UNIT 5

Transformational Geometry

MODULE 12

Transformations and Congruence

TEKS 8.10.A, 8.10.C

MODULE 13

Dilations, Similarity, and Proportionality

TEKS 8.3.A, 8.3.B, 8.3.C, 8.10.A, 8.10.B, 8.10.D

CAREERS IN MATH

Contractor A contractor is engaged in the construction, repair, and dismantling of structures such as buildings, bridges, and roads. Contractors use math when researching and implementing building codes, making measurements and scaling models, and in financial management.

If you are interested in a career as a contractor, you should study the following mathematical subjects:

- Business Math
- Geometry
- Algebra
- Trigonometry

Research other careers that require the use of business math and scaling.

Unit 5 Performance Task

At the end of the unit, check out how **contractors** use math.

UNIT 5

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters within found words to answer the riddle at the bottom of the page.

W	T	C	F	V	A	F	I	L	I	T	U	G	N	S
M	U	J	L	O	C	Z	H	B	R	S	D	E	O	W
F	E	Q	V	H	B	T	E	A	N	T	P	X	I	A
M	Y	J	G	B	O	G	N	P	K	G	G	B	T	Q
N	X	L	H	A	A	S	E	V	Z	R	U	B	C	W
L	O	X	D	M	L	A	G	R	H	H	L	R	U	A
G	P	I	I	A	E	M	X	K	V	A	K	X	D	K
B	D	E	T	N	E	M	E	G	R	A	L	N	E	A
W	R	I	E	C	A	N	R	E	Y	M	A	T	R	P
P	O	G	L	K	E	R	O	T	A	T	I	O	N	R
N	Z	Y	E	A	C	L	R	O	V	Z	S	P	I	U
U	Y	T	J	N	T	Q	F	N	B	X	G	G	C	J
O	N	E	R	Z	I	I	P	E	I	Y	F	A	B	W
I	F	V	U	C	M	S	O	G	R	P	Q	K	B	W
C	Q	T	U	I	C	C	U	N	L	N	T	L	D	Y

The input of a transformation. (Lesson 12.1)

A transformation that flips a figure across a line. (Lesson 12.2)

A transformation that slides a figure along a straight line. (Lesson 12.1)

A transformation that turns a figure around a given point. (Lesson 12.3)

The product of a figure made larger by dilation. (Lesson 13.1)

The product of a figure made smaller by dilation. (Lesson 13.1)

Scaled replicas that change the size but not the shape of a figure. (Lesson 13.1)

Q: What do you call an angle that's broken?

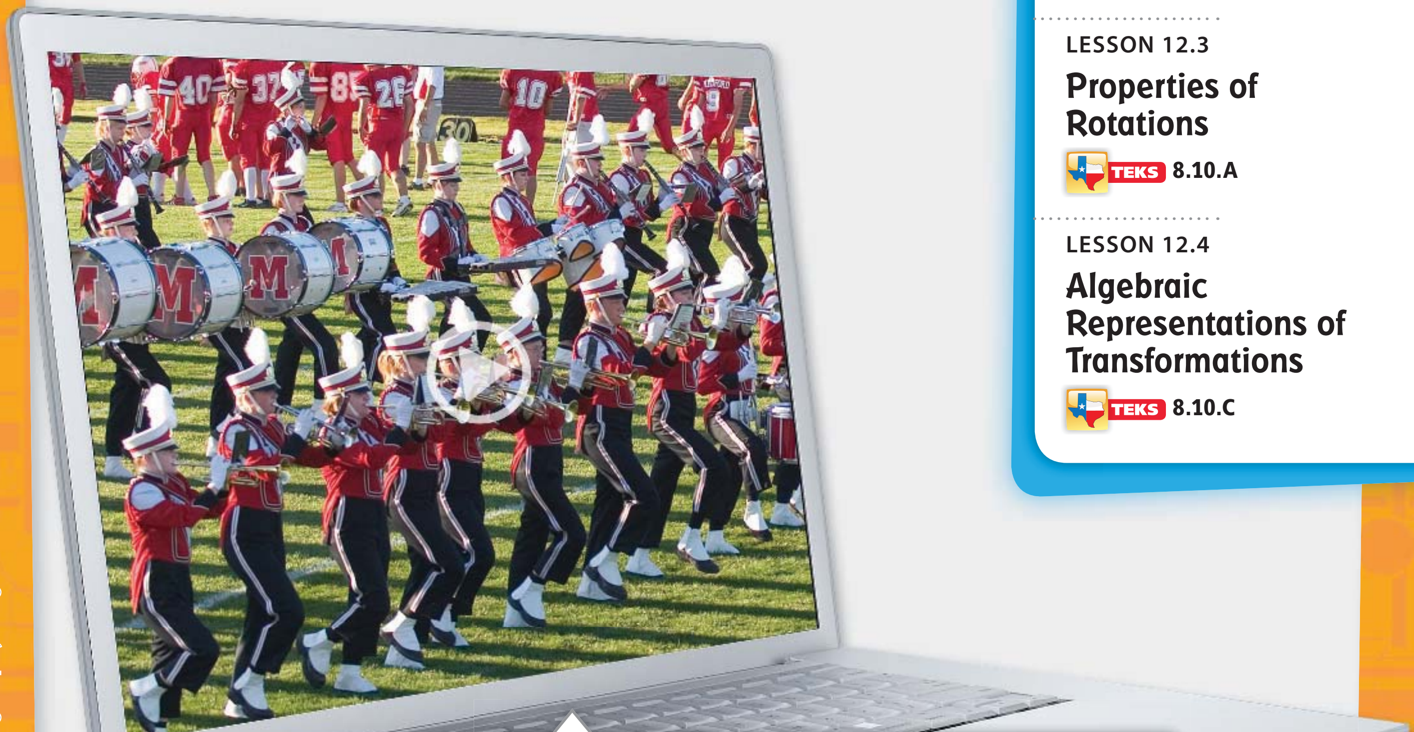
A: A ___ ___ ___ ___ ___ ___ ___ ___ ___!

Transformations and Congruence

MODULE 12

ESSENTIAL QUESTION

How can you use transformations and congruence to solve real-world problems?



Real-World Video

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When a marching band lines up and marches across the field, they are modeling a translation. As they march, they maintain size and orientation. A translation is one type of transformation.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Integer Operations

EXAMPLE

$-3 - (-6) = -3 + 6$ To subtract an integer, add its opposite.

$= |-3| - |6|$ The signs are different, so find the difference of the absolute values: $6 - 3 = 3$. Use the sign of the number with the greater absolute value.

$= 3$

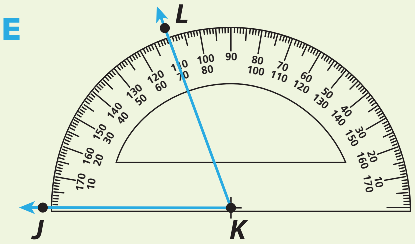
Find each difference.

1. $5 - (-9)$ _____ **2.** $-6 - 8$ _____ **3.** $2 - 9$ _____ **4.** $-10 - (-6)$ _____

5. $3 - (-11)$ _____ **6.** $12 - 7$ _____ **7.** $-4 - 11$ _____ **8.** $0 - (-12)$ _____

Measure Angles

EXAMPLE



$m\angle JKL = 70°$

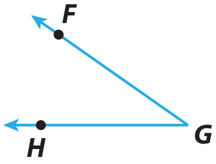
Place the center point of the protractor on the angle's vertex.

Align one ray with the base of the protractor.

Read the angle measure where the other ray intersects the semicircle.

Use a protractor to measure each angle.

9.



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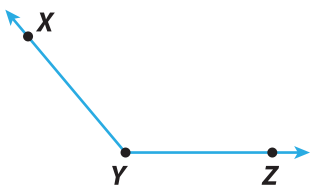
10.



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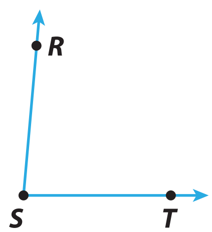
11.



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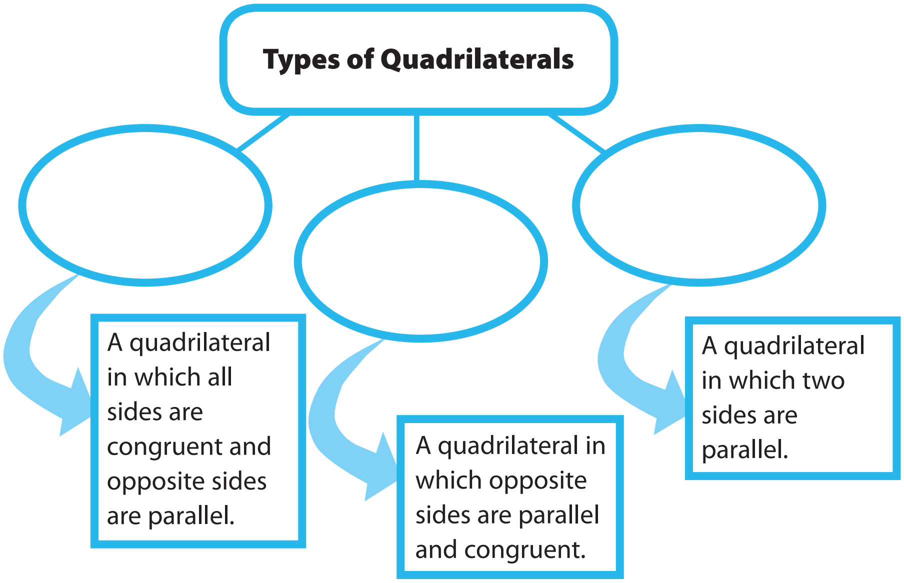
Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic organizer. You will put one word in each oval.



Vocabulary

Review Words

- coordinate plane *(plano cartesiano)*
- ✔ parallelogram *(paralelogramo)*
- quadrilateral *(cuadrilátero)*
- ✔ rhombus *(rombo)*
- ✔ trapezoid *(trapecio)*

Preview Words

- center of rotation *(centro de rotación)*
- image *(imagen)*
- line of reflection *(línea de reflexión)*
- preimage *(imagen original)*
- reflection *(reflexión)*
- rotation *(rotación)*
- transformation *(transformación)*
- translation *(traslación)*

Understand Vocabulary

Match the term on the left to the correct expression on the right.

1. transformation	**A.** A function that describes a change in the position, size, or shape of a figure.
2. reflection	**B.** A function that slides a figure along a straight line.
3. translation	**C.** A transformation that flips a figure across a line.

Active Reading

Booklet Before beginning the module, create a booklet to help you learn the concepts in this module. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.

MODULE 12

Unpacking the TEKS

Understanding the TEKS and the vocabulary terms in the TEKS will help you know exactly what you are expected to learn in this module.

TEKS 8.10.A

Generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

What It Means to You

You will identify a rotation, a reflection, and a translation, and understand that the image has the same shape and size as the preimage.

UNPACKING EXAMPLE 8.10.A

The figure shows triangle *ABC* and its image after three different transformations. Identify and describe the translation, the reflection, and the rotation of triangle *ABC*.

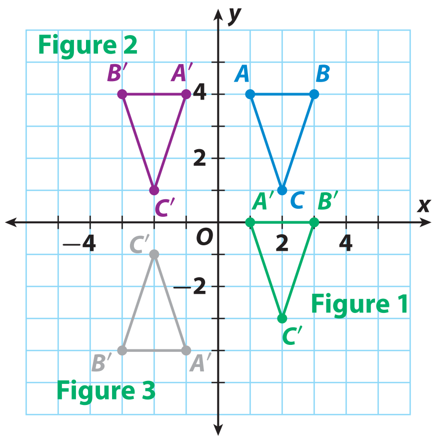


Figure 1 is a translation 4 units down. Figure 2 is a reflection across the *y*-axis. Figure 3 is a rotation of 180°.

TEKS 8.10.C

Explain the effect of translations, reflections over the *x*- or *y*-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation.

What It Means to You

You can use an algebraic representation to translate, reflect, or rotate a two-dimensional figure.

UNPACKING EXAMPLE 8.10.C

Rectangle *RSTU* with vertices (–4, –1), (–1, 1), (–1, –3), and (–4, –3) is reflected across the *y*-axis. Find the coordinates of the image.

The rule to reflect across the *y*-axis is to change the sign of the *x*-coordinate.

Coordinates	Reflect across the *y*-axis (–*x*, *y*)	Coordinates of image
(–4, 1), (–1, 1), (–1, –3), (–4, –3)	(–(–4), 1), (–(–1), 1), (– (–1), –3), (–(–4), –3)	(4, 1), (1, 1), (1, –3), (4, –3)

The coordinates of the image are (4, 1), (1, 1), (1, –3), and (4, –3).

Visit **my.hrw.com** to see all the **TEKS** unpacked.

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LESSON 12.1

Properties of Translations

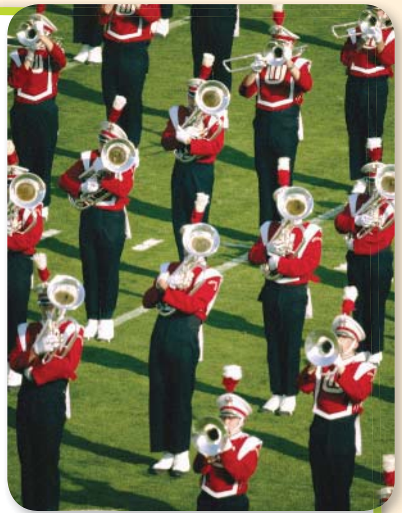
TEKS **Two-dimensional shapes—8.10.A** Generalize the properties of orientation and congruence of . . . translations. . . of two-dimensional shapes on a coordinate plane.

ESSENTIAL QUESTION

How do you describe the properties of orientation and congruence of translations?

EXPLORE ACTIVITY 1 TEKS 8.10.A

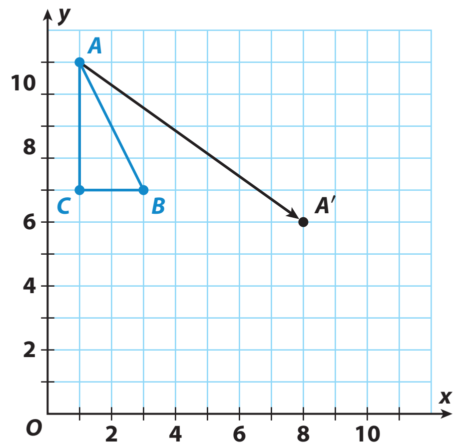
Exploring Translations



You learned that a function is a rule that assigns exactly one output to each input. A **transformation** is a function that describes a change in the position, size, or shape of a figure. The input of a transformation is the **preimage**, and the output of a transformation is the **image**.

A **translation** is a transformation that slides a figure along a straight line. The image has the same size and shape as the preimage.

The triangle shown on the grid is the preimage (input). The arrow shows the motion of a translation and how point *A* is translated to point *A*′.



A Trace triangle *ABC* onto a piece of paper. Cut out your traced triangle.

B Slide your triangle along the arrow to model the translation that maps point *A* to point *A*′.

C The image of the translation is the triangle produced by the translation. Sketch the image of the translation.

