Equations and Inequalities

MODULE 11

Equations and Inequalities with the Variable on Both Sides

TEKS 8.8.A, 8.8.B, 8.8.C

CAREERS IN MATH

Hydraulic Engineer A hydraulic engineer specializes in the behavior of fluids, mainly water. A hydraulic engineer applies the mathematics of fluid dynamics to the collection, transport, measurement, and regulation of water and other fluids.

If you are interested in a career in hydraulic engineering, you should study the following mathematical subjects:

- Algebra
- Geometry
- Trigonometry
- Probability and Statistics
- Calculus

Research other careers that require the understanding of the mathematics of fluid dynamics.

Unit 4 Performance Task

At the end of the unit, check out how **hydraulic engineers** use math.

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters to answer the riddle at the bottom of the page.

1. **FIACALRONT INFEOCIECTF**

2. **LCMADEI CINETFOEFIC**

3. **UQAOTENI**

4. **ROPWE**

5. **TINELQUYAI**

1. A number that is multiplied by the variable in an algebraic expression, where the number is a fraction. (**Lesson** 11.2)

2. A number that is multiplied by the variable in an algebraic expression, where the number is a decimal. (**Lesson** 11.2)

3. A mathematical statement that two expressions are equal. (**Lesson** 11.1)

4. A number that is formed by repeated multiplication of the same factor. Multiply this to remove decimals from an unsolved equation. (**Lesson** 11.2)

5. A statement that two expressions are not equal. (**Lesson** 11.3)

Q: What is the best time to divide a half dollar between two people?

A: at a ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___!

Equations and Inequalities with the Variable on Both Sides

? ESSENTIAL QUESTION

How can you use equations and inequalities with the variable on both sides to solve real-world problems?

Real-World Video

Some employees earn commission plus their salary when they make a sale. There may be options about their pay structure. They can find the best option by solving an equation with the variable on both sides.

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Interactively explore key concepts to see how math works.

Personal Math Trainer

Get immediate feedback and help as you work through practice sets.

Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Find Common Denominators

EXAMPLE Find the LCD of 3, 5, and 10.

3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30,…   *List the multiples of each number.*
5: 5, 10, 15, 20, 25, 30, 35,…   *Choose the least multiple the lists have in common.*
10: 10, 20, 30, 40, 50,… $LCD(3, 5, 10) = 30$

Find the LCD.

1. 8, 12 _____   **2.** 9, 12 _____   **3.** 15, 20 _____   **4.** 8, 10 _____

Multiply Decimals by Powers of 10

EXAMPLE 3.719×100 *Count the zeros in 100: 2 zeros*
$3.719 \times 100 = 371.9$ *Move the decimal point 2 places to the right.*

Find the product.

5. 0.683×100 **6.** $9.15 \times 1,000$ **7.** 0.005×100 **8.** $1,000 \times 1,000$

_____   _____   _____   _____

Connect Words and Equations

EXAMPLE Two times a number decreased by 5 is -6.

Two times x decreased by 5 is -6.   *Represent the unknown with a variable.*
$2x - 5$ is -6 *Times means multiplication. Decreased by means subtraction.*
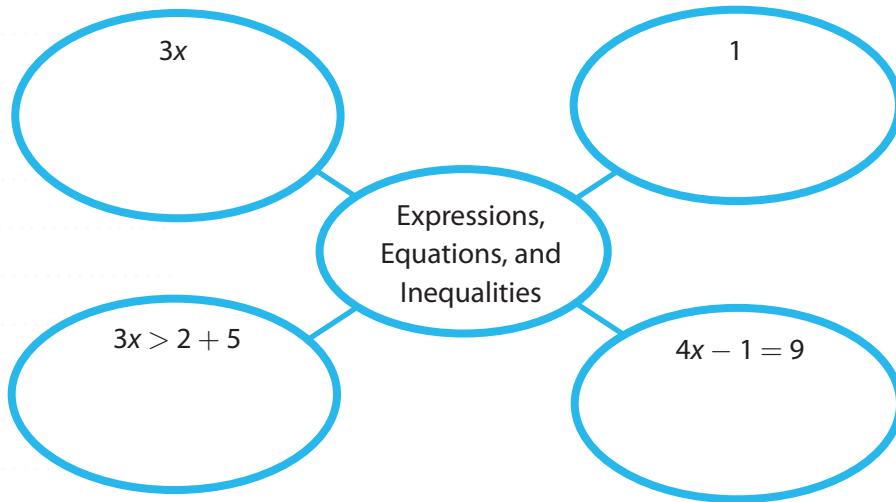$2x - 5 = -6$ *Place the equal sign.*

Write an algebraic equation for the sentence.

9. The difference between three times a number and 7 is 14. _____

10. The quotient of five times a number and 7 is no more than 10. _____

11. 14 less than 3 times a number is 5 more than half of the number. _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the bubble map. You may put more than one word in each oval.

- $3x$
- 1
- Expressions, Equations, and Inequalities
- $3x > 2 + 5$
- $4x - 1 = 9$

Understand Vocabulary

Complete the sentences using the review words.

1. A value of the variable that makes an equation true is a _____.

2. The set of all whole numbers and their opposites are _____.

3. An _____ is an expression that contains at least one variable.

Active Reading

Layered Book Before beginning the module, create a layered book to help you learn the concepts in this module. At the top of the first flap, write the title of the book, "Equations and Inequalities with Variables on Both Sides." Then label each flap with one of the lesson titles in this module. As you study each lesson, write important ideas, such as vocabulary and formulas, under the appropriate flap.

Unpacking the TEKS

Understanding the TEKS and the vocabulary terms in the TEKS will help you know exactly what you are expected to learn in this module.

TEKS 8.8.A

Write one-variable equations or inequalities with variables on both sides that represent problems using rational number coefficients and constants.

Key Vocabulary

coefficient *(coeficiente)*
The number that is multiplied by the variable in an algebraic expression.

constant *(constante)*
A value that does not change.

inequality *(desigualdad)*
A mathematical sentence that shows the relationship between quantities that are not equal.

What It Means to You

You will learn to write an equation that represents a situation, such as comparing costs in a real-world situation.

UNPACKING EXAMPLE 8.8.A

At Work It Out, a gym membership is $50 per month. At Get Fit, the initiation fee is $100 and membership is $30 per month. Write an equation to find the number of months for which the total cost is the same.

The cost of a membership at Work It Out for x months is represented by $50x$.

The cost of a membership at Get Fit for x months is represented by $100 + 30x$.

Total cost at Work It Out	=	Total cost at Get Fit
$50x$	=	$100 + 30x$

TEKS 8.8.C

Model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants.

Key Vocabulary

rational number *(número racional)*
A number that can be expressed as the ratio of two integers.

What It Means to You

You can write and solve an equation that has a variable on both sides of the equal sign.

