ be rational numbers.

| Quotients of Rational Numbers |  |  |
| :---: | :---: | :---: |
| Sign of Dividend $\boldsymbol{p}$ | Sign of Divisor $\boldsymbol{q}$ | Sign of Quotient $\frac{p}{\boldsymbol{q}}$ |
| + | - | - |
| - | + | - |
| + | + | + |
| - | - | + |

Also, $-\left(\frac{p}{q}\right)=\frac{-p}{q}=\frac{p}{-q}$, for $q$ not zero.

## EXPLORE ACTIVITY 2

## Placement of Negative Signs in Quotients

Quotients can have negative signs in different places.
Are the rational numbers $\frac{12}{-4}, \frac{-12}{4}$, a nd $-\left(\frac{12}{4}\right)$ equivalent?
A Find each quotient. Then use the rules you found in Explore Activity 1 to make sure the sign of the quotient is correct.

$$
\frac{12}{-4}=
$$

$$
\frac{-12}{4}=
$$

$$
-\left(\frac{12}{4}\right)=
$$

$\qquad$
B What do you notice about each quotient?

C The rational numbers are / are not equivalent.
D Conjecture Explain how the placement of the negative sign in the rational number affects the sign of the quotients.
$\qquad$
$\qquad$

## Reflect

Write two equivalent expressions for each quotient.
3. $\frac{14}{-7}$ $\qquad$ , $\qquad$ 4. $\frac{-32}{-8}$
$\qquad$ ,

## Quotients of Rational Numbers

The rules for dividing rational numbers are the same as dividing integers.

## EXAMPLE 1



A Over 5 months, Carlos wrote 5 checks for a total of $\$ 323.75$ to pay for his cable TV service. His cable bill is the same amount each month. What was the change in Carlos' bank account each month to pay for cable?
Find the quotient: $\frac{-323.75}{5}$
STEP 1 Use a negative number to represent the withdrawal from his account each month.

STEP 2 Find $\frac{-323.75}{5}$.
STEP 3 Determine the sign of the quotient.
The quotient will be negative because the signs are different.
STEP 2 Divide.
$\frac{-323.75}{5}=-64.75$

- Carlos withdrew $\$ 64.75$ each month to pay for cable TV.

B Find $\frac{\frac{7}{10}}{\frac{-1}{5}}$.
STEP 1 Determine the sign of the quotient.
The quotient will be negative because the signs are different.
STEP 2 Write the complex fraction as division: $\frac{\frac{7}{10}}{-\frac{1}{5}}=\frac{7}{10} \div \frac{-1}{5}$
STEP 3 Rewrite using multiplication: $\frac{7}{10} \times\left(-\frac{5}{1}\right)$

Multiply by the reciprocal.

STEP $3 \quad \frac{7}{10} \times\left(-\frac{5}{1}\right)=-\frac{35}{10} \quad$ Multiply.

$$
=-\frac{7}{2} \quad \text { Simplify } .
$$

$$
\frac{\frac{7}{10}}{-\frac{1}{5}}=-\frac{7}{2}
$$

## YOUR TURN

Find each quotient.
5. $\frac{2.8}{-4}=$ $\qquad$ 6. $\frac{-\frac{5}{8}}{-\frac{6}{7}}=$ $\qquad$ 7. $-\frac{5.5}{0.5}=$
$\qquad$

## Guided Practice

Find each quotient. (Explore Activity 1 and 2, Example 1)