D The vertices of the image are labeled using prime notation. For example, the image of *A* is *A*′. Label the images of points *B* and *C*.

E Describe the motion modeled by the translation.

Move __________ units right and __________ units down.

F Check that the motion you described in part **E** is the same motion that maps point *A* onto *A*′, point *B* onto *B*′, and point *C* onto *C*′.

Reflect

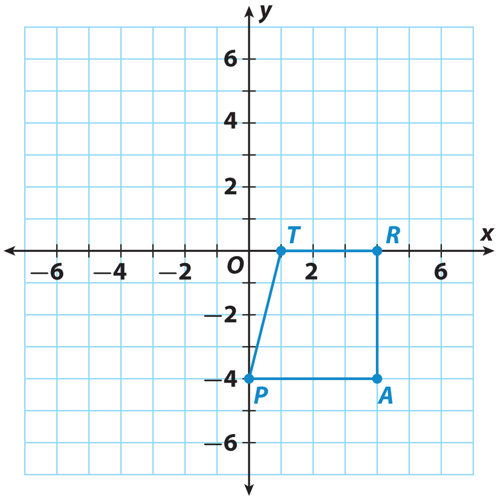
1. How is the orientation of the triangle affected by the translation?

____

EXPLORE ACTIVITY 2 TEKS 8.10.A

Properties of Translations

Use trapezoid *TRAP* to investigate the properties of translations.



A Trace the trapezoid onto a piece of paper. Cut out your traced trapezoid.

B Place your trapezoid on top of the trapezoid in the figure. Then translate your trapezoid 5 units to the left and 3 units up. Sketch the image of the translation by tracing your trapezoid in this new location. Label the vertices of the image T', R', A', and P'.

C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

$TR =$ ________ $RA =$ ________ $AP =$ ________ $TP =$ ________

D Use a ruler to measure the sides of trapezoid $T'R'A'P'$ in centimeters.

$T'R' =$ ________ $R'A' =$ ________ $A'P' =$ ________ $T'P' =$ ________

E What do you notice about the lengths of corresponding sides of the two figures?

____

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T =$ ______ $m\angle R =$ ______ $m\angle A =$ ______ $m\angle P =$ ______

G Use a protractor to measure the angles of trapezoid $T'R'A'P'$.

$m\angle T' =$ ______ $m\angle R' =$ ______ $m\angle A' =$ ______ $m\angle P' =$ ______

H What do you notice about the measures of corresponding angles of the two figures?

____

I Which sides of trapezoid *TRAP* are parallel? How do you know?

____

Which sides of trapezoid $T'R'A'P'$ are parallel? ________________

What do you notice? ________________________

____

Reflect

2. **Make a Conjecture** Use your results from parts **E**, **H**, and **I** to make a conjecture about translations.

____

____

3. What can you say about translations and congruence?

____

____

Graphing Translations

To translate a figure in the coordinate plane, translate each of its vertices. Then connect the vertices to form the image.

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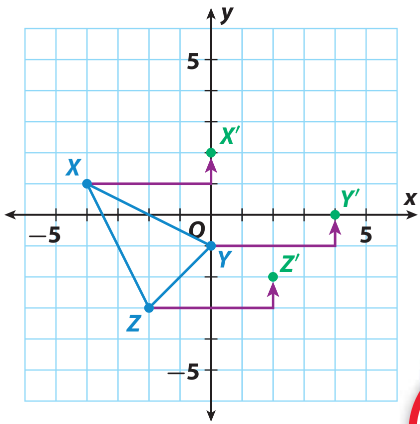
EXAMPLE 1

TEKS 8.10.A

The figure shows triangle *XYZ*. Graph the image of the triangle after a translation of 4 units to the right and 1 unit up.

STEP 1 Translate point *X*.

Count right 4 units and up 1 unit and plot point *X′*.



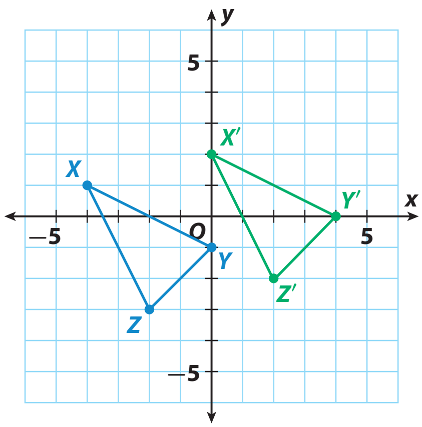
STEP 2 Translate point *Y*.

Count right 4 units and up 1 unit and plot point *Y′*.

STEP 3 Translate point *Z*.

Count right 4 units and up 1 unit and plot point *Z′*.

STEP 4 Connect *X′*, *Y′*, and *Z′* to form triangle *X′Y′Z′*.



Each vertex is moved 4 units right and 1 unit up.

Math Talk
Mathematical Processes

Is the image congruent to the preimage? How do you know?

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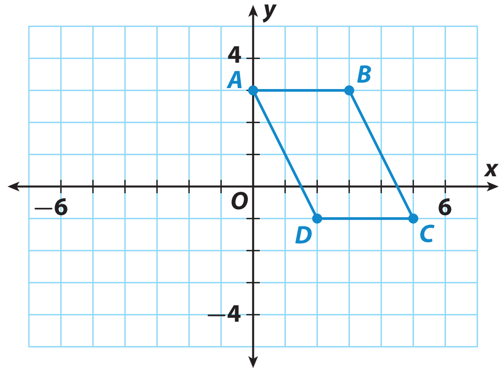
YOUR TURN

4. The figure shows parallelogram *ABCD*. Graph the image of the parallelogram after a translation of 5 units to the left and 2 units down.



Guided Practice

1. **Vocabulary** A ____________________ is a change in the position, size, or shape of a figure.

2. **Vocabulary** When you perform a transformation of a figure on the coordinate plane, the input of the transformation is called

the ____________________, and the output of the transformation is

called the ____________________.

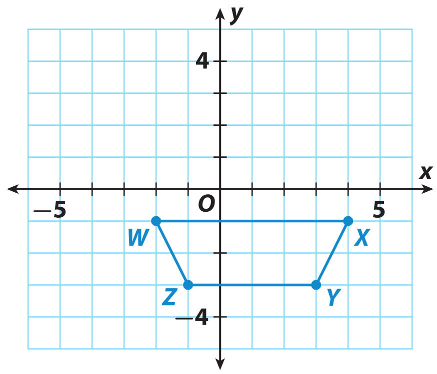
3. Joni translates a right triangle 2 units down and 4 units to the right. How does the orientation of the image of the triangle compare with the orientation of the preimage? (Explore Activity 1)

____

4. Rashid drew rectangle *PQRS* on a coordinate plane. He then translated the rectangle 3 units up and 3 units to the left and labeled the image *P′Q′R′S′*. How do rectangle *PQRS* and rectangle *P′Q′R′S′* compare? (Explore Activity 2)

____

5. The figure shows trapezoid *WXYZ*. Graph the image of the trapezoid after a translation of 4 units up and 2 units to the left. (Example 1)



ESSENTIAL QUESTION CHECK-IN

6. What are the properties of translations?

____

____

Name ____________________ Class ____________ Date ____________

12.1 Independent Practice

TEKS 8.10.A

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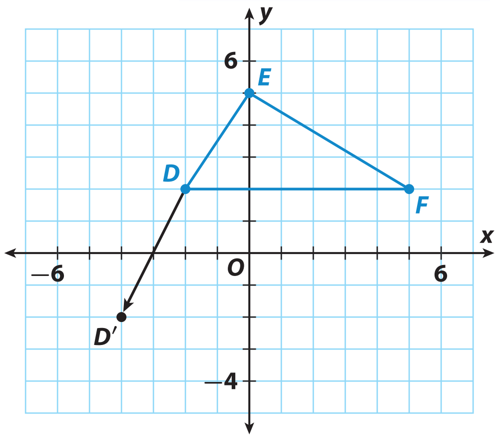
7. The figure shows triangle *DEF*.



a. Graph the image of the triangle after the translation that maps point *D* to point *D*′.

b. How would you describe the translation?

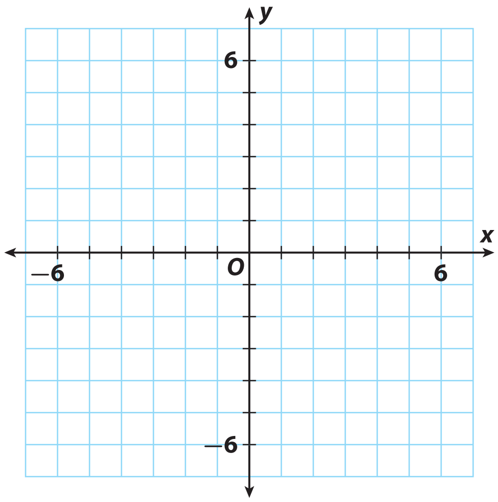
____

____

c. How does the image of triangle *DEF* compare with the preimage?

____

8. a. Graph quadrilateral *KLMN* with vertices *K*(–3, 2), *L*(2, 2), *M*(0, –3), and *N*(–4, 0) on the coordinate grid.



b. On the same coordinate grid, graph the image of quadrilateral *KLMN* after a translation of 3 units to the right and 4 units up.

c. Which side of the image is congruent to side $\overline{LM}$?

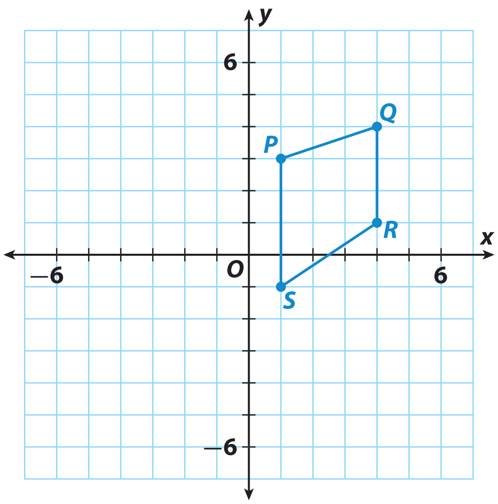
____

Name three other pairs of congruent sides.

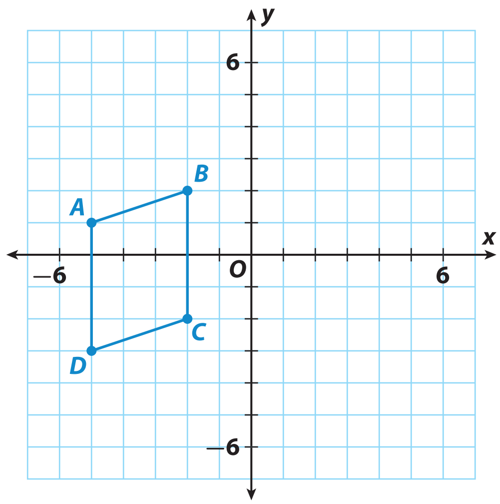
____

Draw the image of the figure after each translation.

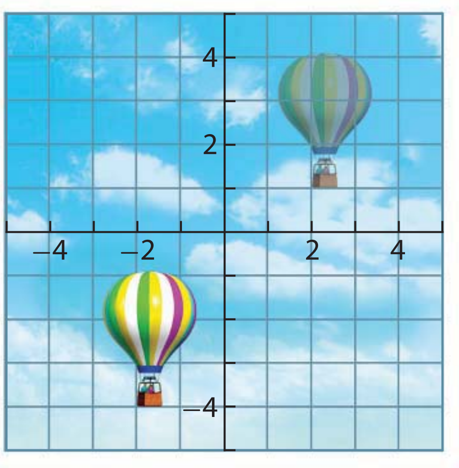
9. 4 units left and 2 units down



10. 5 units right and 3 units up



11. The figure shows the ascent of a hot air balloon. How would you describe the translation?



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12. **Critical Thinking** Is it possible that the orientation of a figure could change after it is translated? Explain.

____

____

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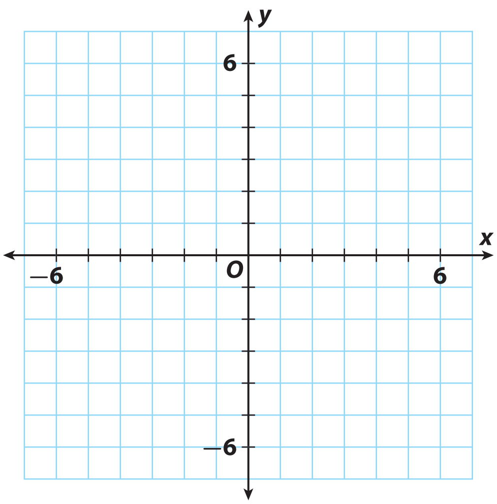
H.O.T. FOCUS ON HIGHER ORDER THINKING

13. a. **Multistep** Graph triangle *XYZ* with vertices *X*(–2, –5), *Y*(2, –2), and *Z*(4, –4) on the coordinate grid.

b. On the same coordinate grid, graph and label triangle *X′Y′Z′*, the image of triangle *XYZ* after a translation of 3 units to the left and 6 units up.

c. Now graph and label triangle *X″Y″Z″*, the image of triangle *X′Y′Z′* after a translation of 1 unit to the left and 2 units down.

d. **Analyze Relationships** How would you describe the translation that maps triangle *XYZ* onto triangle *X″Y″Z″*?



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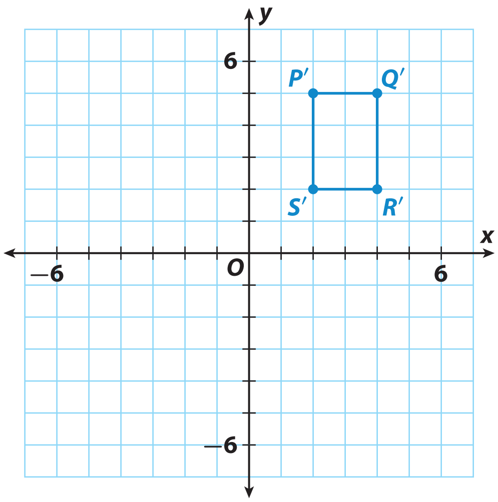
14. **Critical Thinking** The figure shows rectangle *P′Q′R′S′*, the image of rectangle *PQRS* after a translation of 5 units to the right and 7 units up. Graph and label the preimage *PQRS*.



15. **Communicate Mathematical Ideas** Explain why the image of a figure after a translation is congruent to its preimage.

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LESSON 12.2

Properties of Reflections

TEKS **Two-dimensional shapes—8.10.A** Generalize the properties of orientation and congruence of… reflections… of two-dimensional shapes on a coordinate plane.

ESSENTIAL QUESTION

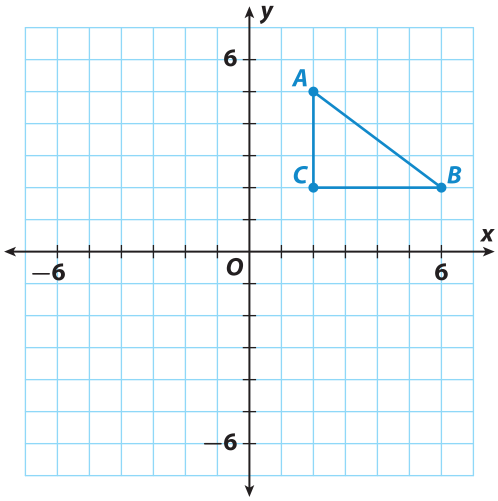
How do you describe the properties of orientation and congruence of reflections?