UNPACKING EXAMPLE 8.8.C

Yellow Taxi has no pickup fee but charges $0.25 per mile. AAA Taxi charges $3 for pickup and $0.15 per mile. Find the number of miles for which the cost of the two taxis is the same.

$$0.25x = 3 + 0.15x$$
$$100(0.25x) = 100(3) + 100(0.15x)$$
$$25x = 300 + 15x$$
$$10x = 300$$
$$x = 30$$

The cost is the same for 30 miles.

Visit **my.hrw.com** to see all the **TEKS** unpacked.

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Equations with the Variable on Both Sides

? ESSENTIAL QUESTION

How can you represent and solve equations with the variable on both sides?

EXPLORE ACTIVITY   **TEKS** 8.8.C

Modeling an Equation with a Variable on Both Sides

Algebra tiles can model equations with a variable on both sides.

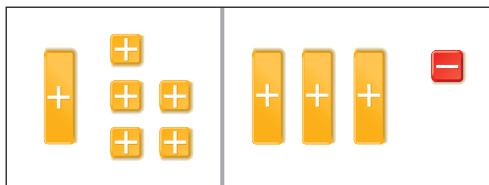**Use algebra tiles to model and solve $x + 5 = 3x - 1$.**

KEY

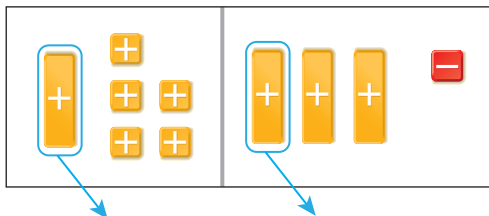$\boxed{+} = 1$

$\boxed{-} = -1$

$\boxed{+} + \boxed{-} = 0$

$\boxed{+} = x$

Model $x + 5$ on the left side of the mat and $3x - 1$ on the right side.
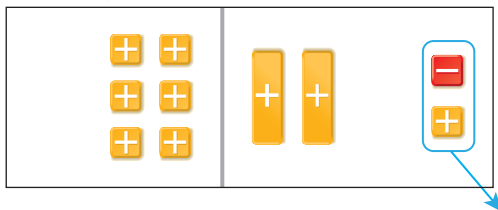Remember that $3x - 1$ is the same as

$3x + \rule{2cm}{0.4pt}$.

Remove one x-tile from both sides. This

represents subtracting $\rule{2cm}{0.4pt}$ from both sides of the equation.

Math Talk
Mathematical Processes

Why is a positive unit tile added to both sides in the third step?

Place one $+1$-tile on both sides. This

represents adding $\rule{2cm}{0.4pt}$ to both sides of the equation. Remove zero pairs.
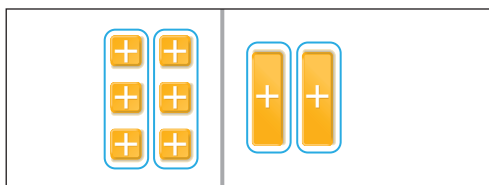
Separate each side into 2 equal groups.

One x-tile is equivalent to $\rule{1.5cm}{0.4pt}$ $+1$-tiles.

The solution is $\rule{1.5cm}{0.4pt}$ = $\rule{1.5cm}{0.4pt}$.

Reflect

1. How can you check the solution to $x + 5 = 3x - 1$ using algebra tiles?

$\rule{12cm}{0.4pt}$

$\rule{12cm}{0.4pt}$
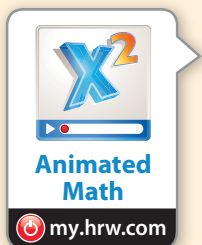
Solving an Equation with the Variable on Both Sides

Equations with the variable on both sides can be used to compare costs of real-world situations. To solve these equations, use inverse operations to get the variable terms on one side of the equation.

EXAMPLE 1 Real World

TEKS 8.8.A, 8.8.C

Andy's Rental Car charges an initial fee of $20 plus an additional $30 per day to rent a car. Buddy's Rental Car charges an initial fee of $36 plus an additional $28 per day. For what number of days is the total cost charged by the companies the same?

STEP 1 Write an expression representing the total cost of renting a car from Andy's Rental Car.

<div align="center">

Initial fee $+$ **Cost for x days**

$20 \quad + \quad 30x$
</div>

STEP 2 Write an expression representing the total cost of renting a car from Buddy's Rental Car.

<div align="center">

Initial fee $+$ **Cost for x days**

$36 \quad + \quad 28x$
</div>

STEP 3 Write an equation that can be solved to find the number of days for which the total cost charged by the companies would be the same.

<div align="center">

Total cost at Andy's $=$ **Total cost at Buddy's**

$20 + 30x \quad = \quad 36 + 28x$
</div>

STEP 4 Solve the equation for x.

$$20 + 30x = 36 + 28x \qquad \text{Write the equation.}$$
$$\underline{-28x \qquad -28x} \qquad \text{Subtract } 28x \text{ from both sides.}$$
$$20 + 2x = 36$$
$$\underline{-20 \qquad\qquad -20} \qquad \text{Subtract 20 from both sides.}$$
$$2x = 16$$
$$\frac{2x}{2} = \frac{16}{2} \qquad \text{Divide both sides by 2.}$$
$$x = 8$$

The total cost is the same if the rental is for 8 days.

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2. A water tank holds 256 gallons but is leaking at a rate of 3 gallons per week. A second water tank holds 384 gallons but is leaking at a rate of 5 gallons per week. After how many weeks will the amount of water in the two tanks be the same?

_____

Writing a Real-World Situation from an Equation

As shown in Example 1, an equation with the variable on both sides can be used to represent a real-world situation. You can reverse this process by writing a real-world situation for a given equation.

EXAMPLE 2 Real World

TEKS 8.8.B

My Notes

Write a real-world situation that could be modeled by the equation $150 + 25x = 55x$.

STEP 1 The left side of the equation consists of a constant plus a variable term. It could represent the total cost for doing a job where there is an initial fee plus an hourly charge.

STEP 2 The right side of the equation consists of a variable term. It could represent the cost for doing the same job based on an hourly charge only.

STEP 3 The equation $150 + 25x = 55x$ could be represented by this situation: A handyman charges $150 plus $25 per hour for house painting. A painter charges $55 per hour. How many hours would a job have to take for the handyman's fee and the painter's fee to be the same?

3. Write a real-world situation that could be modeled by the equation $30x = 48 + 22x$.

_____

_____

_____

_____

Guided Practice

Use algebra tiles to model and solve each equation. (Explore Activity)

1. $x + 4 = -x - 4$ _____

2. $2 - 3x = -x - 8$ _____

3. At Silver Gym, membership is $25 per month, and personal training sessions are $30 each. At Fit Factor, membership is $65 per month, and personal training sessions are $20 each. In one month, how many personal training sessions would Sarah have to buy to make the total cost at the two gyms equal? (Example 1)

_____

4. Write a real-world situation that could be modeled by the equation $120 + 25x = 45x$. (Example 2)

_____

_____

_____

5. Write a real-world situation that could be modeled by the equation $100 - 6x = 160 - 10x$. (Example 2)

_____

_____

_____

_____

_____

? ESSENTIAL QUESTION CHECK-IN

6. How can you solve an equation with the variable on both sides?

_____

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_____

_____

11.1 Independent Practice

TEKS 8.8.A, 8.8.B, 8.8.C

7. Derrick's Dog Sitting and Darlene's Dog Sitting are competing for new business. The companies ran the ads shown.

a. Write and solve an equation to find the number of hours for which the total cost will be the same for the two services.