1. $\frac{0.72}{-0.9}=$ $\qquad$ 2. $\binom{\frac{1}{5}}{-\frac{7}{5}}=$
$\qquad$
2. $\frac{56}{-7}=$ $\qquad$ 4. $\frac{251}{4} \div\left(-\frac{3}{8}\right)=$ $\qquad$
3. $\frac{75}{-\frac{1}{5}}=$ $\qquad$ 6. $\frac{-91}{-13}=$ $\qquad$
4. $\frac{-\frac{3}{7}}{\frac{9}{4}}=$ $\qquad$ 8. $-\frac{12}{0.03}=$ $\qquad$
5. A water pail in your backyard has a small hole in it. You notice that it has drained a total of 3.5 liters in 4 days. What is the average change in water volume each day? (Example 1)
$\qquad$
6. The price of one share of $A B C$ Company declined a total of $\$ 45.75$ in 5 days. What was the average change of the price of one share per day? (Example 1)
7. To avoid a storm, a passenger-jet pilot descended 0.44 mile in 0.8 minute. What was the plane's average change of altitude per minute?
(Example 1)
$\qquad$

## ESSENTIAL QUESTION CHECK-IN

12. Explain how you would find the sign of the quotient $\frac{32 \div(-2)}{-16 \div 4}$.
$\qquad$
$\qquad$
$\qquad$

### 1.6 Independent Practice

## TEKS 7.3.A, 7.3.B

13. $\frac{5}{-\frac{2}{8}}=$ $\qquad$
14. $5 \frac{1}{3} \div\left(-1 \frac{1}{2}\right)=$ $\qquad$
15. $\frac{-120}{-6}=$ $\qquad$
16. $\frac{-\frac{4}{5}}{-\frac{2}{3}}=$ $\qquad$
17. $1.03 \div(-10.3)=$ $\qquad$
18. $\frac{-0.4}{80}=$ $\qquad$
19. $1 \div \frac{9}{5}=$ $\qquad$
20. $\frac{\frac{-1}{4}}{\frac{23}{24}}=$ $\qquad$
21. $\frac{-10.35}{-2.3}=$
22. Alex usually runs for 21 hours a week, training for a marathon. If he is unable to run for 3 days, describe how to find out how many hours of training time he loses, and write the appropriate integer to describe how it affects his time.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
23. A hot air balloon descended 99.6 meters in 12 seconds. What was the balloon's average rate of descent in meters per second?

24. Sanderson is having trouble with his assignment. His work is as follows:
$\frac{-\frac{3}{4}}{\frac{4}{3}}=-\frac{3}{4} \times \frac{4}{3}=-\frac{12}{12}=-1$
However, his answer does not match the answer that his teacher gives him. What is Sanderson's mistake? Find the correct answer.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
25. Science Beginning in 1996, a glacier lost an average of 3.7 meters of thickness each year. Find the total change in its thickness by the end of 2012.

## M.O.5 focus on hicher order thinking

29. Represent Real-World Problems Describe a real-world situation that can be represented by the quotient $-85 \div 15$. Then find the quotient and explain what the quotient means in terms of the real-world situation.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
30. Construct an Argument Divide 5 by 4 . Is your answer a rational number? Explain.
$\qquad$
$\qquad$
$\qquad$
31. Critical Thinking Is the quotient of an integer divided by a nonzero integer always a rational number? Explain.

## Ready to Go On?

### 1.1 Rational Numbers and Decimals

Write each mixed number as a decimal.
Personal Math Trainer

1. $4 \frac{1}{5}$
2. $12 \frac{14}{15}$
3. $5 \frac{5}{32}$

### 1.2 Relationships Between Sets of Numbers

4. Are all whole numbers rational numbers? Explain.
$\qquad$
$\qquad$

### 1.3 Adding Rational Numbers

Find each sum.
5. $4.5+7.1=$ $\qquad$ 6. $5 \frac{1}{6}+\left(-3 \frac{5}{6}\right)=$
$\qquad$

### 1.4 Subtracting Rational Numbers

 Find each difference.7. $-\frac{1}{8}-\left(6 \frac{7}{8}\right)=$ $\qquad$ 8. $14.2-(-4.9)=$
$\qquad$

### 1.5 Multiplying Rational Numbers

Multiply.
9. $-4\left(\frac{7}{10}\right)=$ $\qquad$
10. $-3.2(-5.6)(4)=$ $\qquad$