EXPLORE ACTIVITY 1 TEKS 8.10.A

Exploring Reflections

A **reflection** is a transformation that flips a figure across a line. The line is called the **line of reflection**. Each point and its image are the same distance from the line of reflection.

The triangle shown on the grid is the preimage. You will explore reflections across the *x*- and *y*-axes.



A Trace triangle *ABC* and the *x*- and *y*-axes onto a piece of paper.

B Fold your paper along the *x*-axis and trace the image of the triangle on the opposite side of the *x*-axis. Unfold your paper and label the vertices of the image A', B', and C'.

C What is the line of reflection for this transformation?

____

D Find the perpendicular distance from each point to the line of reflection.

Point *A* __________ Point *B* __________ Point *C* __________

E Find the perpendicular distance from each point to the line of reflection.

Point A' __________ Point B' __________ Point C' __________

F What do you notice about the distances you found in **D** and **E**?

____

Reflect

1. Fold your paper from **A** along the *y*-axis and trace the image of triangle *ABC* on the opposite side. Label the vertices of the image A'', B'', and C''. What is the line of reflection for this transformation? ________________

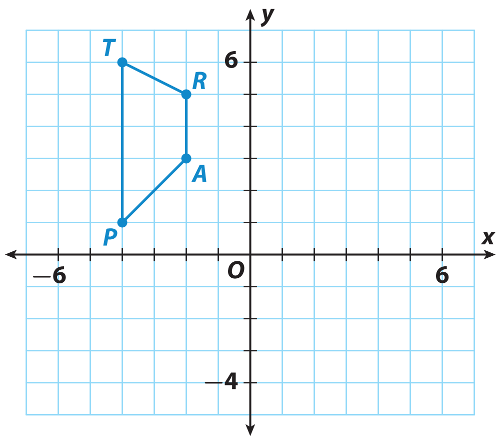
2. How does each image in your drawings compare with its preimage?

____

EXPLORE ACTIVITY 2 TEKS 8.10.A

Properties of Reflections

Use trapezoid *TRAP* to investigate the properties of reflections.



A Trace the trapezoid onto a piece of paper. Cut out your traced trapezoid.

B Place your trapezoid on top of the trapezoid in the figure. Then reflect your trapezoid across the y-axis. Sketch the image of the reflection by tracing your trapezoid in this new location. Label the vertices of the image T', R', A', and P'.

C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

$TR =$ ________ $RA =$ ________ $AP =$ ________ $TP =$ ________

D Use a ruler to measure the sides of trapezoid $T'R'A'P'$ in centimeters.

$T'R' =$ ________ $R'A' =$ ________ $A'P' =$ ________ $T'P' =$ ________

E What do you notice about the lengths of corresponding sides of the two figures?

____

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T =$ ______ $m\angle R =$ ______ $m\angle A =$ ______ $m\angle P =$ ______

G Use a protractor to measure the angles of trapezoid $T'R'A'P'$.

$m\angle T' =$ ______ $m\angle R' =$ ______ $m\angle A' =$ ______ $m\angle P' =$ ______

H What do you notice about the measures of corresponding angles of the two figures?

____

I Which sides of trapezoid *TRAP* are parallel? ____________________

Which sides of trapezoid $T'R'A'P'$ are parallel? ____________________
What do you notice?

____

____

Reflect

3. **Make a Conjecture** Use your results from **E**, **H**, and **I** to make a conjecture about reflections.

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____

____

Math Talk
Mathematical Processes
What can you say about reflections and congruence?

Graphing Reflections

To reflect a figure across a line of reflection, reflect each of its vertices. Then connect the vertices to form the image. Remember that each point and its image are the same distance from the line of reflection.

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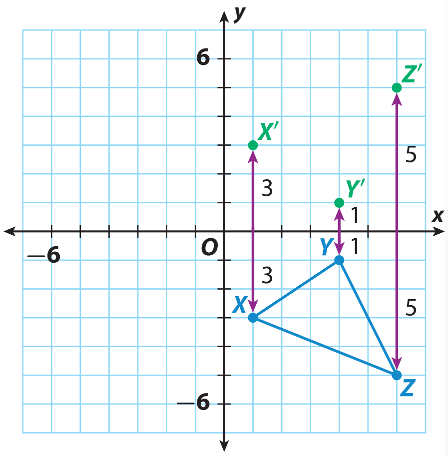
EXAMPLE 1

TEKS 8.10.A

The figure shows triangle *XYZ*. Graph the image of the triangle after a reflection across the *x*-axis.

STEP 1 Reflect point *X*.

Point *X* is 3 units below the *x*-axis. Count 3 units above the *x*-axis and plot point *X*′.



STEP 2 Reflect point *Y*.

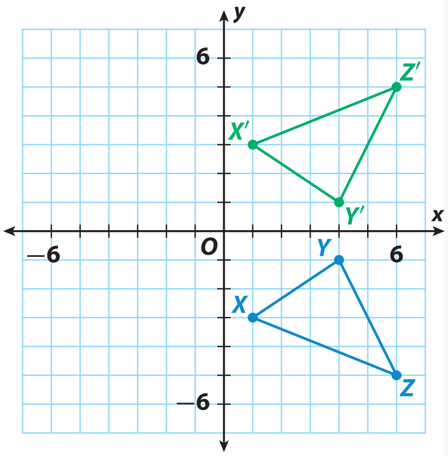
Point *Y* is 1 unit below the *x*-axis. Count 1 unit above the *x*-axis and plot point *Y*′.

STEP 3 Reflect point *Z*.

Point *Z* is 5 units below the *x*-axis. Count 5 units above the *x*-axis and plot point *Z*′.

STEP 4 Connect *X*′, *Y*′, and *Z*′ to form triangle *X*′*Y*′*Z*′.

Each vertex of the image is the same distance from the x-axis as the corresponding vertex in the original figure.



My Notes

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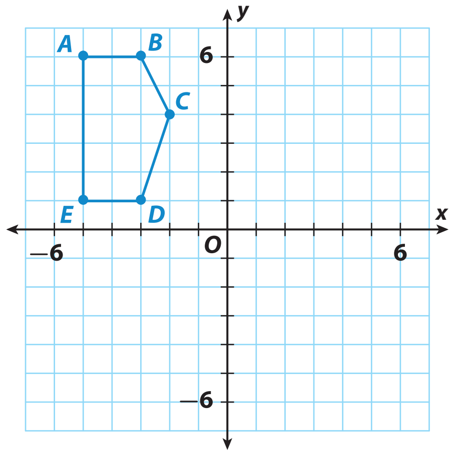
YOUR TURN

4. The figure shows pentagon *ABCDE*. Graph the image of the pentagon after a reflection across the *y*-axis.



Guided Practice

1. **Vocabulary** A reflection is a transformation that flips a figure across a line called the ________________.

2. The figure shows trapezoid *ABCD*. (Explore Activities 1 and 2 and Example 1)

   a. Graph the image of the trapezoid after a reflection across the *x*-axis. Label the vertices of the image.

   b. How do trapezoid *ABCD* and trapezoid *A'B'C'D'* compare?

   ____

   c. **What If?** Suppose you reflected trapezoid *ABCD* across the *y*-axis. How would the orientation of the image of the trapezoid compare with the orientation of the preimage?

   ____

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   ____

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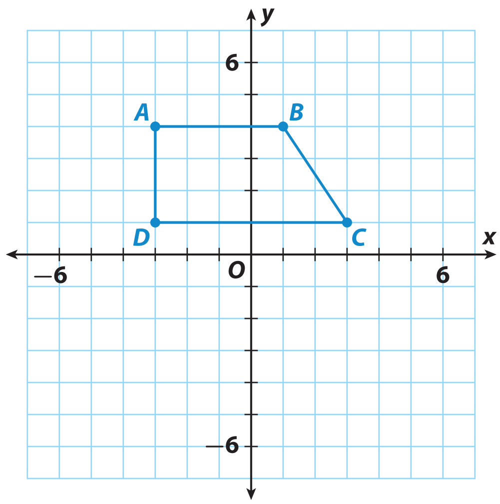

ESSENTIAL QUESTION CHECK-IN

3. What are the properties of reflections?

____

____

Name ______________________ Class ______________ Date ______________

12.2 Independent Practice

TEKS 8.10.A

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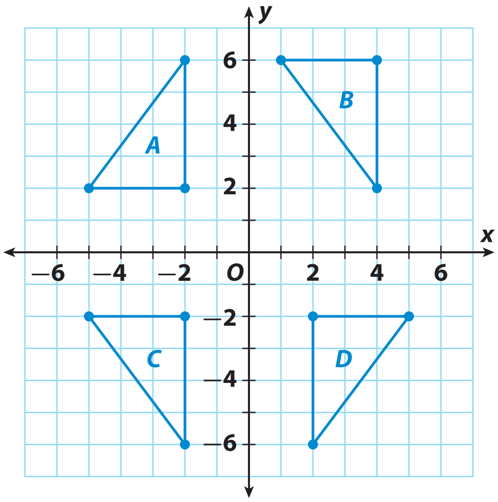
The graph shows four right triangles. Use the graph for Exercises 4–7.



4. Which two triangles are reflections of each other across the *x*-axis?

____

5. For which two triangles is the line of reflection the *y*-axis?

____

6. Which triangle is a translation of triangle *C*? How would you describe the translation?

____

____

7. Which triangles are congruent? How do you know?

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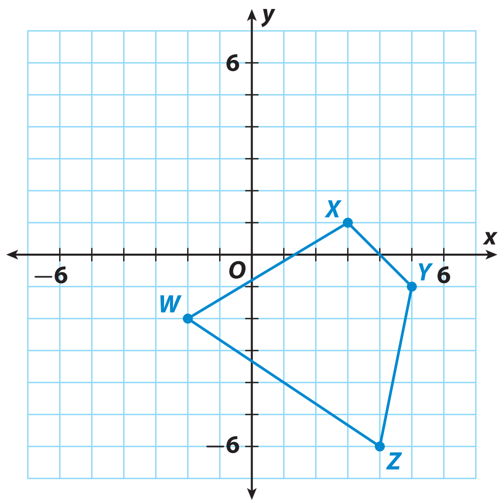
8. a. Graph quadrilateral *WXYZ* with vertices *W*(−2, −2), *X*(3, 1), *Y*(5, −1), and *Z*(4, −6) on the coordinate grid.



b. On the same coordinate grid, graph quadrilateral *W′X′Y′Z′*, the image of quadrilateral *WXYZ* after a reflection across the *x*-axis.

c. Which side of the image is congruent to side $\overline{YZ}$?

____

Name three other pairs of congruent sides.

____

____

d. Which angle of the image is congruent to ∠*X*?

____

Name three other pairs of congruent angles.

____

____

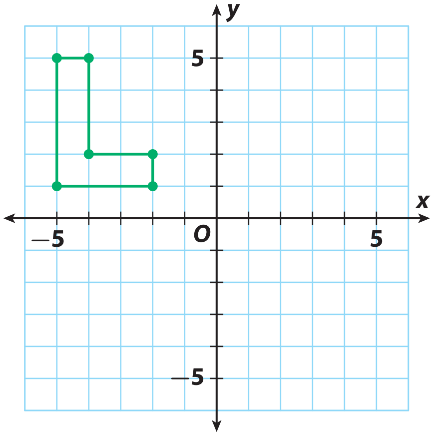
9. **Critical Thinking** Is it possible that the image of a point after a reflection could be the same point as the preimage? Explain.

____

____

H.O.T. FOCUS ON HIGHER ORDER THINKING

10. a. Graph the image of the figure shown after a reflection across the y-axis.



b. On the same coordinate grid, graph the image of the figure you drew in part **a** after a reflection across the x-axis.

c. **Make a Conjecture** What other sequence of transformations would produce the same final image from the original preimage? Check your answer by performing the transformations. Then make a conjecture that generalizes your findings.

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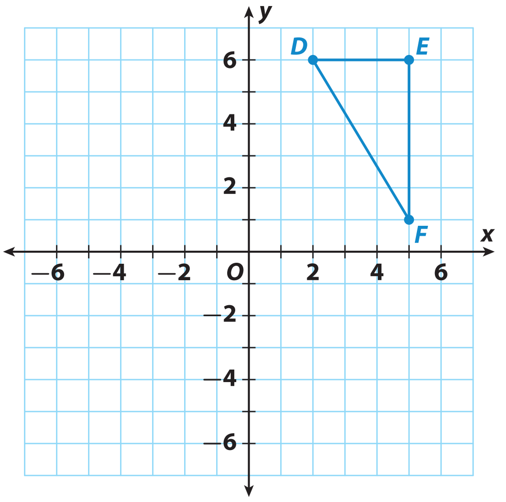
11. a. Graph triangle DEF with vertices $D(2, 6)$, $E(5, 6)$, and $F(5, 1)$ on the coordinate grid.



b. Next graph triangle $D'E'F'$, the image of triangle DEF after a reflection across the y-axis.

c. On the same coordinate grid, graph triangle $D''E''F''$, the image of triangle $D'E'F'$ after a translation of 7 units down and 2 units to the right.

d. **Analyze Relationships** Find a different sequence of transformations that will transform triangle DEF to triangle $D''E''F''$.

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LESSON 12.3

Properties of Rotations

TEKS **Two-dimensional shapes—8.10.A** Generalize the properties of orientation and congruence of rotations. . . of two-dimensional shapes on a coordinate plane.

ESSENTIAL QUESTION

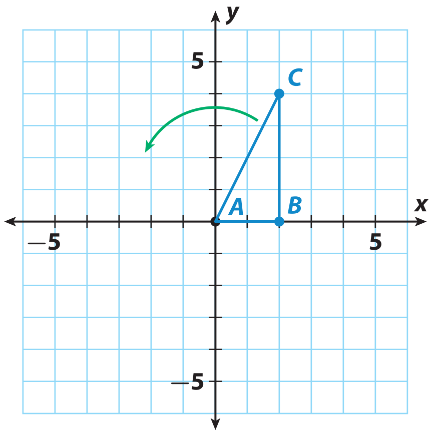
How do you describe the properties of orientation and congruence of rotations?

EXPLORE ACTIVITY 1 TEKS 8.10.A

Exploring Rotations

A **rotation** is a transformation that turns a figure around a given point called the **center of rotation**. The image has the same size and shape as the preimage.

The triangle shown on the grid is the preimage. You will use the origin as the center of rotation.



A Trace triangle *ABC* onto a piece of paper. Cut out your traced triangle.

B Rotate your triangle 90° counterclockwise about the origin. The side of the triangle that lies along the *x*-axis should now lie along the *y*-axis.

C Sketch the image of the rotation. Label the images of points *A*, *B*, and *C* as *A*′, *B*′, and *C*′.

D Describe the motion modeled by the rotation.

Rotate ________________ degrees ____________________ about the origin.

E Check that the motion you described in **D** is the same motion that maps point *A* onto *A*′, point *B* onto *B*′, and point *C* onto *C*′.

Reflect

1. **Communicate Mathematical Ideas** How are the size and the orientation of the triangle affected by the rotation?