_____

b. **Analyze Relationships** Which dog sitting service is more economical to use if you need 5 hours of service? Explain.

_____

_____

Derrick's Dog Sitting

$12 plus $5 per hour

Darlene's Dog Sitting

$18 plus $3 per hour

8. Country Carpets charges $22 per square yard for carpeting, and an additional installation fee of $100. City Carpets charges $25 per square yard for the same carpeting, and an additional installation fee of $70.

a. Write and solve an equation to find the number of square yards of carpeting for which the total cost charged by the two companies will be the same.

_____

b. **Justify Reasoning** Mr. Shu wants to hire one of the two carpet companies to install carpeting in his basement. Is he more likely to hire Country Carpets or City Carpets? Explain your reasoning.

_____

_____

_____

_____

Write an equation to represent each relationship. Then solve the equation.

9. Two less than 3 times a number is the same as the number plus 10.

_____

10. A number increased by 4 is the same as 19 minus 2 times the number.

_____

11. Twenty less than 8 times a number is the same as 15 more than the number.

_____

12. The charges for an international call made using the calling card for two phone companies are shown in the table.

Phone Company	Charges
Company A	35¢ plus 3¢ per minute
Company B	45¢ plus 2¢ per minute

a. What is the length of a phone call that would cost the same no matter which company is used?

_____

b. **Analyze Relationships** When is it better to use the card from Company B?

_____

_____

13. **Draw Conclusions** Liam is setting up folding chairs for a meeting. If he arranges the chairs in 9 rows of the same length, he has 3 chairs left over. If he arranges the chairs in 7 rows of that same length, he has 19 left over. How many chairs does Liam have?

_____

14. **Explain the Error** Rent-A-Tent rents party tents for a flat fee of $365 plus $125 a day. Capital Rentals rents party tents for a flat fee of $250 plus $175 a day. Delia wrote the following equation to find the number of days for which the total cost charged by the two companies would be the same:

$$365x + 125 = 250x + 175$$

Find and explain the error in Delia's work. Then write the correct equation.

_____

_____

_____

15. **Persevere in Problem Solving** Lilliana is training for a marathon. She runs the same distance every day for a week. On Monday, Wednesday, and Friday, she runs 3 laps on a running trail and then runs 6 more miles. On Tuesday and Sunday, she runs 5 laps on the trail and then runs 2 more miles. On Saturday, she just runs laps. How many laps does Lilliana run on Saturday?

_____

Work Area

Equations with Rational Numbers

? ESSENTIAL QUESTION
How can you solve equations with rational number coefficients and constants?

Math On the Spot
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Solving an Equation that Involves Fractions

To solve an equation with the variable on both sides that involves fractions, start by eliminating the fractions from the equation.

EXAMPLE 1
TEKS 8.8.C

Solve $\frac{7}{10}n + \frac{3}{2} = \frac{3}{5}n + 2$.

$$10 = 10 \times 1$$
$$= 5 \times 2$$
$$= 2 \times 5$$

STEP 1 Determine the least common multiple of the denominators: LCM(**10**, **5**, **2**) = 10

STEP 2 Multiply both sides of the equation by the LCM.

$$10\left(\frac{7}{10}n + \frac{3}{2}\right) = 10\left(\frac{3}{5}n + 2\right)$$

$${}^1 10\left(\frac{7}{10}n\right) + {}^5 10\left(\frac{3}{2}\right) = {}^2 10\left(\frac{3}{5}n\right) + 10(2)$$

$$7n + 15 = 6n + 20$$

STEP 3 Use inverse operations to solve the equation.

$$
\begin{array}{rcl}
7n + 15 & = & 6n + 20 \\
\underline{-15} & & \underline{-15} \quad \text{Subtract 15 from both sides.}\\
7n & = & 6n + 5 \\
\underline{-6n} & & \underline{-6n} \\
n & = & 5 \quad \text{Subtract } 6n \text{ from both sides.}
\end{array}
$$

Math Talk
Mathematical Processes

The constant on the right side, 2, is not a fraction. Why do you still need to multiply it by the LCM, 10?

Reflect

1. What is the advantage of multiplying both sides of the equation by the least common multiple of the denominators in the first step?

_____

2. What If? What happens in the first step if you multiply both sides by a common multiple of the denominators that is not the LCM?

_____

_____

YOUR TURN

Solve.

3. $\frac{1}{7}k - 6 = \frac{3}{7}k + 4$ _____

4. $\frac{5}{6}y + 1 = -\frac{1}{2}y + \frac{1}{4}$ _____

Solving an Equation that Involves Decimals

Solving an equation with the variable on both sides that involves decimals is similar to solving an equation with fractions. But instead of first multiplying both sides by the LCM, multiply by a power of 10 to eliminate the decimals.

EXAMPLE 2

TEKS 8.8.A, 8.8.C

Javier walks from his house to the zoo at a constant rate. After walking 0.75 mile, he meets his brother, Raul, and they continue walking at the same constant rate. When they arrive at the zoo, Javier has walked for 0.5 hour and Raul has walked for 0.2 hour. What is the rate in miles per hour at which the brothers walked to the zoo?

STEP 1 Write an equation for the distance from the brothers' house to the zoo, using the fact that distance equals rate times time. Let $r =$ the brothers' walking rate.

$$\underbrace{\text{distance to zoo}}_{0.2r + 0.75} = \underbrace{\text{distance to zoo}}_{0.5r}$$

STEP 2 Multiply both sides of the equation by $10^2 = 100$.

> Multiplying by 100 clears the equation of decimals. Multiplying by 10 does not: $10 \times 0.75 = 7.5$.

$$100(0.2r) + 100(0.75) = 100(0.5r)$$
$$20r + 75 = 50r$$

STEP 3 Use inverse operations to solve the equation.

$$
\begin{array}{rcl}
20r + 75 & = & 50r \\
-20r & & -20r \\
\hline
75 & = & 30r \\
\frac{75}{30} & = & \frac{30r}{30} \\
2.5 & = & r
\end{array}
$$

Write the equation.
Subtract $20r$ from both sides.

Divide both sides by 30.

So, the brothers' constant rate of speed was 2.5 miles per hour.