### 1.6 Dividing Rational Numbers

Find each quotient.
11. $-\frac{19}{2} \div \frac{38}{7}=$ $\qquad$ 12. $\frac{-32.01}{-3.3}=$
$\qquad$

## ESSENTIAL QUESTION

13. How can you use rational numbers to represent real-world problems?

## Selected Response

1. What is $-7 \frac{5}{12}$ written as a decimal?
(A) -7.25
(B) $-7.333 \ldots$
(C) $-7.41666 \ldots$
(D) -7.512
2. Which set or sets does the number $-9 \frac{1}{2}$ belong to?
(A) Integers only
(B) Rational numbers only
(C) Integers and rational numbers only
(D) Whole numbers, integers, and rational numbers
3. Renee ate $\frac{1}{4}$ of a pizza, and Sumi ate $\frac{1}{3}$ of the same pizza. How much of the pizza did they eat in all?
(A) $\frac{1}{7}$ of the pizza
(B) $\frac{2}{7}$ of the pizza
(C) $\frac{5}{12}$ of the pizza
(D) $\frac{7}{12}$ of the pizza
4. Kareem had $\$ 25$ in his bank account on Monday. The table shows his account activity for the next four days. What was the balance in Kareem's account on Friday?

| Day | Deposit | Withdrawal |
| :--- | :--- | :--- |
| Monday | none | $\$ 13.50$ |
| Tuesday | $\$ 85.10$ | none |
| Wednesday | none | $\$ 55.32$ |
| Thursday | $\$ 17.95$ | none |

(A) $\$ 59.23$
(C) $-\$ 9.23$
(B) $\$ 9.23$
(D) $-\$ 59.23$
5. A used boat is on sale for $\$ 2,400$. Austin makes an offer equal to $\frac{2}{3}$ of this price. How much does Austin offer for the boat?
(A) $\$ 3,600$
(C) $\$ 1,600$
(B) $\$ 1,800$
(D) $\$ 800$
6. Working together, 9 friends pick $23 \frac{2}{5}$ bags of apples at an orchard. They divide the bags of apples equally between them. How many bags does each friend get?
(A) $32 \frac{2}{5}$ bags
(C) $2 \frac{3}{5}$ bags
(B) $14 \frac{2}{5}$ bags
(D) $2 \frac{5}{9}$ bags
7. The Flathead Rail Tunnel in Montana is about $7 \frac{3}{4}$ miles long. A train travels at a speed of $\frac{3}{4}$ mile per minute. How long will it take the train to go through the tunnel?
(A) $\frac{7}{16}$ minute
(B) $5 \frac{3}{16}$ minutes
(C) $8 \frac{1}{3}$ minutes
(D) $10 \frac{1}{3}$ minutes

## Gridded Response

8. What is the value of $(-2.75)(-1.16)$ ?

|  |  |  |  | - |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (0) | (0) | (0) | (0) |  | (0) | (0) |
| (1) | (1) | (1) | (1) |  | (1) | (1) |
| (2) | (2) | (2) | (2) |  | (2) | (2) |
| (3) | (3) | (3) | (3) |  | (3) | (3) |
| (4) | (4) | (4) | (4) |  | (4) | (4) |
| (5) | (5) | (5) | (5) |  | (5) | (5) |
| (6) | (6) | (6) | (6) |  | (6) | (6) |
| (7) | (7) | (7) | (7) |  | (7) | (7) |
| (8) | (8) | (8) | (8) |  | (8) | (8) |
| (9) | (9) | (9) | (9) |  | (9) | (9) |

## Sturdy Guide Review

## movie 1 Rational Numbers

## ESSENTIAL QUESTION

How can you use rational numbers to solve real-world problems?

## EXAMPLE 1

Eddie walked $1 \frac{2}{3}$ miles on a hiking trail. Write $1 \frac{2}{3}$ as a decimal. Use the decimal to classify $1 \frac{2}{3}$ by naming the set or sets to which it belongs.