____

____

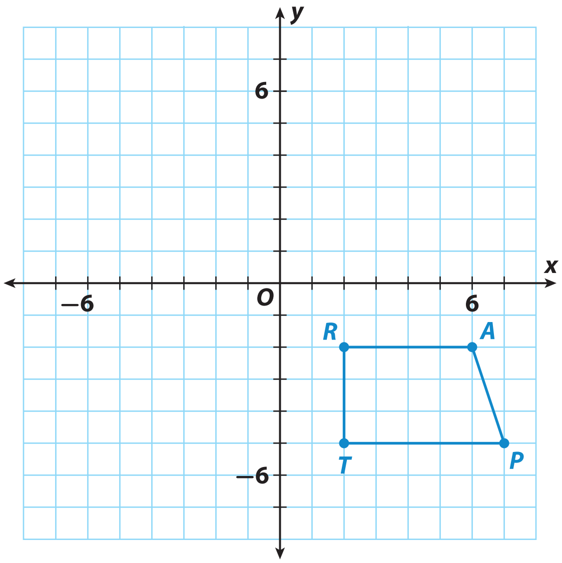
____

2. Rotate triangle *ABC* 90° clockwise about the origin. Sketch the result on the coordinate grid above. Label the image vertices *A*″, *B*″, and *C*″.

EXPLORE ACTIVITY 2 TEKS 8.10.A

Properties of Rotations

Use trapezoid *TRAP* to investigate the properties of rotations.



A Trace the trapezoid onto a piece of paper. Include the portion of the *x*- and *y*-axes bordering the third quadrant. Cut out your tracing.

B Place your trapezoid and axes on top of those in the figure. Then use the axes to help rotate your trapezoid 180° counterclockwise about the origin. Sketch the image of the rotation of your trapezoid in this new location. Label the vertices of the image T', R', A', and P'.

C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

$TR =$ ________ $RA =$ ________

$AP =$ ________ $TP =$ ________

D Use a ruler to measure the sides of trapezoid $T'R'A'P'$ in centimeters.

$T'R' =$ ________ $R'A' =$ ________

$A'P' =$ ________ $T'P' =$ ________

E What do you notice about the lengths of corresponding sides of the two figures?

____

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T =$ ________ $m\angle R =$ ________ $m\angle A =$ ________ $m\angle P =$ ________

G Use a protractor to measure the angles of trapezoid $T'R'A'P'$.

$m\angle T' =$ ________ $m\angle R' =$ ________ $m\angle A' =$ ________ $m\angle P' =$ ________

H What do you notice about the measures of corresponding angles of the two figures?

____

I Which sides of trapezoid *TRAP* are parallel? ________________

Which sides of trapezoid $T'R'A'P'$ are parallel? ________________

What do you notice? ____

____

Reflect

3. **Make a Conjecture** Use your results from **E**, **H**, and **I** to make a conjecture about rotations.

____

____

4. Place your tracing back in its original position. Then perform a 180° *clockwise* rotation about the origin. Compare the result.

____

____

Graphing Rotations

To rotate a figure in the coordinate plane, rotate each of its vertices. Then connect the vertices to form the image.

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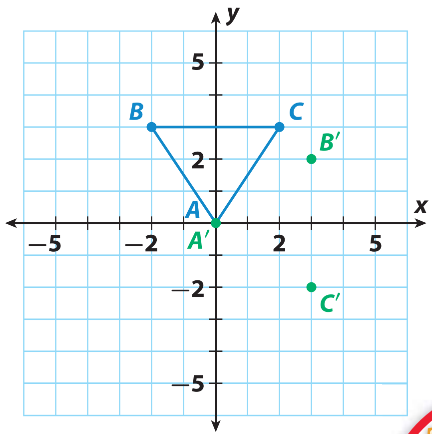
EXAMPLE 1

TEKS 8.10.A

The figure shows triangle *ABC*. Graph the image of triangle *ABC* after a rotation of 90° clockwise.

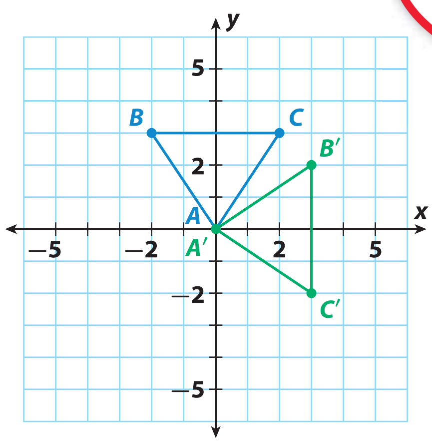


STEP 1 Rotate the figure clockwise from the *y*-axis to the *x*-axis. Point *A* will still be at (0, 0).

Point *B* is 2 units to the left of the *y*-axis, so point *B*′ is 2 units above the *x*-axis.

Point *C* is 2 units to the right of the *y*-axis, so point C′ is 2 units below the *x*-axis.

STEP 2 Connect *A*′, *B*′, and *C*′ to form the image triangle *A*′*B*′*C*′.



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Math Talk
Mathematical Processes
How is the orientation of the triangle affected by the rotation?

Reflect

5. Is the image congruent to the preimage? How do you know?

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____

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YOUR TURN

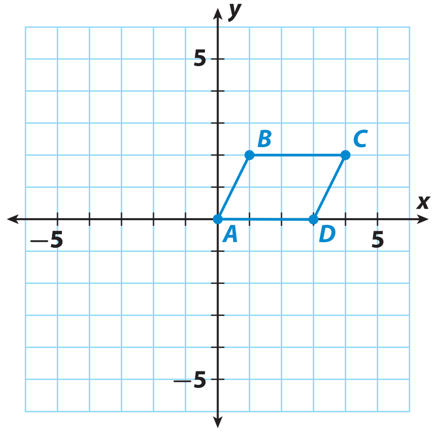
Graph the image of quadrilateral *ABCD* after each rotation.

6. 180°

7. 270° clockwise

8. Find the coordinates of Point *C* after a 90° counterclockwise rotation followed by a 180° rotation.

____



Guided Practice

1. **Vocabulary** A rotation is a transformation that turns a figure around a given ____________________ called the center of rotation.

Siobhan rotates a right triangle 90° counterclockwise about the origin.

2. How does the orientation of the image of the triangle compare with the orientation of the preimage? (Explore Activity 1)

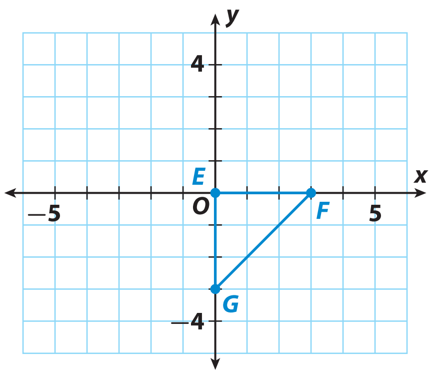
____

3. Is the image of the triangle congruent to the preimage? (Explore Activity 2)

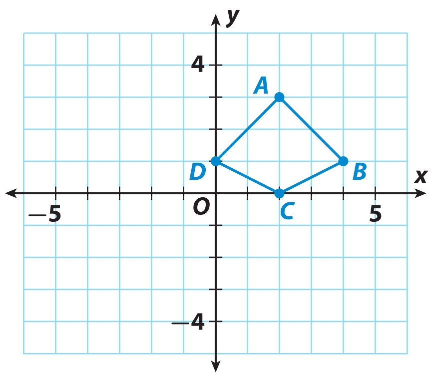
____

Draw the image of the figure after the given rotation about the origin. (Example 1)

4. 90° counterclockwise



5. 180°



ESSENTIAL QUESTION CHECK-IN

6. What are the properties of rotations?

____

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12.3 Independent Practice

TEKS 8.10.A

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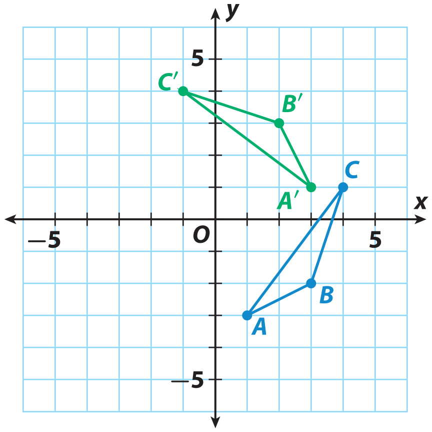
7. The figure shows triangle *ABC* and a rotation of the triangle about the origin.



a. How would you describe the rotation?

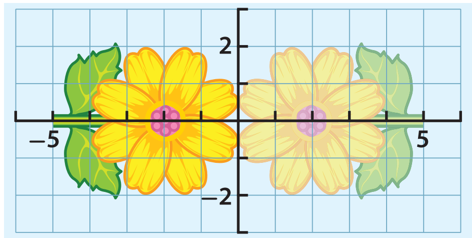
____

____

b. What are the coordinates of the image?

__________, __________, __________

8. The graph shows a figure and its image after a transformation.



a. How would you describe this as a rotation?

____

____

b. Can you describe this as a transformation other than a rotation? Explain.

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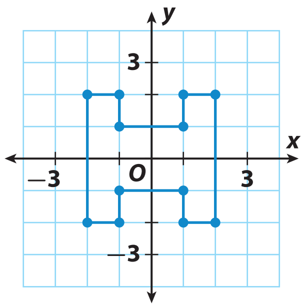
9. What type of rotation will preserve the orientation of the H-shaped figure in the grid?



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10. A point with coordinates $(-2, -3)$ is rotated $90°$ clockwise about the origin. What are the coordinates of its image?

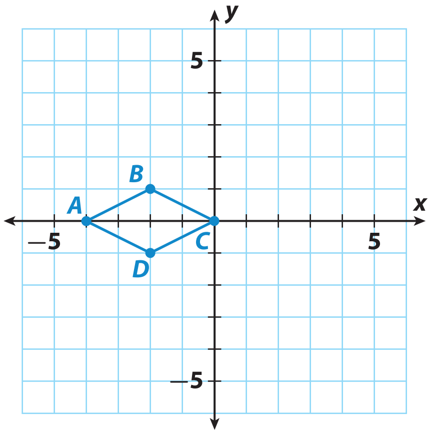
____

Complete the table with rotations of 180° or less. Include the direction of rotation for rotations of less than 180°.

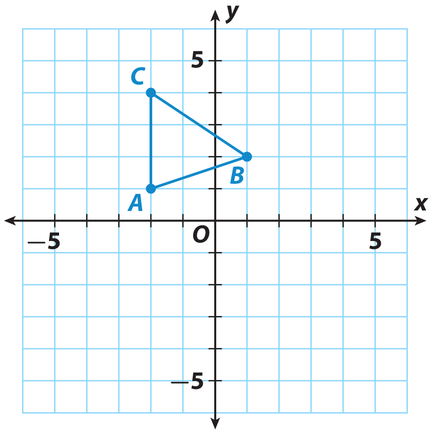
	Shape in quadrant	Image in quadrant	Rotation
11.	I	IV	
12.	III	I	
13.	IV	III	

Draw the image of the figure after the given rotation about the origin.

14. 180°



15. 270° counterclockwise



16. Is there a rotation for which the orientation of the image is always the same as that of the preimage? If so, what?

____

H.O.T. FOCUS ON HIGHER ORDER THINKING

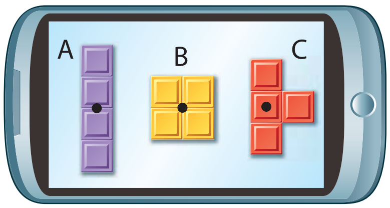
Work Area

17. **Problem Solving** Lucas is playing a game where he has to rotate a figure for it to fit in an open space. Every time he clicks a button, the figure rotates 90 degrees clockwise. How many times does he need to click the button so that each figure returns to its original orientation?

Figure A ____________

Figure B ____________

Figure C ____________



18. **Make a Conjecture** Triangle *ABC* is reflected across the *y*-axis to form the image *A′B′C′*. Triangle *A′B′C′* is then reflected across the *x*-axis to form the image *A″B″C″*. What type of rotation can be used to describe the relationship between triangle *A″B″C″* and triangle *ABC*?

____

19. **Communicate Mathematical Ideas** Point *A* is on the *y*-axis. Describe all possible locations of image *A′* for rotations of 90°, 180°, and 270°. Include the origin as a possible location for *A*.

____

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LESSON 12.4

Algebraic Representations of Transformations

TEKS **Two-dimensional shapes—8.10.C** Explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation.

ESSENTIAL QUESTION

How can you describe the effect of a translation, rotation, or reflection on coordinates using an algebraic representation?

Algebraic Representations of Translations

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The rules shown in the table describe how coordinates change when a figure is translated up, down, right, and left on the coordinate plane.

Translations	
Right a units	Add a to the x-coordinate: $(x, y) \rightarrow (x + a, y)$
Left a units	Subtract a from the x-coordinate: $(x, y) \rightarrow (x - a, y)$
Up b units	Add b to the y-coordinate: $(x, y) \rightarrow (x, y + b)$
Down b units	Subtract b from the y-coordinate: $(x, y) \rightarrow (x, y - b)$

EXAMPLE 1

TEKS 8.10.C

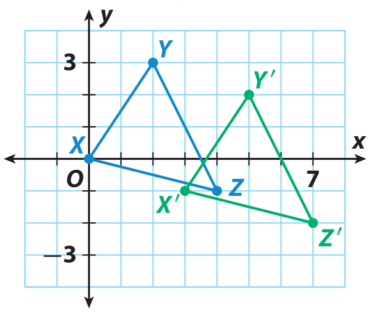
Triangle XYZ has vertices $X(0, 0)$, $Y(2, 3)$, and $Z(4, -1)$. Find the vertices of triangle $X'Y'Z'$ after a translation of 3 units to the right and 1 unit down. Then graph the triangle and its image.

STEP 1 Apply the rule to find the vertices of the image.

Add 3 to the x-coordinate of each vertex and subtract 1 from the y-coordinate of each vertex.

Vertices of $\triangle XYZ$	Rule: $(x + 3, y - 1)$	Vertices of $\triangle X'Y'Z'$
$X(0, 0)$	$(0 + 3, 0 - 1)$	$X'(3, -1)$
$Y(2, 3)$	$(2 + 3, 3 - 1)$	$Y'(5, 2)$
$Z(4, -1)$	$(4 + 3, -1 - 1)$	$Z'(7, -2)$

STEP 2 Graph triangle XYZ and its image.



Math Talk
Mathematical Processes

When you translate a figure to the left or right, which coordinate do you change?

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YOUR TURN

1. A rectangle has vertices at (0, −2), (0, 3), (3, −2), and (3, 3). What are the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x - 6, y - 3)$? Describe the translation.

____

____

Algebraic Representations of Reflections

The signs of the coordinates of a figure change when the figure is reflected across the *x*-axis and *y*-axis. The table shows the rules for changing the signs of the coordinates after a reflection.