5. Logan has two aquariums. One aquarium contains 1.3 cubic feet of water and the other contains 1.9 cubic feet of water. The water in the larger aquarium weighs 37.44 pounds more than the water in the smaller aquarium. Write an equation with a variable on both sides to represent the situation. Then find the weight of 1 cubic foot of water.

_____

Writing a Real-World Situation from an Equation

Real-world situations can often be represented by equations involving fractions and decimals. Fractions and decimals can represent quantities such as weight, volume, capacity, time, and temperature. Decimals can also be used to represent dollars and cents.

EXAMPLE 3 · Real World · TEKS 8.8.B

My Notes

Write a real-world situation that can be modeled by the equation $0.95x = 0.55x + 60$.

The left side of the equation consists of a variable term. It could represent the total cost for x items.

The right side of the equation consists of a variable term plus a constant. It could represent the total cost for x items plus a flat fee.

The equation $0.95x = 0.55x + 60$ could be represented by this situation: Toony Tunes charges $0.95 for each song you download. Up With Downloads charges $0.55 for each song but also charges an annual membership fee of $60. How many songs must a customer download in a year so that the cost will be the same at both websites?

6. Write a real-world problem that can be modeled by the equation $\frac{1}{3}x + 10 = \frac{3}{5}x$.

_____

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_____

1. Sandy is upgrading her Internet service. Fast Internet charges $60 for installation and $50.45 per month. Quick Internet has free installation but charges $57.95 per month. **(Example 2)**

a. Write an equation that can be used to find the number of months after which the Internet service would cost the same.

_____

b. Solve the equation.

_____

Solve. **(Examples 1 and 2)**

2. $\frac{3}{4}n - 18 = \frac{1}{4}n - 4$

3. $6 + \frac{4}{5}b = \frac{9}{10}b$

4. $\frac{2}{11}m + 16 = 4 + \frac{6}{11}m$

_____ _____ _____

5. $2.25t + 5 = 13.5t + 14$

6. $3.6w = 1.6w + 24$

7. $-0.75p - 2 = 0.25p$

_____ _____ _____

8. Write a real-world problem that can be modeled by the equation $1.25x = 0.75x + 50$. **(Example 3)**

_____

_____

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_____

_____

? ESSENTIAL QUESTION CHECK-IN

9. How does the method for solving equations with fractional or decimal coefficients and constants compare with the method for solving equations with integer coefficients and constants?

_____

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_____

11.2 Independent Practice

TEKS 8.8.A, 8.8.B, 8.8.C

Personal Math Trainer

Online Assessment and Intervention

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10. Members of the Wide Waters Club pay $105 per summer season, plus $9.50 each time they rent a boat. Nonmembers must pay $14.75 each time they rent a boat. How many times would a member and a non-member have to rent a boat in order to pay the same amount?

_____

11. Margo can purchase tile at a store for $0.79 per tile and rent a tile saw for $24. At another store she can borrow the tile saw for free if she buys tiles there for $1.19 per tile. How many tiles must she buy for the cost to be the same at both stores?

_____

12. The charges for two shuttle services are shown in the table. Find the number of miles for which the cost of both shuttles is the same.

	Pickup Charge ($)	Charge per Mile ($)
Easy Ride	10	0.10
Best	0	0.35

_____

13. Multistep Rapid Rental Car charges a $40 rental fee, $15 for gas, and $0.25 per mile driven. For the same car, Capital Cars charges $45 for rental and gas and $0.35 per mile.

**a.** For how many miles is the rental cost at both companies the same?

_____

b. What is that cost?

_____

14. Write an equation with the solution $x = 20$. The equation should have the variable on both sides, a fractional coefficient on the left side, and a fraction anywhere on the right side.

_____

15. Write an equation with the solution $x = 25$. The equation should have the variable on both sides, a decimal coefficient on the left side, and a decimal anywhere on the right side. One of the decimals should be written in tenths, the other in hundredths.

_____

16. Geometry The perimeters of the rectangles shown are equal. What is the perimeter of each rectangle?
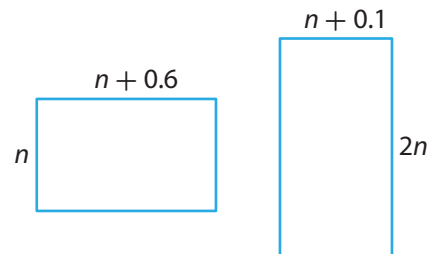
17. Analyze Relationships The formula $F = 1.8C + 32$ gives the temperature in degrees Fahrenheit (F) for a given temperature in degrees Celsius (C). There is one temperature for which the number of degrees Fahrenheit is equal to the number of degrees Celsius. Write an equation you can solve to find that temperature and then use it to find the temperature.

_____

18. Explain the Error Agustin solved an equation as shown. What error did Agustin make? What is the correct answer?

_____

_____

_____

_____

$$\frac{1}{3}x - 4 = \frac{3}{4}x + 1$$

$$12\left(\frac{1}{3}x\right) - 4 = 12\left(\frac{3}{4}x\right) + 1$$

$$4x - 4 = 9x + 1$$

$$-5 = 5x$$

$$x = -1$$

H.O.T. FOCUS ON HIGHER ORDER THINKING

19. Draw Conclusions Solve the equation $\frac{1}{2}x - 5 + \frac{2}{3}x = \frac{7}{6}x + 4$. Explain your results.

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_____

20. Look for a Pattern Describe the pattern in the equation. Then solve the equation.

$$0.3x + 0.03x + 0.003x + 0.0003x + \ldots = 3$$

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21. Critique Reasoning Jared wanted to find three consecutive even integers whose sum was 4 times the first of those integers. He let k represent the first integer, then wrote and solved this equation: $k + (k + 1) + (k + 2) = 4k$. Did he get the correct answer? Explain.

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_____

_____

_____

Work Area

Inequalities with the Variable on Both Sides

? ESSENTIAL QUESTION

How can you use inequalities to represent real-world problems?

EXPLORE ACTIVITY Real World

TEKS 8.8.A

Modeling a Real-World Situation with an Inequality

Many real-world situations can be modeled by inequalities. Some phrases that indicate an inequality are "less than," "greater than," "no more than," and "at least."

Super-Clean house cleaning company charges a fee of $384 to power wash a house plus $2 per linear foot. Power Bright charges $6 per linear foot, but no flat fee. Write an inequality that can be solved to find the number of linear feet a house must have to make the total cost charged by Super-Clean less than the cost charged by Power Bright.

A Translate from words into symbols.

Let *l* represent the number of _____ of the house.

Super-Clean fee	plus	$2 per linear foot	times	number of linear feet	is less than	Power Bright's charge per linear foot	times	number of linear feet
☐	☐	☐	☐	☐	☐	☐	☐	☐

B Write the inequality.

_____

Reflect

1. How did you decide which inequality symbol to use?

_____

_____

_____

Writing an Inequality for a Real-World Situation

Once you have modeled a real-world situation with an inequality, you can solve the inequality to answer the question posed by the situation. Use inverse operations to get the variable terms on one side of the inequality and the constant terms on the other side. Recall that if you multiply or divide both sides by a negative number, you need to reverse the inequality symbol.