## Key Vocabulary

rational number (número racional)
repeating decimal (decimal periódico)
set (conjunto)
subset (subconjunto)
terminating decimal
(decimal cerrado)

$$
\begin{array}{llll} 
& \frac{2}{3}=\frac{5}{3} & \begin{array}{l}
3 \longdiv { 5 . 0 6 } \\
\text { Write } 1 \frac{2}{3} \\
\text { fraction. }
\end{array} & \frac{-3}{20}
\end{array} \quad \text { Divide the numerator by }
$$

The decimal equivalent of $1 \frac{2}{3}$ is $1.66 \ldots$, or $1 . \overline{6}$. It is a repeating decimal, and therefore can be classified as a rational number.

## EXAMPLE 2

Find each sum or difference.
A. $-2+4.5$


Start at -2 and move 4.5 units to the right. $-2+4.5=2.5$
B. $-\frac{2}{5}-\left(-\frac{4}{5}\right)$


Start at $-\frac{2}{5}$. Move $\left|-\frac{4}{5}\right|=\frac{4}{5}$ unit to the right because you are subtracting a negative number. $-\frac{2}{5}-\left(-\frac{4}{5}\right)=\frac{2}{5}$

## EXAMPLE 3

Find the product: $3\left(-\frac{1}{6}\right)\left(-\frac{2}{5}\right)$.
$3\left(-\frac{1}{6}\right)=-\frac{1}{2}$
Find the product of the first two factors. One is positive and one is negative, so the product is negative.
$-\frac{1}{2}\left(-\frac{2}{5}\right)=\frac{1}{5}$
Multiply the result by the third factor. Both are
negative, so the product is positive.
$3\left(-\frac{1}{6}\right)\left(-\frac{2}{5}\right)=\frac{1}{5}$

## EXAMPLE 4

Find the quotient: $\frac{15.2}{-2}$.
$\frac{15.2}{-2}=-7.6$
The quotient is negative because the signs are different.

## EXERCISES

Write each mixed number as a whole number or decimal. Classify each number by naming the set or sets to which it belongs: rational numbers, integers, or whole numbers. (Lessons 1.1, 1.2)

1. $\frac{3}{4}$ $\qquad$
2. $\frac{11}{3}$ $\qquad$
3. $\frac{8}{2}$
4. $\frac{5}{2}$ $\qquad$

Find each sum or difference. (Lessons 1.3, 1.4)
5. $-5+9.5$ $\qquad$
6. $\frac{1}{6}+\left(-\frac{5}{6}\right)$ $\qquad$
7. $-0.5+(-8.5)$ $\qquad$
8. $-3-(-8)$ $\qquad$ 9. $5.6-(-3.1)$
10. $3 \frac{1}{2}-2 \frac{1}{4}$

Find each product or quotient. (Lessons 1.5, 1.6)
11. $-9 \times(-5)$ $\qquad$
12. $0 \times(-7)$ $\qquad$ 13. $-8 \times 8$ $\qquad$
14. $\frac{-56}{8}$
15. $\frac{-130}{-5}$
16. $\frac{34.5}{1.5}$
17. $-\frac{2}{5}\left(-\frac{1}{2}\right)\left(-\frac{5}{6}\right)$ $\qquad$ 18. $\left(\frac{1}{5}\right)\left(-\frac{5}{7}\right)\left(\frac{3}{4}\right)$ $\qquad$
19. Lei withdrew $\$ 50$ from her bank account every day for a week. What was the change in her account in that week?
20. In 5 minutes, a seal descended 24 feet. What was the average rate of change in the seal's elevation per minute?

## Unit 1 Performance Tasks

1. CAREERS IN MATH Urban Planner Armand is an urban planner, and he has proposed a site for a new town library. The site is between City Hall and the post office on Main Street.