Reflections	
Across the *x*-axis	Multiply each *y*-coordinate by −1: $(x, y) \rightarrow (x, -y)$
Across the *y*-axis	Multiply each *x*-coordinate by −1: $(x, y) \rightarrow (-x, y)$

EXAMPLE 2

TEKS 8.10.C

My Notes

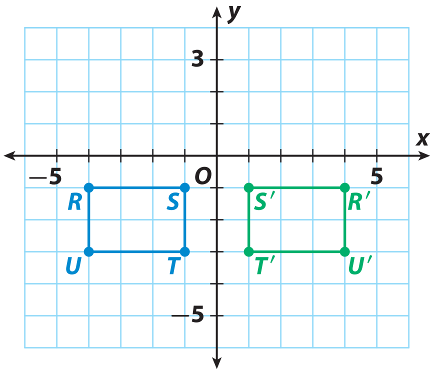
Rectangle *RSTU* has vertices *R*(−4, −1), *S*(−1, −1), *T*(−1, −3), and *U*(−4, −3). Find the vertices of rectangle *R′S′T′U′* after a reflection across the *y*-axis. Then graph the rectangle and its image.

STEP 1 Apply the rule to find the vertices of the image.

Multiply the *x*-coordinate of each vertex by −1.

Vertices of *RSTU*	Rule: $(-1 \cdot x, y)$	Vertices of *R′S′T′U′*
R(−4, −1)	$(-1 \cdot (-4), -1)$	*R′*(4, −1)
S(−1, −1)	$(-1 \cdot (-1), -1)$	*S′*(1, −1)
T(−1, −3)	$(-1 \cdot (-1), -3)$	*T′*(1, −3)
U(−4, −3)	$(-1 \cdot (-4), -3)$	*U′*(4, −3)

STEP 2 Graph rectangle *RSTU* and its image.



YOUR TURN

2. Triangle *ABC* has vertices $A(-2, 6)$, $B(0, 5)$, and $C(3, -1)$. Find the vertices of triangle $A'B'C'$ after a reflection across the *x*-axis.

____

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Algebraic Representations of Rotations

When points are rotated about the origin, the coordinates of the image can be found using the rules shown in the table.

Rotations	
90° clockwise	Multiply each *x*-coordinate by -1; then switch the *x*- and *y*-coordinates: $(x, y) \rightarrow (y, -x)$
90° counterclockwise	Multiply each *y*-coordinate by -1; then switch the *x*- and *y*-coordinates: $(x, y) \rightarrow (-y, x)$
180°	Multiply both coordinates by -1: $(x, y) \rightarrow (-x, -y)$

EXAMPLE 3

TEKS 8.10.C

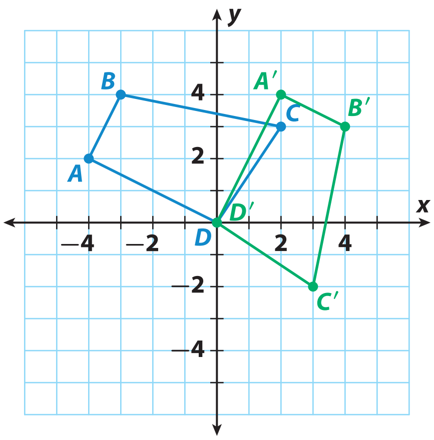
Quadrilateral *ABCD* has vertices at $A(-4, 2)$, $B(-3, 4)$, $C(2, 3)$, and $D(0, 0)$. Find the vertices of quadrilateral $A'B'C'D'$ after a 90° clockwise rotation. Then graph the quadrilateral and its image.

STEP 1 Apply the rule to find the vertices of the image.

Multiply the x-coordinate of each vertex by -1, and then switch the x- and y-coordinates.

Vertices of *ABCD*	Rule: $(y, -x)$	Vertices of $A'B'C'D'$
$A(-4, 2)$	$(2, -1 \cdot (-4))$	$A'(2, 4)$
$B(-3, 4)$	$(4, -1 \cdot (-3))$	$B'(4, 3)$
$C(2, 3)$	$(3, -1 \cdot 2)$	$C'(3, -2)$
$D(0, 0)$	$(0, -1 \cdot 0)$	$D'(0, 0)$

STEP 2 Graph the quadrilateral and its image.



Math Talk
Mathematical Processes

Explain how to use the 90° rotation rule to develop a rule for a 360° rotation.

Reflect

3. **Communicate Mathematical Ideas** How would you find the vertices of an image if a figure were rotated 270° clockwise? Explain.

____

____

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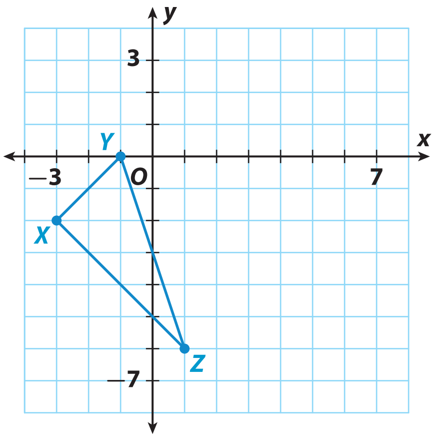
YOUR TURN

4. A triangle has vertices at $J(-2, -4)$, $K(1, 5)$, and $L(2, 2)$. What are the coordinates of the vertices of the image after the triangle is rotated 90° counterclockwise?

____

Guided Practice

1. Triangle XYZ has vertices $X(-3, -2)$, $Y(-1, 0)$, and $Z(1, -6)$. Find the vertices of triangle $X'Y'Z'$ after a translation of 6 units to the right. Then graph the triangle and its image. (Example 1)



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2. Describe what happens to the x- and y-coordinates after a point is reflected across the x-axis. (Example 2)

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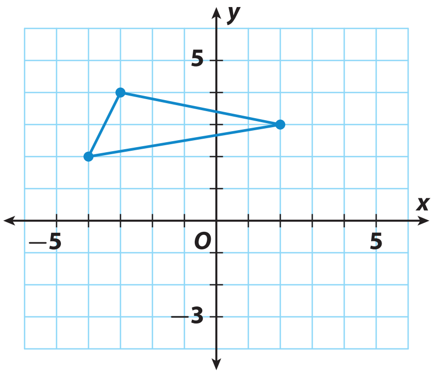
____

3. Use the rule $(x, y) \rightarrow (y, -x)$ to graph the image of the triangle at right. Then describe the transformation. (Example 3)



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ESSENTIAL QUESTION CHECK-IN

4. How do the x- and y-coordinates change when a figure is translated right a units and down b units?

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Name ______________________ Class ______________ Date ____________

12.4 Independent Practice

TEKS 10.8C

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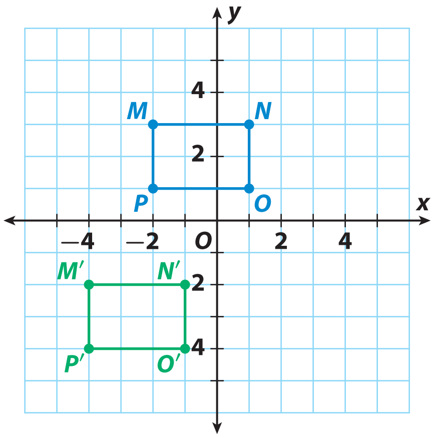
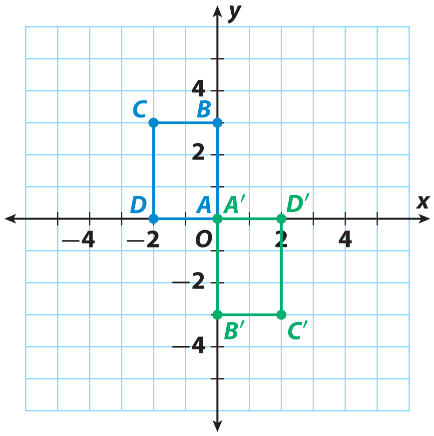
Write an algebraic rule to describe each transformation. Then describe the transformation.

5.



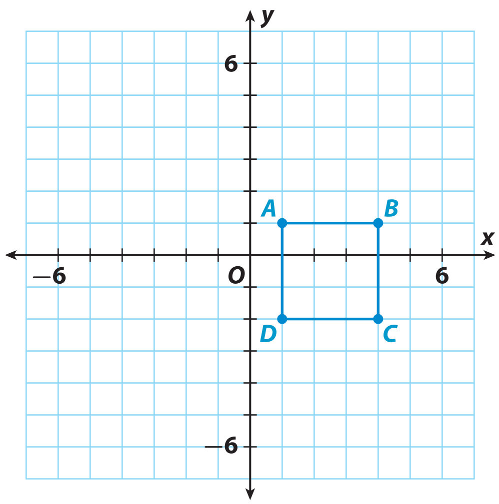
6.



7. Triangle XYZ has vertices $X(6, -2.3)$, $Y(7.5, 5)$, and $Z(8, 4)$. When translated, X' has coordinates $(2.8, -1.3)$. Write a rule to describe this transformation. Then find the coordinates of Y' and Z'.

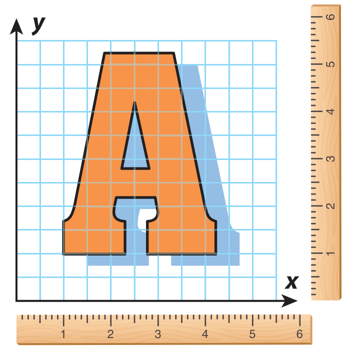
8. Point L has coordinates $(3, -5)$. The coordinates of point L' after a reflection are $(-3, -5)$. Without graphing, tell which axis point L was reflected across. Explain your answer.

9. Use the rule $(x, y) \rightarrow (x - 2, y - 4)$ to graph the image of the rectangle. Then describe the transformation.



10. Parallelogram $ABCD$ has vertices $A(-2, -5\frac{1}{2})$, $B(-4, -5\frac{1}{2})$, $C(-3, -2)$, and $D(-1, -2)$. Find the vertices of parallelogram $A'B'C'D'$ after a translation of $2\frac{1}{2}$ units down.

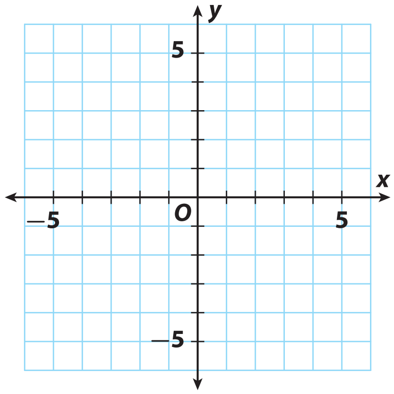
11. Alexandra drew the logo shown on half-inch graph paper. Write a rule that describes the translation Alexandra used to create the shadow on the letter A.



12. Kite *KLMN* has vertices at *K*(1, 3), *L*(2, 4), *M*(3, 3), and *N*(2, 0). After the kite is rotated, *K*′ has coordinates (−3, 1). Describe the rotation, and include a rule in your description. Then find the coordinates of *L*′, *M*′, and *N*′.

H.O.T. FOCUS ON HIGHER ORDER THINKING

13. Make a Conjecture Graph the triangle with vertices (−3, 4), (3, 4), and (−5, −5). Use the transformation (*y*, *x*) to graph its image.



a. Which vertex of the image has the same coordinates as a vertex of the original figure? Explain why this is true.

b. What is the equation of a line through the origin and this point?

c. Describe the transformation of the triangle.

14. Critical Thinking Mitchell says the point (0, 0) does not change when reflected across the *x*- or *y*-axis or when rotated about the origin. Do you agree with Mitchell? Explain why or why not.

15. Analyze Relationships Triangle *ABC* with vertices *A*(−2, −2), *B*(−3, 1), and *C*(1, 1) is translated by $(x, y) \rightarrow (x - 1, y + 3)$. Then the image, triangle *A*′*B*′*C*′, is translated by $(x, y) \rightarrow (x + 4, y - 1)$, resulting in *A*″*B*″*C*″.

a. Find the coordinates for the vertices of triangle *A*″*B*″*C*″.

b. Write a rule for one translation that maps triangle *ABC* to triangle *A*″*B*″*C*″.

Work Area

MODULE QUIZ

Ready to Go On?

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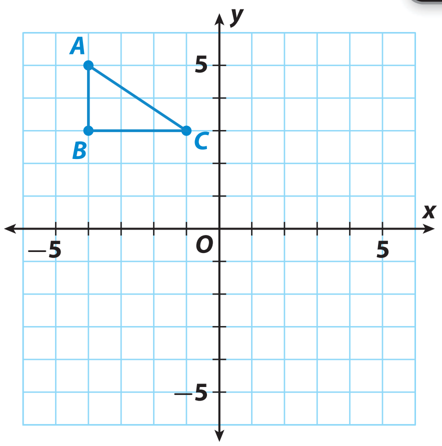
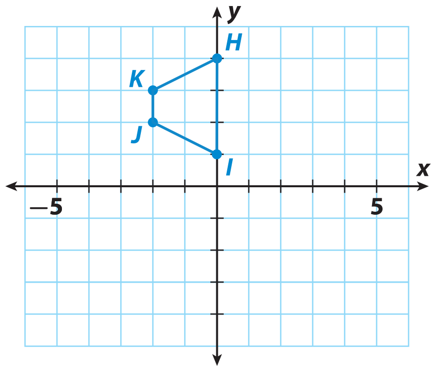
12.1–12.3 Properties of Translations, Reflections, and Rotations

Use the graph for Exercises 1–2.



1. Graph the image of triangle *ABC* after a translation of 6 units to the right and 4 units down. Label the vertices of the image *A′*, *B′*, and *C′*.

2. On the same coordinate grid, graph the image of triangle *ABC* after a reflection across the *y*-axis. Label the vertices of the image *A″*, *B″*, and *C″*.



3. Graph the image of trapezoid *HIJK* after it is rotated 180° about the origin. Label the vertices of the image. Find the vertices if the trapezoid *HIJK* is rotated 360°.

____

4. **Vocabulary** Translations, reflections, and rotations produce a figure that is

________________ to the original figure.

12.4 Algebraic Representations of Transformations

5. A triangle has vertices at (2, 3), (−2, 2), and (−3, 5). What are the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x + 4, y - 3)$? Describe the transformation.

____

____

ESSENTIAL QUESTION

6. How can you use transformations to solve real-world problems?

____

____

MODULE 12 MIXED REVIEW

Texas Test Prep

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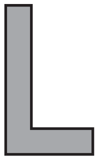
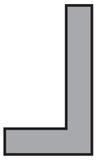
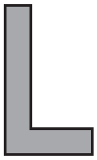
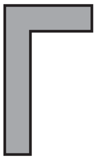
Selected Response

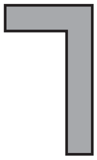
1. What would be the orientation of the figure L after a translation of 8 units to the right and 3 units up?



Ⓐ 

Ⓒ 

Ⓑ 

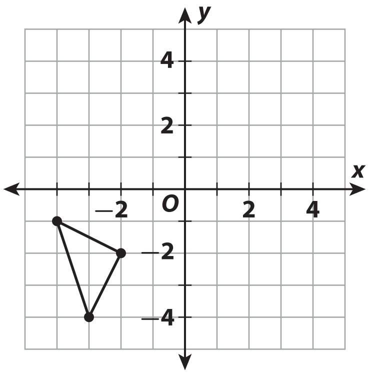
Ⓓ 

2. A straw has a diameter of 0.6 cm and a length of 19.5 cm. What is the surface area of the straw to the nearest tenth? Use 3.14 for π.