My Notes

EXAMPLE 1 Real World   TEKS 8.8.A

A yellow hot-air balloon is 100 feet off the ground and rising at a rate of 8 feet per second. An orange hot-air balloon is 160 feet off the ground and rising at a rate of 5 feet per second. After how long will the yellow balloon be higher than the orange balloon?

STEP 1 Write an expression representing the height of the yellow balloon.

Current height (ft)	+	Number of feet it rises in s seconds
100	+	$8s$

> Let s represent the number of seconds the balloons are rising.

STEP 2 Write an expression representing the height of the orange balloon.

Current height (ft)	+	Number of feet it rises in s seconds
160	+	$5s$

STEP 3 Write an inequality that can be solved to find the number of seconds it will take for the height of the yellow balloon to be greater than the height of the orange balloon.

Height of yellow balloon	>	Height of orange balloon
$100 + 8s$	>	$160 + 5s$

STEP 4 Solve the inequality for s.

$$100 + 8s > 160 + 5s$$
$$\underline{ -5s \qquad -5s}$$ Subtract $5s$ from both sides.
$$100 + 3s > 160$$
$$\underline{-100 \qquad\quad -100}$$ Subtract 100 from both sides.
$$3s > 60$$
$$\frac{3s}{3} > \frac{60}{3}$$ Divide both sides by 3.
$$s > 20$$

The yellow balloon will be higher than the orange balloon after 20 seconds.

YOUR TURN

2. The temperature in Amarillo is 74°F and is increasing at a rate of 2°F per hour. In Houston, it is 68°F and increasing 4°F per hour. Write and solve an inequality to find how long it will take for the temperature in Houston to exceed the temperature in Amarillo.

_____

Writing a Real-World Situation from an Inequality

As shown in Example 1, inequalities with the variable on both sides can be used to represent real-world situations. You can reverse this process by writing a real-world situation for a given inequality.

EXAMPLE 2 Real World

TEKS 8.8.B

Write a real-world situation that can be modeled by the inequality $50 - 3d < 30 - 2d$.

Each side of the inequality consists of a constant with a variable term subtracted from it.

This side can represent a lunch account that begins with $50 and has $3 taken out each day.

This side can represent a lunch account that begins with $30 and has $2 taken out each day.

$$50 - 3d \quad < \quad 30 - 2d$$

The inequality symbol is $<$, so find when the balance in the first account is *less than* the balance in the second account.

The inequality $50 - 3d < 30 - 2d$ can represent this situation: Joe has $50 in his lunch account and spends $3 each day. Renee has $30 in her lunch account and spends $2 each day. After how many days will the balance in Joe's account be less than the balance in Renee's account?

YOUR TURN

3. Write a real-world situation that can be modeled by the inequality $46h > 84 + 25h$.

_____

_____

_____

1. *The Daily Record* charges a fee of $525 plus $75 per week to run an ad. *The Chronicle* charges $150 per week. (Explore Activity and Example 1)

    a. Write an inequality that can be solved to find the number of weeks an ad must run to make the total cost of running an ad in *The Daily Record* less than the cost in *The Chronicle*.

    _____

    b. Solve your inequality.

    _____

2. The inventory report at Jacob's Office Supplies shows that there are 150 packages of pencils and 120 packages of markers. If the store sells 7 packages of pencils each day and 5 packages of markers each day, write and solve an inequality to find in how many days the number of packages of pencils will be fewer than the number of packages of markers. (Example 1)

_____

3. Write a real-world situation that can be modeled by the inequality $834 + 14s > 978 - 10s$. Then solve the inequality. (Example 2)

_____

_____

_____

_____

_____

_____

? ESSENTIAL QUESTION CHECK-IN

4. How can you use inequalities to represent real-world problems?

_____

_____

_____

_____

11.3 Independent Practice

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5. The school band is selling pizzas for $7 each to raise money for new uniforms. The supplier charges $100 plus $4 per pizza.

a. Write an inequality that can be solved to find the number of pizzas the band members must sell to make a profit.

_____

b. Solve your inequality.

_____

6. Nadia was offered a job selling ads for a magazine. She must agree to one of the following payment options:

| **Choice A** | $110 for each ad she sells |
| **Choice B** | A weekly salary of $150, plus $85 for each ad she sells |

Write and solve an inequality to determine the number of ads Nadia would need to sell per week in order for Choice A to be the better choice.

_____

7. Represent Real-World Problems Write a real-world situation that can be modeled by the inequality $20x > 30 + 15x$.

_____

_____

_____

_____

_____

Write an inequality to represent each relationship. Then solve your inequality.

8. Ten less than five times a number is less than six times the number decreased by eight.

_____

9. The sum of a number and twenty is greater than four times the number decreased by one.

_____

10. Bob's Bagels offers pre-paid cards and has the specials shown. Carol has a $50 card she uses to buy coffee and a bagel each week. Diego has a $60 card he uses to buy tea and a breakfast sandwich each week. Write and solve an inequality to find the number of weeks in which the balance on Carol's card will be greater than the balance on Diego's card.

H.O.T. FOCUS ON HIGHER ORDER THINKING

11. Critique Reasoning Ahmad and Cameron solved the same inequality but got different answers. One of the solutions is incorrect. Find and correct the error.

Ahmad's Solution

$$28 - 7x < 40 - 3x$$
$$\underline{+\ 7x \qquad\quad +\ 7x}$$
$$28 < 40 + 4x$$
$$\underline{-\ 40 \quad -\ 40}$$
$$-12 < \qquad 4x$$
$$\frac{-12}{4} < \frac{4x}{4}$$
$$-3 < x$$

Cameron's Solution

$$28 - 7x < 40 - 3x$$
$$\underline{+\ 3x \qquad\quad +\ 3x}$$
$$28 - 4x < 40$$
$$\underline{-\ 28 \qquad\quad -\ 28}$$
$$-4x < 12$$
$$\frac{-4x}{-4} < \frac{12}{-4}$$
$$x < -3$$

12. Represent Real-World Problems Meena sells apple pies at the farmer's market. She charges $12 for each pie. It costs her $5 to make each pie, and there is a $35 fee she must pay each week to have a booth at the market.

a. Write and solve an inequality to find the number of pies Meena must sell each week in order to make a profit.

b. What If? Suppose the fee to have a booth at the farmer's market increases to $40. Will the new fee increase the number of pies Meena will have to sell in order to make a profit? Explain your reasoning.

? **ESSENTIAL QUESTION**

How can you use inequalities with rational number coefficients and constants to represent real-world problems?

Modeling with an Inequality that Involves Fractions

If an inequality contains fractions, you can to multiply both sides by the least common multiple of the denominators to clear the fractions.