The distance between City Hall and the post office is $6 \frac{1}{2}$ miles. The library site is $1 \frac{1}{4}$ miles closer to City Hall than it is to the post office.
a. Write $6 \frac{1}{2}$ miles and $1 \frac{1}{4}$ miles as decimals.
b. Let $d$ represent the distance from City Hall to the library site. Write an expression for the distance from the library site to the post office.
c. Write an equation that represents the following statement: The distance from City Hall to the library site plus the distance from the library site to the post office is equal to the distance from City Hall to the post office.
d. Solve your equation from part c to determine the distance from City Hall to the library site, and the distance from the post office to the library site.
2. Sumaya is reading a book with 240 pages. She has already read 90 pages. She plans to read 20 more pages each day until she finishes the book.
a. Sumaya writes the equation $330=-20 d$ to find the number of days she will need to finish the book. Identify the errors that Sumaya made.
$\qquad$
$\qquad$
b. Write and solve an equation to determine how many days Sumaya will need to finish the book. In your answer, count part of a day as a full day.
c. Estimate how many days you would need to read a book about the same length as Sumaya's book. What information did you use to find the estimate?
$\qquad$
$\qquad$
$\qquad$
3. Jackson works as a veterinary technician and earns $\$ 12.20$ per hour.
a. Jackson normally works 40 hours a week. In a normal week, what is his total pay before taxes and other deductions?
b. Last week, Jackson was ill and missed some work. His total pay before deductions was $\$ 372.10$. Write and solve an equation to find the number of hours Jackson worked.
c. Jackson records his hours each day on a time sheet. Last week when he was ill, his time sheet was incomplete. How many

| Mon | Tues | Wed | Thurs | Fri |
| :---: | :---: | :---: | :---: | :---: |
| 8 | $7 \frac{1}{4}$ | $8 \frac{1}{2}$ |  |  | hours are missing? Show your work.

$\qquad$
$\qquad$
d. When Jackson works more than 40 hours in a week, he earns
1.5 times his normal hourly rate for each of the extra hours.

Jackson worked 43 hours one week. What was his total pay before deductions? Justify your answer.
$\qquad$
$\qquad$
e. What is a reasonable range for Jackson's expected yearly pay before deductions? Describe any assumptions you made in finding your answer.

## Selected Response

1. What is $-6 \frac{9}{16}$ written as a decimal?
(A) -6.625
(B) -6.5625
(C) -6.4375
(D) -6.125
2. Working together, 6 friends pick $14 \frac{2}{5}$ pounds of pecans at a pecan farm. They divide the pecans equally among themselves. How many pounds does each friend get?
(A) $20 \frac{2}{5}$ pounds
(B) $8 \frac{2}{5}$ pounds
(C) $2 \frac{3}{5}$ pounds
(D) $2 \frac{2}{5}$ pounds
3. What is the value of $(-3.25)(-1.56)$ ?
(A) -5.85
(B) -5.07
(C) 5.07
(D) 5.85
4. Ruby ate $\frac{1}{3}$ of a pizza, and Angie ate $\frac{1}{5}$ of the pizza. How much of the pizza did they eat in all?
(A) $\frac{1}{15}$ of the pizza
(B) $\frac{1}{8}$ of the pizza
(C) $\frac{3}{8}$ of the pizza
(D) $\frac{8}{15}$ of the pizza
5. Jaime had $\$ 37$ in his bank account on Sunday. The table shows his account activity for the next four days. What was the balance in Jaime's account after his deposit on Thursday?

| Jamie's Bank Account |  |  |
| :---: | :---: | :---: |
| Day | Deposit | Withdrawal |
| Monday | $\$ 17.42$ | none |
| Tuesday | none | $-\$ 12.60$ |
| Wednesday | none | $-\$ 9.62$ |
| Thursday | $\$ 62.29$ | none |