Ⓐ 5.5 cm^2 Ⓒ 36.7 cm^2

Ⓑ 11.7 cm^2 Ⓓ 37.3 cm^2

3. In what quadrant would the triangle be located after a rotation of 270° clockwise about the origin?



Ⓐ I Ⓒ III

Ⓑ II Ⓓ IV

4. Which rational number is greater than $-3\frac{1}{3}$ but less than $-\frac{4}{5}$?

Ⓐ -0.4 Ⓒ -0.19

Ⓑ $-\frac{9}{7}$ Ⓓ $-\frac{22}{5}$

5. Which of the following is **not** true of a trapezoid that has been reflected across the *x*-axis?

Ⓐ The new trapezoid is the same size as the original trapezoid.

Ⓑ The new trapezoid is the same shape as the original trapezoid.

Ⓒ The new trapezoid is in the same orientation as the original trapezoid.

Ⓓ The *x*-coordinates of the new trapezoid are the same as the *x*-coordinates of the original trapezoid.

6. A triangle with coordinates (6, 4), (2, −1), and (−3, 5) is translated 4 units left and rotated 180° about the origin. What are the coordinates of its image?

Ⓐ (2, 4), (−2, −1), (−7, 5)

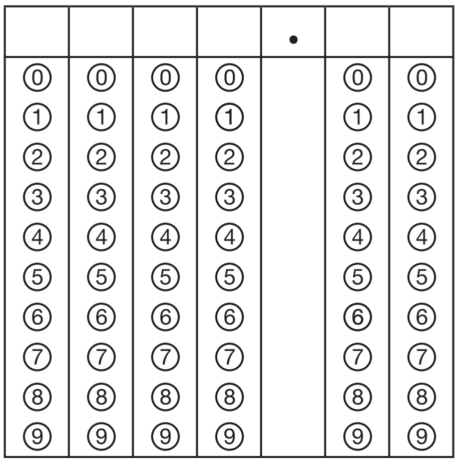
Ⓑ (4, 6), (−1, 2), (5, −3)

Ⓒ (4, −2), (−1, 2), (5, 7)

Ⓓ (−2, −4), (2, 1), (7, −5)

Gridded Response

7. Solve the equation $3y + 17 = -2y + 25$ for y.



Dilations, Similarity, and Proportionality

MODULE 13

ESSENTIAL QUESTION

How can you use dilations, similarity, and proportionality to solve real-world problems?

LESSON 13.1
Properties of Dilations
TEKS 8.3.A, 8.3.B, 8.10.A, 8.10.B

LESSON 13.2
Algebraic Representations of Dilations
TEKS 8.3.B, 8.3.C, 8.10.D

LESSON 13.3
Dilations and Measurement
TEKS 8.3.B, 8.3.C, 8.10.B, 8.10.D

Real-World Video

To plan a mural, the artist first makes a smaller drawing showing what the mural will look like. Then the image is enlarged by a scale factor on the mural canvas. This enlargement is called a dilation.

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Simplify Ratios

EXAMPLE $\frac{35}{21} = \frac{35 \div 7}{21 \div 7}$ *To write a ratio in simplest form, find the greatest common factor of the numerator and denominator.*

$= \frac{5}{3}$ *Divide the numerator and denominator by the GCF.*

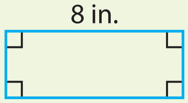
Write each ratio in simplest form.

1. $\frac{6}{15}$ ________ **2.** $\frac{8}{20}$ ________ **3.** $\frac{30}{18}$ ________ **4.** $\frac{36}{30}$ ________

Find Perimeter

EXAMPLE

8 in.



3 in.

$P = 8 + 3 + 8 + 3 = 22$ in.

Find the sum of the lengths of the sides.

Find the perimeter.

5. square with sides of 8.9 cm ________

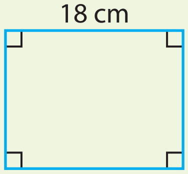
6. rectangle with length $5\frac{1}{2}$ ft and width $2\frac{3}{4}$ ft ________

7. equilateral triangle with sides of $8\frac{3}{8}$ in. ________

Area of Squares, Rectangles, Triangles

EXAMPLE

18 cm



14 cm

$A = bh$ *Use the formula for the area of a rectangle.*

$A = 18 \times 14$ *Substitute for the variables.*

$= 252 \text{ cm}^2$ *Multiply.*

Find the area.

8. Square with sides of 6.5 cm: ________

9. Triangle with base 10 in. and height 6 in.: ________

10. Rectangle with length $3\frac{1}{2}$ ft and width $2\frac{1}{2}$ ft: ________

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic organizer. You will put one word in each rectangle.

The four regions on a coordinate plane.

The point where the axes intersect to form the coordinate plane.

Reviewing the Coordinate Plane

The horizontal axis of a coordinate plane.

The vertical axis of a coordinate plane.

Vocabulary

Review Words

- coordinate plane *(plano cartesiano)*
- image *(imagen)*
- ✔ origin *(origen)*
- preimage *(imagen original)*
- ✔ quadrants *(cuadrante)*
- ratio *(razón)*
- scale *(escala)*
- ✔ *x*-axis *(eje x)*
- ✔ *y*-axis *(eje y)*

Preview Words

- center of dilation *(centro de dilatación)*
- dilation *(dilatación)*
- enlargement *(agranda-miento)*
- reduction *(reducción)*
- scale factor *(factor de escala)*

Understand Vocabulary

Complete the sentences using the review words.

1. A figure larger than the original, produced through dilation, is an ________________.

2. A figure smaller than the original, produced through dilation, is a ________________.

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write the highlighted vocabulary words on one side of the flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions used in this module.

MODULE 13

Unpacking the TEKS

Understanding the TEKS and the vocabulary terms in the TEKS will help you know exactly what you are expected to learn in this module.

TEKS 8.3.C

Use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation.

Key Vocabulary

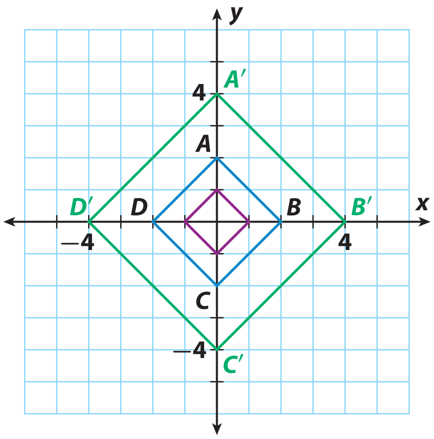
scale factor *(factor de escala)*
The ratio used to enlarge or reduce similar figures.

What It Means to You

You will use an algebraic representation to describe a dilation.

UNPACKING EXAMPLE 8.3.C

The blue square *ABCD* is the preimage. Write two algebraic representations, one for the dilation to the green square and one for the dilation to the purple square.



The coordinates of the vertices of the original image are multiplied by 2 for the green square.

Green square: $(x, y) \rightarrow (2x, 2y)$

The coordinates of the vertices of the original image are multiplied by $\frac{1}{2}$ for the purple square.

Purple square: $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$

TEKS 8.10.D

Model the effect on linear and area measurements of dilated two-dimensional shapes.

Key Vocabulary

similar shapes *(formas)*
Figures with the same shape but not necessarily the same size.

What It Means to You

You can find the effect of a dilation on the perimeter and area of a figure.

UNPACKING EXAMPLE 8.10.D

The length of the side of a square is 3 inches. If the square is dilated by a scale factor of 5, what are the perimeter and the area of the new square?

The length of a side of the original square is 3 inches. The length of a side of the dilated square is $5 \cdot 3$, or 15 inches.

Original Square	Dilated square
Perimeter: $4(3) = 12$ in.	Perimeter: $4(15) = 60$ in.
Area: $3^2 = 9$ in^2	Area: $15^2 = 225$ in^2

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LESSON 13.1

Properties of Dilations

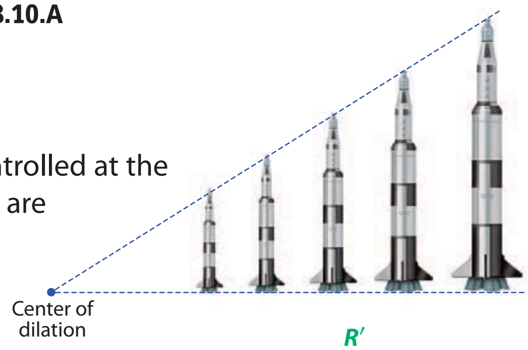
TEKS **Proportionality—8.3.B** Compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane. *Also 8.3.A, 8.10.A, 8.10.B*

ESSENTIAL QUESTION

How do you describe the properties of dilations?

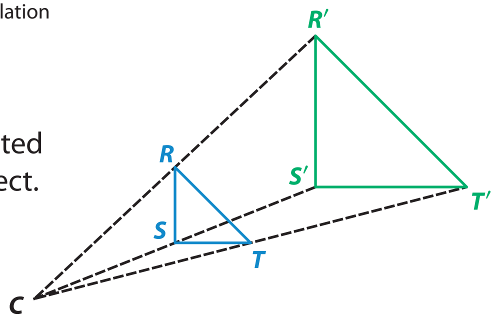
EXPLORE ACTIVITY 1 Real World TEKS 8.3.A, 8.10.A

Exploring Dilations



The missions that placed 12 astronauts on the moon were controlled at the Johnson Space Center in Houston. The toy models at the right are scaled-down replicas of the Saturn V rocket that powered the moon flights. Each replica is a transformation called a **dilation**. Unlike the other transformations you have studied—translations, rotations, and reflections—dilations change the size (but not the shape) of a figure.

Every dilation has a fixed point called the **center of dilation** located where the lines connecting corresponding parts of figures intersect.



Triangle $R'S'T'$ is a dilation of triangle RST. Point C is the center of dilation.

A Use a ruler to measure segments $\overline{CR}$, $\overline{CR'}$, $\overline{CS}$, $\overline{CS'}$, $\overline{CT}$, and $\overline{CT'}$ to the nearest millimeter. Record the measurements and ratios in the table.

CR'	CR	$\frac{CR'}{CR}$	CS'	CS	$\frac{CS'}{CS}$	CT'	CT	$\frac{CT'}{CT}$

B Write a conjecture based on the ratios in the table.

____

C Measure and record the corresponding side lengths of the triangles.

$R'S'$	RS	$\frac{R'S'}{RS}$	$S'T'$	ST	$\frac{S'T'}{ST}$	$R'T'$	RT	$\frac{R'T'}{RT}$

D Write a conjecture based on the ratios in the table.

____

E Measure the corresponding angles and describe your results.

____

EXPLORE ACTIVITY 1 *(cont'd)*

Reflect

1. Are triangles *RST* and *R′S′T′* similar? Why or why not?

___

___

2. Compare the orientation of a figure with the orientation of its dilation.

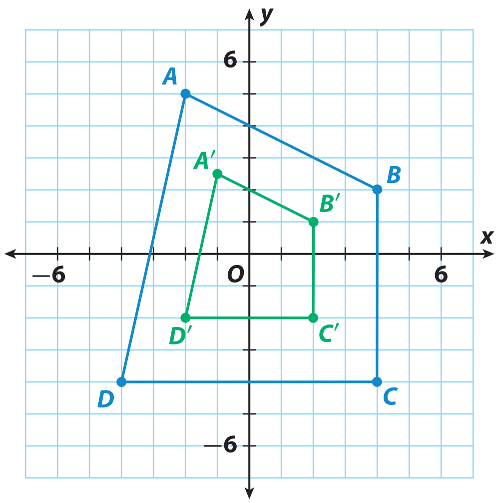
___

___

EXPLORE ACTIVITY 2 TEKS 8.3.B

Exploring Dilations on a Coordinate Plane

In this activity you will explore how the coordinates of a figure on a coordinate plane are affected by a dilation.



A Complete the table. Record the *x*- and *y*-coordinates of the points in the two figures and the ratios of the *x*-coordinates and the *y*-coordinates.

Vertex	*x*	*y*	Vertex	*x*	*y*	Ratio of *x*-coordinates (*A′B′C′D′* ÷ *ABCD*)	Ratio of *y*-coordinates (*A′B′C′D′* ÷ *ABCD*)
A′			*A*				
B′			*B*				
C′			*C*				
D′			*D*				

B Write a conjecture about the ratios of the coordinates of a dilation image to the coordinates of the original figure.

___

___

Reflect

3. In Explore Activity 1, triangle $R'S'T'$ was larger than triangle RST. How is the relationship between quadrilateral $A'B'C'D'$ and quadrilateral $ABCD$ different?

____

Math Talk
Mathematical Processes
How are dilations different from the other transformations you have learned about?

Finding a Scale Factor

As you have seen in the two activities, a dilation can produce a larger figure (an **enlargement**) or a smaller figure (a **reduction**). The **scale factor** describes how much the figure is enlarged or reduced. The scale factor is the ratio of a length of the image to the corresponding length on the original figure.

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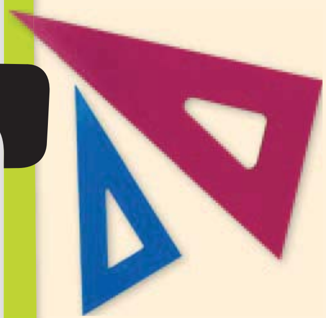
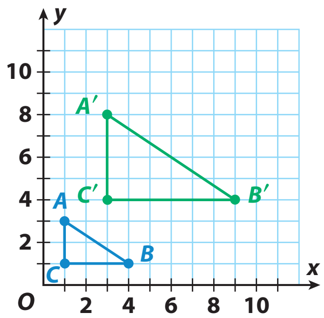
In Explore Activity 1, the side lengths of triangle $R'S'T'$ were twice the length of those of triangle RST, so the scale factor was 2. In Explore Activity 2, the side lengths of quadrilateral $A'B'C'D'$ were half those of quadrilateral $ABCD$, so the scale factor was 0.5.



EXAMPLE 1 Real World

TEKS 8.3.B

An art supply store sells several sizes of drawing triangles. All are dilations of a single basic triangle. The basic triangle and one of its dilations are shown on the grid. Find the scale factor of the dilation.



STEP 1 Use the coordinates to find the lengths of the sides of each triangle.

Triangle ABC: $AC = 2$ $CB = 3$

Triangle $A'B'C'$: $A'C' = 4$ $C'B' = 6$

Since the scale factor is the same for all corresponding sides, you can record just two pairs of side lengths. Use one pair as a check on the other.

STEP 2 Find the ratios of the corresponding sides.

$\frac{A'C'}{AC} = \frac{4}{2} = 2$ $\frac{C'B'}{CB} = \frac{6}{3} = 2$

The scale factor of the dilation is 2.