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EXAMPLE 1
TEKS 8.8.A

Write an inequality to represent the relationship "Twice a number plus four is greater than two thirds of the number". Then solve your inequality.

STEP 1 Write an inequality.

Twice a number	plus	four	is greater than	two thirds of the number.
$2x$	$+$	4	$>$	$\frac{2}{3}x$

An inequality is $2x + 4 > \frac{2}{3}x$.

STEP 2 Multiply both sides of the inequality by the LCM, 3.

$$3(2x) + 3(4) > 3\left(\frac{2}{3}x\right)$$

$$6x + 12 > \overset{1}{3}\left(\frac{2}{\underset{1}{3}}x\right)$$

$$6x + 12 > 2x$$

STEP 3 Use inverse operations to solve the inequality.

$$\begin{array}{ccc} 6x + 12 & > & 2x \\ \underline{-6x} & & \underline{-6x} \\ 12 & > & -4x \end{array}$$
Subtract 6x from both sides.

$$\frac{12}{-4} < \frac{-4x}{-4}$$
Divide both sides by −4, reversing the direction of the inequality symbol.

$$-3 < x$$

Math Talk
Mathematical Processes
What could you do differently in Step 3 so you would not have to reverse the inequality symbol?

YOUR TURN

1. Write an inequality to represent the relationship "Three-fourths of a number is greater than five less than the number." Then solve your inequality. _____

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Modeling with an Inequality that Involves Decimals

To solve an inequality with the variable on both sides that involves decimals, multiply both sides by a power of 10 to eliminate the decimals.

EXAMPLE 2

TEKS 8.8.A, 8.8.C

Two water tanks hold 28.62 gallons and 31.2 gallons of water. The larger tank is leaking at a rate of 0.12 gallon per hour. The smaller tank is leaking at a rate of 0.08 gallon per hour. After how many hours will there be less water in the larger tank than in the smaller tank?

STEP 1 Write an inequality. Let h represent the number of hours.

amount in larger tank $<$ amount in smaller tank

$$31.2 - 0.12h < 28.62 - 0.08h$$

STEP 2 Multiply both sides of the inequality by $10^2 = 100$.

$$100(31.2) - 100(0.12h) < 100(28.62) - 100(0.08h)$$

$$3{,}120 - 12h < 2{,}862 - 8h$$

STEP 3 Use inverse operations to solve the inequality.

$$
\begin{array}{rcl}
3{,}120 - 12h & < & 2{,}862 - 8h \\
-2{,}862 & & -2{,}862 \\
\hline
258 - 12h & < & -8h \\
+12h & & +12h \\
\hline
258 & < & 4h
\end{array}
$$
Subtract 2,862 from both sides.

Add 12h to both sides.

$$\frac{258}{4} < \frac{4h}{4}$$
Divide both sides by 4.

$$64.5 < h$$

So, after 64.5 hours, there will be less water in the larger tank.

Reflect

2. In Step 2, why do you multiply both sides by 100 rather than by 10?

_____

_____

YOUR TURN

3. Bamboo Plant A is 1.2 meters tall and growing at a rate of 0.45 meter per day. Bamboo Plant B is 0.85 meter tall and growing 0.5 meter per day. After how many days will Plant B be taller than Plant A? _____

Writing a Real-World Situation from an Inequality

By studying the way that a given inequality is constructed, you can write a real-world situation that the inequality models. The table gives phrases you can use for the symbols that appear in inequalities.

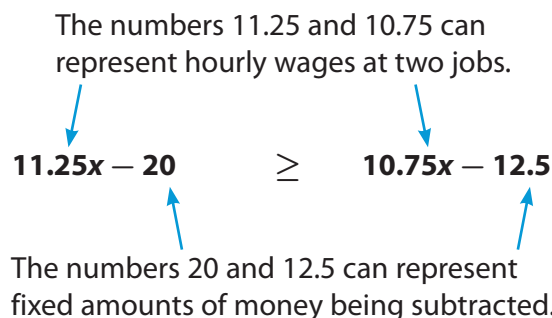| Symbol	$<$	$>$	$\leq$	$\geq$
Phrases	less than; fewer than	greater than; more than	less than or equal to; at most; no more than; a maximum of	greater than or equal to; at least; no less than; a minimum of

EXAMPLE 3

TEKS 8.8.B

Write a real-world situation that can be modeled by the inequality $11.25x - 20 \geq 10.75x - 12.5$.

Each side of the inequality consists of a variable term with a constant subtracted from it. The left side must exceed or be equal to the right side.

The numbers 11.25 and 10.75 can represent hourly wages at two jobs.

$$11.25x - 20 \qquad \geq \qquad 10.75x - 12.5$$

The numbers 20 and 12.5 can represent fixed amounts of money being subtracted.

The inequality $11.25x - 20 \geq 10.75x - 12.5$ can represent this situation: Ryan earns $11.25 per hour. His transit cost to and from work is $20 per week. Tony earns $10.75 per hour. His weekly transit cost is $12.50. After how many hours of work in a week do Ryan's earnings minus transit cost exceed Tony's earnings minus transit cost?

YOUR TURN

4. Write a real-world problem that can be modeled by the inequality $-10 - \frac{1}{4}x > 20 - \frac{1}{2}x$.

_____

_____

_____

_____

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My Notes

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Write an inequality to represent each relationship. Then solve your inequality. (Example 1)

1. Three fourths of a number is less than six plus the number.

_____

2. One fifth of a number added to eleven is greater than three fourths of the number.

_____

3. Ian wants to promote his band on the Internet. Site A offers website hosting for $4.95 per month with a $49.95 startup fee. Site B offers website hosting for $9.95 per month with no startup fee. Write and solve an inequality to determine how many months Ian could have his website on Site B and still keep his total cost less than on Site A. (Example 2)

_____

4. Write a real-world problem that can be modeled by the inequality $10x > 5.5x + 31.5$. (Example 3)

_____

_____

_____

_____

_____

? ESSENTIAL QUESTION CHECK-IN

5. How can you use inequalities with rational number coefficients and constants to represent real-world problems?

_____

_____

_____

_____

_____

11.4 Independent Practice

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6. Rugs Emporium installs carpet for $80 plus $9.50 per square yard of carpet. Carpets-4-U charges $120 for installation and $7.50 per square yard of carpet. Write and solve an inequality to find the number of square yards of carpet for which Rugs Emporium charges more than Carpets-4-U.

_____

7. Billie needs to have her refrigerator repaired. She contacts two appliance repair companies and is given the rates shown in the table. Write and solve an inequality to find the number of hours for which Ace's charge is less than or the same as Acme's charge.

	Flat Fee Charge ($)	Charge per Hour ($)
Acme Repair	49	22.50
Ace Repair	0	34.75

_____

8. Write an inequality with the solution $x < 12$. The inequality should have the variable on both sides, a decimal coefficient of the variable on the left side, and a decimal anywhere on the right side. One of the decimals should be written in tenths, the other in hundredths.