(A) $\$ 57.49$
(B) $\$ 59.65$
(C) $\$ 94.49$
(D) $\$ 138.93$
6. A used motorcycle is on sale for $\$ 3,600$. Erik makes an offer equal to $\frac{3}{4}$ of this price. How much does Erik offer for the motorcycle?
(A) $\$ 4,800$
(B) $\$ 2,700$
(C) $\$ 2,400$
(D) $\$ 900$
7. To which set or sets does the number -18 belong?
(A) integers only
(B) rational numbers only
(C) integers and rational numbers only
(D) whole numbers, integers, and rational numbers
8. Mrs. Rodriguez is going to use $6 \frac{1}{3}$ yards of material to make two dresses. The larger dress requires $3 \frac{2}{3}$ yards of material. How much material will Mrs. Rodriguez have left to use on the smaller dress?
(A) $1 \frac{2}{3}$ yards
(B) $2 \frac{1}{3}$ yards
(C) $2 \frac{2}{3}$ yards
(D) $3 \frac{1}{3}$ yards
9. Winslow buys 1.2 pounds of bananas. The bananas cost $\$ 1.29$ per pound. To the nearest cent, how much does Winslow pay for the bananas?
(A) $\$ 1.08$
(B) $\$ 1.20$
(C) $\$ 1.55$
(D) $\$ 2.49$

## Gridded Response

10. Roberta earns $\$ 7.65$ per hour. How many hours does Roberta need to work to earn \$24.48?

|  |  |  |  | - |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (0) | (0) | (0) | (0) |  | (0) | (0) |
| (1) | (1) | (1) | (1) |  | (1) | (1) |
| (2) | (2) | (2) | (2) |  | (2) | (2) |
| (3) | (3) | (3) | (3) |  | (3) | (3) |
| (4) | (4) | (4) | (4) |  | (4) | (4) |
| (5) | (5) | (5) | (5) |  | (5) | (5) |
| (6) | (6) | (6) | (6) |  | (6) | (6) |
| (7) | (7) | (7) | (7) |  | (7) | (7) |
| (8) | (8) | (8) | (8) |  | (8) | (8) |
| (9) | (9) | (9) | (9) |  | (9) | (9) |

11. What is the product of the following expression?

$$
(-2.2)(1.5)(-4.2)
$$

|  |  |  |  | - |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (0) | (0) | (0) | (0) |  | (0) | (0) |
| (1) | (1) | (1) | (1) |  | (1) | (1) |
| (2) | (2) | (2) | (2) |  | (2) | (2) |
| (3) | (3) | (3) | (3) |  | (3) | (3) |
| (4) | (4) | (4) | (4) |  | (4) | (4) |
| (5) | (5) | (5) | (5) |  | (5) | (5) |
| (6) | (6) | (6) | (6) |  | (6) | (6) |
| (7) | (7) | (7) | (7) |  | (7) | (7) |
| (8) | (8) | (8) | (8) |  | (8) | (8) |
| (9) | (9) | (9) | (9) |  | (9) | (9) |

> Gridded responses cannot be negative numbers. If you get a negatice value, you likely made an error. Check your work!
12. Victor is ordering pizzas for a party. He would like to have $\frac{1}{4}$ of a pizza for each guest. He can only order whole pizzas, not part of a pizza. If he expects 27 guests, how many pizzas should he order?

|  |  |  |  | - |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (0) | (0) | (0) | (0) |  | (0) | (0) |
| (1) | (1) | (1) | (1) |  | (1) | (1) |
| (2) | (2) | (2) | (2) |  | (2) | (2) |
| (3) | (3) | (3) | (3) |  | (3) | (3) |
| (4) | (4) | (4) | (4) |  | (4) | (4) |
| (5) | (5) | (5) | (5) |  | (5) | (5) |
| (6) | (6) | (6) | (6) |  | (6) | (6) |
| (7) | (7) | (7) | (7) |  | (7) | (7) |
| (8) | (8) | (8) | (8) |  | (8) | (8) |
| (9) | (9) | (9) | (9) |  | (9) | (9) |