Reflect

4. Is the dilation an enlargement or a reduction? How can you tell?

____

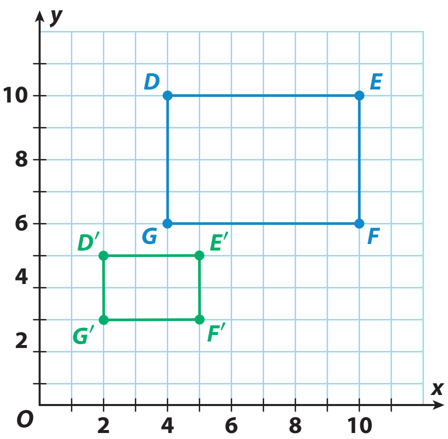
____

YOUR TURN

5. Find the scale factor of the dilation.

____

____

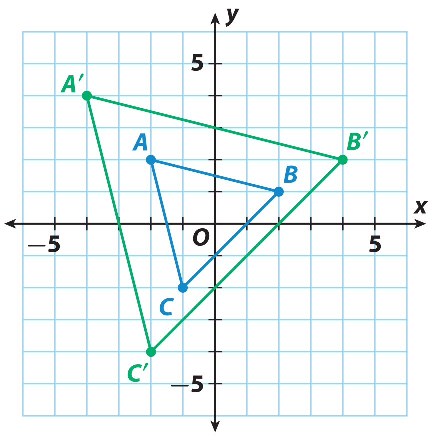


Math Talk Mathematical Processes

Which scale factors lead to enlargements? Which scale factors lead to reductions?

Guided Practice

Use triangles *ABC* and *A′B′C′* for 1–5. (Explore Activities 1 and 2, Example 1)



1. For each pair of corresponding vertices, find the ratio of the *x*-coordinates and the ratio of the *y*-coordinates.

ratio of *x*-coordinates = _______________

ratio of *y*-coordinates = _______________

2. I know that triangle *A′B′C′* is a dilation of triangle *ABC* because the ratios of the corresponding *x*-coordinates are _______________ and the ratios of the corresponding *y*-coordinates are _______________.

3. The ratio of the lengths of the corresponding sides of triangle *A′B′C′* and triangle *ABC* equals _______________.

4. The corresponding angles of triangle *ABC* and triangle *A′B′C′* are _______________.

5. The scale factor of the dilation is _______________.

ESSENTIAL QUESTION CHECK-IN

6. How can you find the scale factor of a dilation?

____

____

Name ____________ Class ____________ Date ____________

13.1 Independent Practice

TEKS 8.3.A, 8.10.A, 8.3.B, 8.10.B

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For 7–11, tell whether one figure is a dilation of the other or not. Explain your reasoning.

7. Quadrilateral *MNPQ* has side lengths of 15 mm, 24 mm, 21 mm, and 18 mm. Quadrilateral *M′N′P′Q′* has side lengths of 5 mm, 8 mm, 7 mm, and 4 mm.

8. Triangle *RST* has angles measuring 38° and 75°. Triangle *R′S′T′* has angles measuring 67° and 38°.

9. Two triangles, Triangle 1 and Triangle 2, are similar.

10. Quadrilateral *MNPQ* is the same shape but a different size than quadrilateral *M′N′P′Q*.

11. On a coordinate plane, triangle *UVW* has coordinates *U*(20, −12), *V*(8, 6), and *W*(−24, −4). Triangle *U′V′W′* has coordinates *U′*(15, −9), *V′*(6, 4.5), and *W′*(−18, −3).

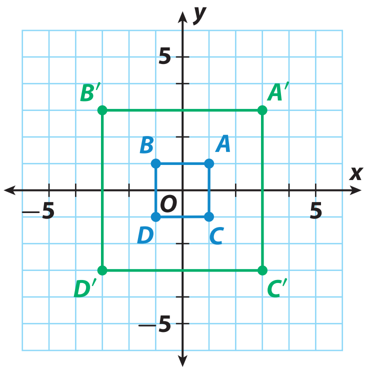
Complete the table by writing "same" or "changed" to compare the image with the original figure in the given transformation.

	Image Compared to Original Figure		
	Orientation	Size	Shape
12. Translation			
13. Reflection			
14. Rotation			
15. Dilation			

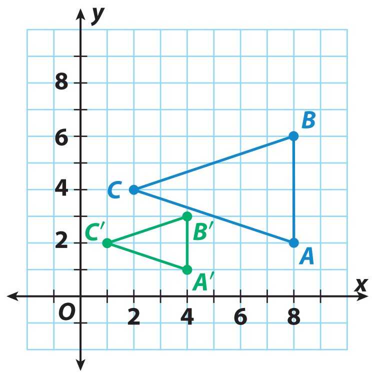
16. Describe the image of a dilation with a scale factor of 1.

Identify the scale factor used in each dilation.

17.



18.



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H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

19. **Critical Thinking** Explain how you can find the center of dilation of a triangle and its dilation.

____

____

____

20. **Make a Conjecture**

a. A square on the coordinate plane has vertices at (−2, 2), (2, 2), (2, −2), and (−2, −2). A dilation of the square has vertices at (−4, 4), (4, 4), (4, −4), and (−4, −4). Find the scale factor and the perimeter of each square.

____

____

b. A square on the coordinate plane has vertices at (−3, 3), (3, 3), (3, −3), and (−3, −3). A dilation of the square has vertices at (−6, 6), (6, 6), (6, −6), and (−6, −6). Find the scale factor and the perimeter of each square.

____

____

c. Make a conjecture about the relationship of the scale factor to the perimeter of a square and its image.

____

____

LESSON 13.2

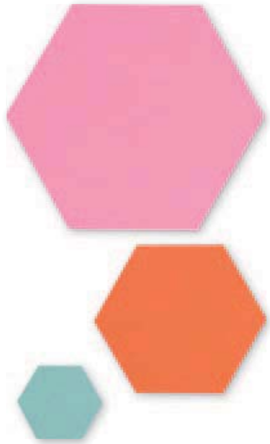
Algebraic Representations of Dilations

TEKS **Proportionality—8.3.C** Use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation. *Also* 8.3.B, 8.10.D

ESSENTIAL QUESTION

How can you describe the effect of a dilation on coordinates using an algebraic representation?

EXPLORE ACTIVITY 1 TEKS 8.3.C



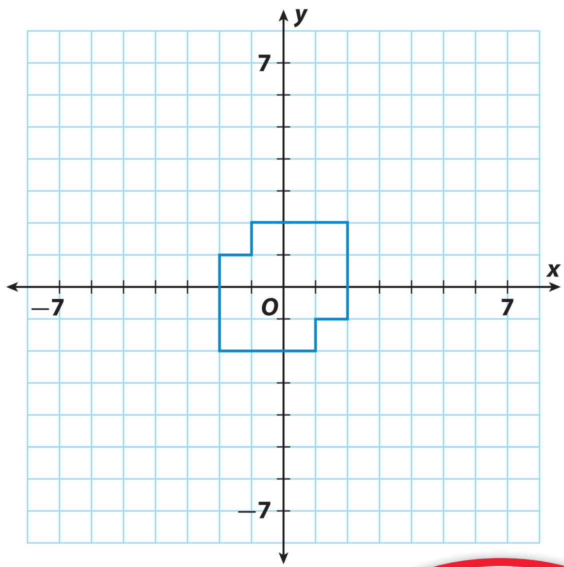
Graphing Enlargements

When a dilation in the coordinate plane has the origin as the center of dilation, you can find points on the dilated image by multiplying the x- and y-coordinates of the original figure by the scale factor. For scale factor k, the algebraic representation of the dilation is $(x, y) \rightarrow (kx, ky)$. For enlargements, $k > 1$.

The figure shown on the grid is the preimage. The center of dilation is the origin.

A List the coordinates of the vertices of the preimage in the first column of the table.

Preimage (x, y)	Image $(3x, 3y)$
(2, 2)	(6, 6)



B What is the scale factor for the dilation? ________

C Apply the dilation to the preimage and write the coordinates of the vertices of the image in the second column of the table.

D Sketch the image after the dilation on the coordinate grid.

Math Talk
Mathematical Processes

What effect would the dilation $(x, y) \rightarrow (4x, 4y)$ have on the radius of a circle?

EXPLORE ACTIVITY 1 *(cont'd)*

Reflect

1. How does the dilation affect the length of line segments?

____

____

2. How does the dilation affect angle measures?

____

EXPLORE ACTIVITY 2 TEKS 8.3.C

Graphing Reductions

For scale factors between 0 and 1, the image is smaller than the preimage. This is called a reduction.

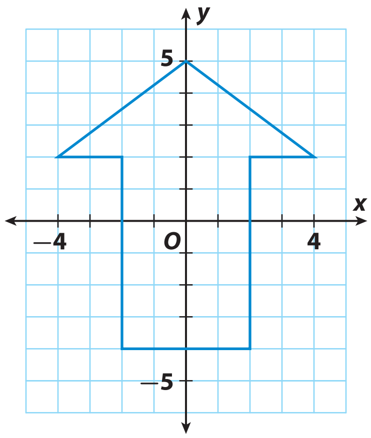
The arrow shown is the preimage. The center of dilation is the origin.

A List the coordinates of the vertices of the preimage in the first column of the table.

B What is the scale factor for the dilation? _______

C Apply the dilation to the preimage and write the coordinates of the vertices of the image in the second column of the table.

Preimage (x, y)	Image $\left(\frac{1}{2}x, \frac{1}{2}y\right)$



D Sketch the image after the dilation on the coordinate grid.

Reflect

3. How does the dilation affect the length of line segments?

____

4. How would a dilation with scale factor 1 affect the preimage?

____

Center of Dilation Outside the Image

The center of dilation can be inside *or* outside the original image and the dilated image. The center of dilation can be anywhere on the coordinate plane as long as the lines that connect each pair of corresponding vertices between the original and dilated image intersect at the center of dilation.

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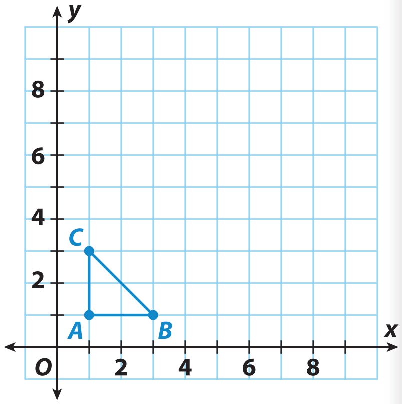
EXAMPLE 1

TEKS 8.3.C

Graph the image of $\triangle ABC$ after a dilation with the origin as its center and a scale factor of 3. What are the vertices of the image?

STEP 1 Multiply each coordinate of the vertices of $\triangle ABC$ by 3 to find the vertices of the dilated image.



$\triangle ABC\ (x, y) \rightarrow (3x, 3y)\ \triangle A'B'C'$

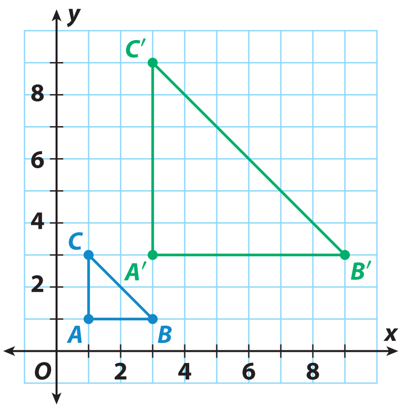
$A(1, 1) \rightarrow A'(1 \cdot 3, 1 \cdot 3) \rightarrow A'(3, 3)$

$B(3, 1) \rightarrow B'(3 \cdot 3, 1 \cdot 3) \rightarrow B'(9, 3)$

$C(1, 3) \rightarrow C'(1 \cdot 3, 3 \cdot 3) \rightarrow C'(3, 9)$

The vertices of the dilated image are $A'(3, 3)$, $B'(9, 3)$, and $C'(3, 9)$.

STEP 2 Graph the dilated image.



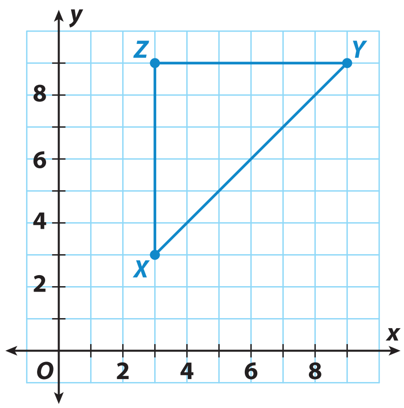
Math Talk
Mathematical Processes

Describe how you can check graphically that you have drawn the image triangle correctly.

YOUR TURN

5. Graph the image of $\triangle XYZ$ after a dilation with a scale factor of $\frac{1}{3}$ and the origin as its center. Then write an algebraic rule to describe the dilation.

____



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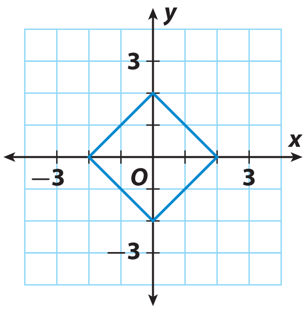
Guided Practice

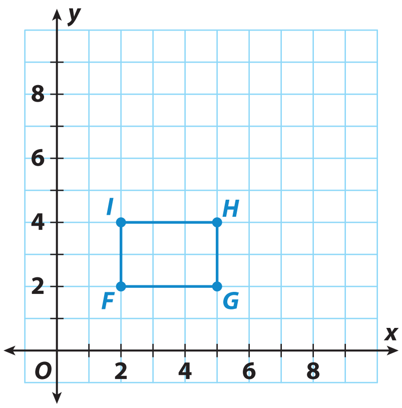
1. The grid shows a diamond-shaped preimage. Write the coordinates of the vertices of the preimage in the first column of the table. Then apply the dilation $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$ and write the coordinates of the vertices of the image in the second column. Sketch the image of the figure after the dilation. (Explore Activities 1 and 2)

Preimage	Image
(2, 0)	(3, 0)



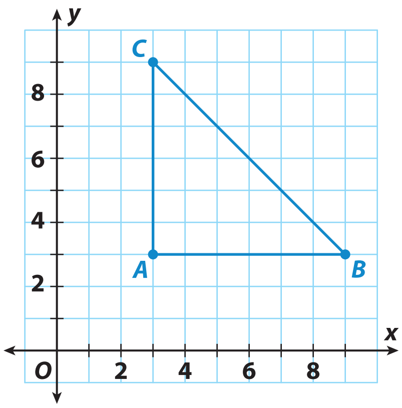
Graph the image of each figure after a dilation with the origin as its center and the given scale factor. Then write an algebraic rule to describe the dilation. (Example 1)

2. scale factor of 1.5



____

3. scale factor of $\frac{1}{3}$



____

ESSENTIAL QUESTION CHECK-IN

4. A dilation of $(x, y) \rightarrow (kx, ky)$ when $0 < k < 1$ has what effect on the figure? What is the effect on the figure when $k > 1$?

____

____

____

Name ____________________ Class ____________________ Date ____________________

13.2 Independent Practice

TEKS 8.3.B, 8.3.C, 8.10.D

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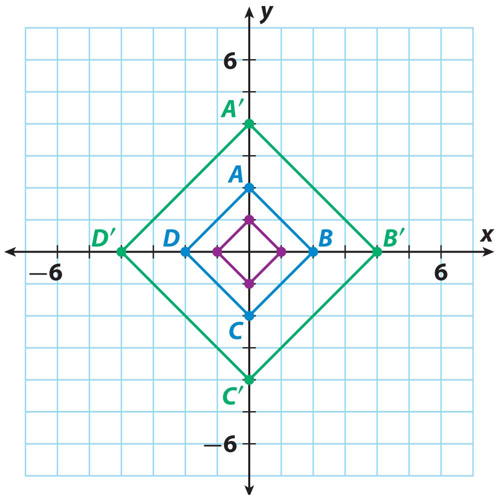
5. The blue square is the preimage. Write two algebraic representations, one for the dilation to the green square and one for the dilation to the purple square.