_____

9. Write an inequality with the solution $x \geq 4$. The inequality should have the variable on both sides, a fractional coefficient of the variable on the left side, and a fraction anywhere on the right side.

_____

10. Multistep According to the Triangle Inequality, the length of the longest side of a triangle must be less than the sum of the lengths of the other two sides. The two shortest sides of a triangle measure $\frac{3}{10}$ and $x + \frac{1}{5}$, and the longest side measures $\frac{3}{2}x$.

a. Write an inequality that uses the Triangle Inequality to relate the three sides.

_____

b. For what values of x is the Triangle Inequality true for this triangle?

_____

11. José and Maris work for different car dealerships. José earns a monthly salary of $3,500 plus a 6% commission on sales. Maris earns a monthly salary of $4,000 plus a 4% commission on sales. Above what value of sales are José's monthly earnings more than those of Maris?

_____

12. Multistep Consider the figures shown.
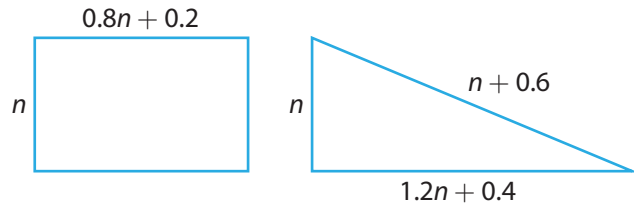
a. Write an expression for the perimeter of each figure.

$0.8n + 0.2$

n n $n + 0.6$

$1.2n + 0.4$

_____

_____

b. For what values of *n* will the rectangle have a greater perimeter than the triangle?

_____

Work Area

13. Draw Conclusions The inequality $y < \frac{2}{3}x + 5$ represents all points on the coordinate plane that are below the line $y = \frac{2}{3}x + 5$.

a. If you substitute the *x*- and *y*-values of the point (6, 11) into the inequality, do you get a true statement? What does this mean?

_____

b. If $y = 2\frac{1}{2}$, what are the possible values of *x*? What does this mean?

_____

_____

_____

14. Communicate Mathematical Ideas Describe what you can do when solving an inequality to assure that the coefficient of the variable term will be positive.

_____

_____

_____

15. Make a Conjecture For what values of *x* is the absolute value of *x* less than or equal to 1.5? (*Hint*: Substitute numbers into the inequality $|x| \leq 1.5$ and see which numbers make the inequality true.) Choose three other positive numbers and answer the question using those numbers. Make a conjecture about when the absolute value of *x* is less than or equal to a given positive number *a*.

_____

_____

_____

_____

Ready to Go On?
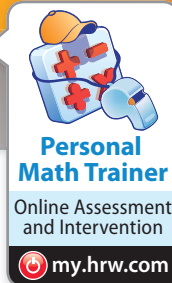
11.1 Equations with the Variable on Both Sides

Solve.

1. $4a - 4 = 8 + a$ _____

2. $4x + 5 = x + 8$ _____

3. Hue is arranging chairs. She can form 6 rows of a given length with 3 chairs left over, or 8 rows of that same length if she gets 11 more chairs. Write and solve an equation to find how many chairs are in that row length. _____

11.2 Equations with Rational Numbers

Solve.

4. $\frac{2}{3}s - \frac{2}{3} = \frac{s}{6} + \frac{4}{3}$ _____

5. $1.5d + 3.25 = 1 + 2.25d$ _____

6. Happy Paws charges $19.00 plus $1.50 per hour to keep a dog during the day. Woof Watchers charges $15.00 plus $2.75 per hour. Write and solve an equation to find for how many hours the total cost of the services is equal.

_____

11.3 Inequalities with the Variable on Both Sides

7. Write an inequality to represent the relationship "Two less than 2 times a number is greater than the number plus 64." Then solve your inequality.

_____

11.4 Inequalities with Rational Numbers

8. One prepaid cell phone company charges $0.028 per minute and a $3 monthly fee. Another company charges $0.034 per minute with no monthly fee. For what numbers of minutes per month are the charges for the first company cheaper? _____

? ESSENTIAL QUESTION

9. How can you use equations with the variable on both sides to solve real-world problems?

_____

_____

Selected Response

1. Two cars are traveling in the same direction. The first car is going 40 mi/h and the second car is going 55 mi/h. The first car left 3 hours before the second car. Which equation could you solve to find how many hours it will take for the second car to catch up to the first car?

Ⓐ $55t + 3 = 40t$

Ⓑ $55t + 165 = 40t$

Ⓒ $40t + 3 = 55t$

Ⓓ $40t + 120 = 55t$

2. A wide screen television display measures approximately 15 inches high and 27 inches wide. A television is advertised by giving the approximate length of the diagonal of its screen. How should the television be advertised?

Ⓐ 36 inches   Ⓒ 30 inches

Ⓑ 31 inches   Ⓓ 21 inches

3. Shawn's Rentals charges $27.50 per hour to rent a surfboard and a wet suit. Darla's Surf Shop charges $23.25 per hour to rent a surfboard plus $17 extra for a wetsuit. For what total number of hours are the charges for Shawn's Rentals the same as the charges for Darla's Surf Shop?

Ⓐ 3   Ⓒ 5

Ⓑ 4   Ⓓ 6

4. Which of the following is an irrational number?

Ⓐ -8 Ⓒ $\sqrt{11}$

Ⓑ 4.63   Ⓓ $\frac{1}{3}$

5. Carlos has at least as many action figures in his collection as Josh. Carlos has 5 complete sets plus 4 individual figures. Josh has 3 complete sets plus 14 individual figures. Which inequality represents how many action figures can be in a complete set?
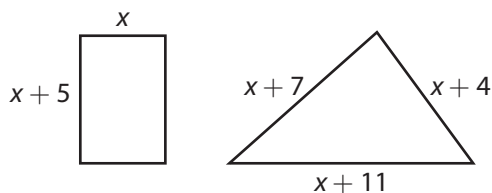
Ⓐ $x \geq 7$ Ⓒ $x \geq 5$

Ⓑ $x \leq 7$ Ⓓ $x \leq 5$

6. Which inequality represents the solution to $1.25x + 2.5 < 2.75x - 6.5$?

Ⓐ $x > 6$ Ⓒ $x > 2.25$

Ⓑ $x < 6$ Ⓓ $x < 2.25$

Gridded Response

7. If both figures have the same perimeter, what is the perimeter of each figure?

x

$x + 5$

$x + 7$ $x + 4$

$x + 11$

Study Guide Review

Equations and Inequalities with the Variable on Both Sides

? ESSENTIAL QUESTION

How can you use equations and inequalities with variables on both sides to solve real-world problems?

EXAMPLE 1

A tutor gives students a choice of how to pay: a base rate of $20 plus $8 per hour, or a set rate of $13 per hour. Find the number of hours of tutoring for which the cost is the same for either choice.

Plan 1 cost: $20 + 8x$ Plan 2 cost: $13x$

$$20 + 8x = 13x \qquad \text{Write the equation.}$$
$$\underline{-8x \qquad -8x} \qquad \text{Subtract } 8x \text{ from both sides.}$$
$$20 = 5x \qquad \text{Divide both sides by 5.}$$
$$x = 4$$

The cost is the same for 4 hours of tutoring.

EXAMPLE 2

Solve $0.6y - 12.3 < 7.4 - 1.9y$.

$$0.6y - 12.3 < 7.4 - 1.9y \qquad\qquad \text{Write the inequality.}$$
$$10(0.6y) - 10(12.3) < 10(7.4) - 10(1.9y) \qquad \text{Multiply both sides by 10.}$$
$$6y - 123 < 74 - 19y$$
$$\underline{+123 \qquad +123} \qquad\qquad\qquad \text{Add 123 to both sides.}$$
$$6y < 197 - 19y$$
$$\underline{+19y \qquad\qquad +19y} \qquad\qquad\quad \text{Add } 19y \text{ to both sides.}$$
$$25y < 197$$
$$y < 7.88 \qquad\qquad\qquad\qquad \text{Divide both sides by 25.}$$

EXERCISES

Solve. (Lessons 11.1, 11.2, 11.3, 11.4)

1. $13 - 6y = 8y$

2. $\frac{1}{5}x + 5 = 19 - \frac{1}{2}x$

3. $7.3t + 22 \leq 2.1t - 22.2$

_____

_____

_____

4. $7 - 45z < 5z + 13$

5. $1.4 + \frac{2}{5}e \geq \frac{3}{15}e - 0.8$

6. $0.75x - 6.5 = -0.5 - 0.25x$

_____

_____

_____

7. Write a real-world situation that could be modeled by the equation $650 + 10m = 60m + 400$. (Lesson 11.1)

_____

_____

_____

_____

8. John is trying to decide which carpeting company to use to put carpet in his living room. Carla's Carpeting charges $45 plus $5.50 per square foot. Fred's Flooring charges $195 plus $4.25 per square foot. For what size room is Carla's Carpeting cheaper than Fred's Flooring? (Lesson 11.3)

_____

Unit 4 Performance Tasks

1. **CAREERS IN MATH** **Hydraulic Engineer** A hydraulic engineer is studying the pressure in a particular fluid. The pressure is equal to the atmospheric pressure 101 kN/m plus 8 kN/m for every meter below the surface, where kN/m is kilonewtons per meter, a unit of pressure.

a. Write an expression for the pressure at a depth of d_1 meters below the liquid surface.   _____

b. Write and solve an equation to find the depth at which the pressure is 200 kN/m.

_____

c. The hydraulic engineer alters the density of the fluid so that the pressure at depth d_2 below the surface is atmospheric pressure 101 kN/m plus 9 kN/m for every meter below the surface. Write an expression for the pressure at depth d_2.   _____

d. If the pressure at depth d_1 in the first fluid is equal to the pressure at depth d_2 in the second fluid, what is the relationship between d_1 and d_2? Explain how you found your answer.

_____

_____

_____

UNIT 4 **MIXED REVIEW**

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Selected Response

1. Ricardo and John start swimming from the same location. Ricardo starts 15 seconds before John and swims at a rate of 3 feet per second. John swims at a rate of 4 feet per second in the same direction as Ricardo. Which equation could you solve to find how long it will take John to catch up with Ricardo?

Ⓐ $4t + 3 = 3t$

Ⓑ $4t + 60 = 3t$

Ⓒ $3t + 3 = 4t$

Ⓓ $3t + 45 = 4t$

2. Gina and Rhonda work for different real estate agencies. Gina earns a monthly salary of $5,000 plus a 6% commission on her sales. Rhonda earns a monthly salary of $6,500 plus a 4% commission on her sales. How much must each sell to earn the same amount in a month?

Ⓐ $1,500

Ⓑ $15,000

Ⓒ $75,000

Ⓓ $750,000

3. Which is the measure of $\angle BAC$ in the triangle below?


Ⓐ 51°

Ⓑ 61°

Ⓒ 68°
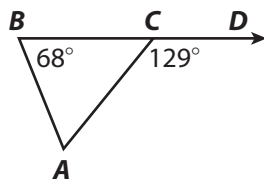
Ⓓ 71°

4. A movie theater has two membership plans. Under Plan A you pay $6 a month plus $4 for each ticket you buy. Under Plan B you pay $18 a month plus $2 for each ticket you buy. Which inequality represents the situation when Plan B becomes less expensive than Plan A?

Ⓐ $x > 2$

Ⓑ $x < 2$

Ⓒ $x > 6$

Ⓓ $x < 6$

5. Which inequality represents the solution to $1.75x + 3.5 > 3.25x - 8.5$?
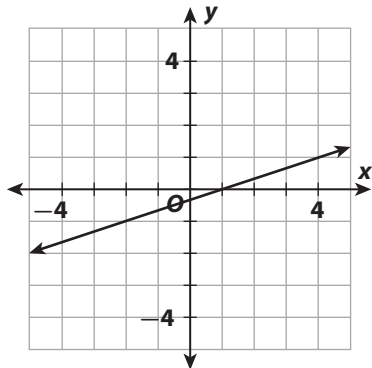
Ⓐ $x > 8$

Ⓑ $x < 8$

Ⓒ $x > 3.3$

Ⓓ $x < 3.3$

6. The triangle and the rectangle have the same perimeter.

Find the value of x.

Ⓐ 2

Ⓑ 10

Ⓒ 18
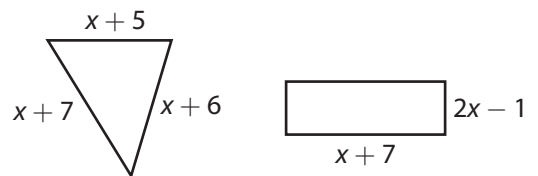
Ⓓ 24

7. What is the slope of the line?

Ⓐ −3   Ⓒ $\frac{1}{3}$

Ⓑ −$\frac{1}{3}$ Ⓓ 3

8. A square wall tile has an area of 58,800 square millimeters. Between which two measurements is the length of one side?

Ⓐ between 24 and 25 millimeters

Ⓑ between 76 and 77 millimeters

Ⓒ between 242 and 243 millimeters

Ⓓ between 766 and 767 millimeters

Gridded Response

9. For the inequality $3x - 5 \geq 5x - 25$, what is the greatest value of x that makes the statement true?

10. Two cars are traveling in the same direction. The first car is going 45 mi/h and the second car is going 60 mi/h. The first car left 2 hours before the second car. How many hours will it take for the second car to travel the same distance as the first car?

11. Mickey is ordering some clothes online. One website charges $1.25 per pound for shipping. The other website charges $0.75 per pound for shipping plus a $2 handling fee. How many pounds of clothing would Mickey need to order for the shipping costs to be the same?