____

____

6. **Critical Thinking** A triangle has vertices $A(-5, -4)$, $B(2, 6)$, and $C(4, -3)$. The center of dilation is the origin and $(x, y) \rightarrow (3x, 3y)$. What are the vertices of the dilated image?

____

____

7. **Critical Thinking** $M'N'O'P'$ has vertices at $M'(3, 4)$, $N'(6, 4)$, $O'(6, 7)$, and $P'(3, 7)$. The center of dilation is the origin. $MNOP$ has vertices at $M(4.5, 6)$, $N(9, 6)$, $O'(9, 10.5)$, and $P'(4.5, 10.5)$. What is the algebraic representation of this dilation?

____

8. **Critical Thinking** A dilation with center (0, 0) and scale factor k is applied to a polygon. What dilation can you apply to the image to return it to the original preimage?

____

9. **Represent Real-World Problems** The blueprints for a new house are scaled so that $\frac{1}{4}$ inch equals 1 foot. The blueprint is the preimage and the house is the dilated image. The blueprints are plotted on a coordinate plane.

a. What is the scale factor in terms of inches to inches?

____

b. One inch on the blueprint represents how many inches in the actual house? How many feet?

____

c. Write the algebraic representation of the dilation from the blueprint to the house.

____

d. A rectangular room has coordinates $Q(2, 2)$, $R(7, 2)$, $S(7, 5)$, and $T(2, 5)$ on the blueprint. The homeowner wants this room to be 25% larger. What are the coordinates of the new room?

____

____

e. What are the dimensions of the new room, in inches, on the blueprint? What will the dimensions of the new room be, in feet, in the new house?

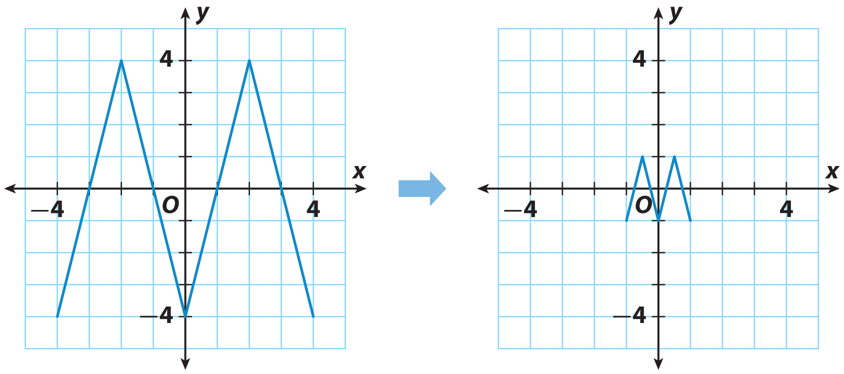
____

____

____

____

10. Write the algebraic representation of the dilation shown.



____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

11. **Critique Reasoning** The set for a school play needs a replica of a historic building painted on a backdrop that is 20 feet long and 16 feet high. The actual building measures 400 feet long and 320 feet high. A stage crewmember writes $(x, y) \rightarrow \left(\frac{1}{12}x, \frac{1}{12}y\right)$ to represent the dilation. Is the crewmember's calculation correct if the painted replica is to cover the entire backdrop? Explain.

____

____

12. **Communicate Mathematical Ideas** Explain what each of these algebraic transformations does to a figure.

a. $(x, y) \rightarrow (y, -x)$ ______________________

b. $(x, y) \rightarrow (-x, -y)$ ______________________

c. $(x, y) \rightarrow (x, 2y)$ ______________________

____

d. $(x, y) \rightarrow \left(\frac{2}{3}x, y\right)$ ______________________

____

e. $(x, y) \rightarrow (0.5x, 1.5y)$ ______________________

____

13. **Communicate Mathematical Ideas** Triangle *ABC* has coordinates *A*(1, 5), *B*(−2, 1), and *C*(−2, 4). Sketch triangle *ABC* and *A′B′C′* for the dilation $(x, y) \rightarrow (-2x, -2y)$. What is the effect of a negative scale factor?

____

____

LESSON 13.3

Dilations and Measurement

TEKS **Two-dimensional shapes—8.10.D** Model the effect on linear and area measurements of dilated two-dimensional shapes. *Also 8.3.B, 8.10.A, 8.10.B*

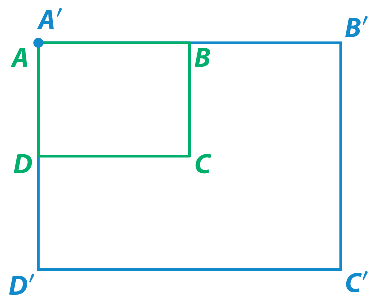
ESSENTIAL QUESTION

How do you describe the effects of dilation on linear and area measurements?

EXPLORE ACTIVITY TEKS 8.10.D

Exploring Dilations and Measurement

The blue rectangle is a dilation (enlargement) of the green rectangle.



A Using a centimeter ruler, measure and record the length of each side of both rectangles. Then calculate the ratios of all pairs of corresponding sides.

$AB =$ ______ $BC =$ ______ $CD =$ ______ $DA =$ ______

$A'B' =$ ______ $B'C' =$ ______ $C'D' =$ ______ $D'A' =$ ______

$\frac{A'B'}{AB} =$ ______ $\frac{B'C'}{BC} =$ ______ $\frac{C'D'}{CD} =$ ______ $\frac{D'A'}{DA} =$ ______

What is true about the ratios that you calculated?

______

What scale factor was used to dilate the green rectangle to the blue rectangle? ______

How are the side lengths of the blue rectangle related to the side lengths of the green rectangle?

______

B What is the perimeter of the green rectangle? ______

What is the perimeter of the blue rectangle? ______

How is the perimeter of the blue rectangle related to the perimeter of the green rectangle?

______

______

______

EXPLORE ACTIVITY *(cont'd)*

C What is the area of the green rectangle? ____________________

What is the area of the blue rectangle? ____________________

How is the area of the blue rectangle related to the area of the green rectangle?

____

____

Reflect

1. **Make a Conjecture** The perimeter and area of two shapes before and after dilation are given. How are the perimeter and area of a dilated figure related to the perimeter and area of the original figure?

	Perimeter	Area
Original	8	4
Dilation	16	16

Scale factor $= 2$

	Perimeter	Area
Original	30	54
Dilation	5	1.5

Scale factor $= \frac{1}{6}$

____

____

____

____

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Problem-Solving Application

Understanding how dilations affect the linear and area measurements of shapes will enable you to solve many real-world problems.

EXAMPLE 1 Problem Solving TEKS 8.3.B

A souvenir shop sells standard-sized decks of cards and mini-decks of cards. A card in the standard deck is a rectangle that has a length of 3.5 inches and a width of 2.5 inches. The perimeter of a card in the mini-deck is 6 inches. What is the area of a card in the mini-deck?

Analyze Information

I need to find the area of a mini-card. I know the length and width of a standard card and the perimeter of a mini-card.

Formulate a Plan

Since the mini-card is a dilation of the standard card, the figures are similar. Find the perimeter of the standard card, and use that to find the scale factor. Then use the scale factor to find the area of the mini-card.

Solve

STEP 1 Find the perimeter of the standard card.

$P_s = 2l + 2w$

$P_s = 2(3.5) + 2(2.5)$

$P_s = 12$ in.

STEP 2 Find the scale factor.

$P_m = P_s \cdot k$

$6 = 12 \cdot k$

$\frac{1}{2} = k$

Multiply the perimeter of the standard card by the scale factor to get the perimeter of the mini-card.

STEP 3 Find the area of the standard card.

$A_s = l_s \cdot w_s$

Use the formula for the area of a rectangle.

$A_s = 3.5 \cdot 2.5$

$A_s = 8.75 \text{ in}^2$

STEP 4 Find the area of the mini-card.

$A_m = A_s \cdot k^2$

Multiply the area of the standard card by the scale factor squared to get the area of the mini-card.

$A_m = 8.75 \cdot \left(\frac{1}{2}\right)^2$

$A_m = 8.75 \cdot \frac{1}{4}$

$A_m = 2.1875 \text{ in}^2$

The area of the mini card is about 2.2 square inches.

Justify and Evaluate

To find the area of the mini-card, find its length and width by multiplying the dimensions of the standard card by the scale factor. The length of the mini-card is $l_s \cdot \frac{1}{2} = 3.5 \cdot \frac{1}{2} = 1.75$ in., and the width is $w_s \cdot \frac{1}{2} = 2.5 \cdot \frac{1}{2} = 1.25$ in. So, $A_m = l_m \cdot w_m = 1.75 \cdot 1.25 = 2.1875 \text{ in}^2$. The answer is correct.

My Notes

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YOUR TURN

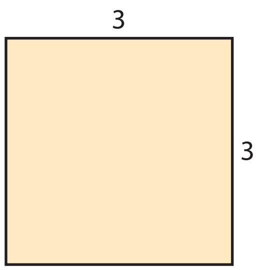
2. Johnson Middle School is selling mouse pads that are replicas of a student's award-winning artwork. The rectangular mouse pads are dilated from the original artwork and have a length of 9 inches and a width of 8 inches. The perimeter of the original artwork is 136 inches. What is the area of the original artwork?

____

Guided Practice

Find the perimeter and area of the image after dilating the figures shown with the given scale factor. (Explore Activity and Example 1)

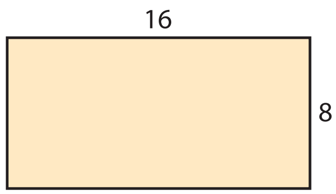
1. Scale factor $= 5$



$P = 12$ $\quad$ $A = 9$

$P' =$ ______ $\quad$ $A' =$ ______

2. Scale factor $= \frac{3}{4}$



$P = 48$ $\quad$ $A = 128$

$P' =$ ______ $\quad$ $A' =$ ______

A group of friends is roping off a soccer field in a back yard. A full-size soccer field is a rectangle with a length of 100 yards and a width of 60 yards. To fit the field in the back yard, the group needs to reduce the size of the field so its perimeter is 128 yards. (Example 1)

3. What is the perimeter of the full-size soccer field? ______

4. What is the scale factor of the dilation? ______

5. What is the area of the soccer field in the back yard? ______

ESSENTIAL QUESTION CHECK-IN

6. When a rectangle is dilated, how do the perimeter and area of the rectangle change?

____

____

____

____

Name ____________________ Class ____________ Date ____________

13.3 Independent Practice

TEKS 8.3.B, 8.10.A, 8.10.B, 8.10.D

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7. When you make a photocopy of an image, is the photocopy a dilation? What is the scale factor? How do the perimeter and area change?

____

____

8. **Problem Solving** The universally accepted film size for movies has a width of 35 millimeters. If you want to project a movie onto a square sheet that has an area of 100 square meters, what is the scale factor that is needed for the projection of the movie? Explain.

____

____

____

9. The perimeter of a square is 48 centimeters. If the square is dilated by a scale factor of 0.75, what is the length of each side of the new square?

____

10. The screen of an eReader has a length of 8 inches and a width of 6 inches. Can the page content from an atlas that measures 19 inches by 12 inches be replicated in the eReader? If not, propose a solution to move the atlas content into the eReader format.

____

____

____

____

____

11. **Represent Real-World Problems** There are 64 squares on a chessboard. Each square on a tournament chessboard measures 2.25×2.25 inches. A travel chessboard is a dilated replica of the tournament chessboard using a scale factor of $\frac{1}{3}$.

a. What is the size of each square on the travel chessboard? ____________

b. How long is each side of the travel board? ____________

c. How much table space do you need to play on the travel chessboard?

____

12. **Draw Conclusions** The legs of a right triangle are 3 units and 4 units long. Another right triangle is dilated from this triangle using a scale factor of 3. What are the side lengths and the perimeter of the dilated triangle?

____

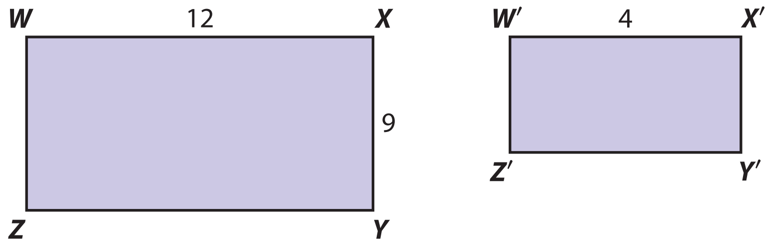
____

____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

Work Area

13. **Critique Reasoning** Rectangle $W'X'Y'Z'$ below is a dilation of rectangle $WXYZ$. A student calculated the area of rectangle $W'X'Y'Z'$ to be 36 square units. Do you agree with this student's calculation? If not, explain and correct the mistake.



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14. **Multistep** Rectangle $A'B'C'D'$ is a dilation of rectangle $ABCD$, and the scale factor is 2. The perimeter of $ABCD$ is 18 mm. The area of $ABCD$ is 20 mm^2.

a. Write an equation for, and calculate, the perimeter of $A'B'C'D'$.

____

____

b. Write an equation for, and calculate, the area of $A'B'C'D'$.

____

____

c. The side lengths of both rectangles are whole numbers of millimeters. What are the side lengths of $ABCD$ and $A'B'C'D'$?

____

Ready to Go On?

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13.1 Properties of Dilations

Determine whether one figure is a dilation of the other. Justify your answer.

1. Triangle *XYZ* has angles measuring 54° and 29°. Triangle *X′Y′Z′* has angles measuring 29° and 92°.

____

2. Quadrilateral *DEFG* has sides measuring 16 m, 28 m, 24 m, and 20 m. Quadrilateral *D′E′F′G′* has sides measuring 20 m, 35 m, 30 m, and 25 m.

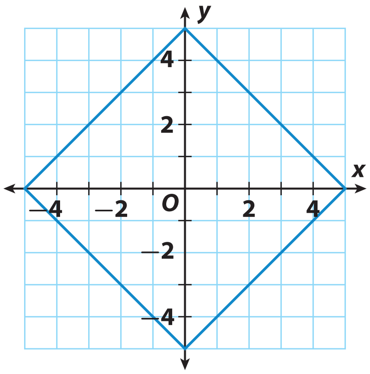
____

____

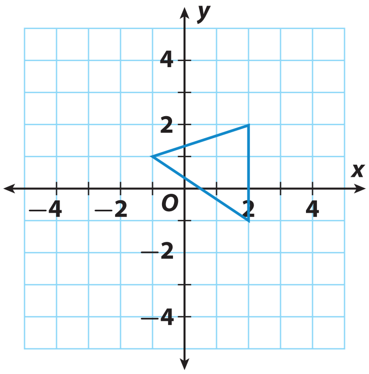
13.2 Algebraic Representations of Dilations

Dilate each figure with the origin as the center of dilation.

3. $(x, y) \rightarrow (0.8x, 0.8y)$



4. $(x, y) \rightarrow (2.5x, 2.5y)$



13.3 Dilations and Measurement

5. A rectangle with length 8 cm and width 5 cm is dilated by a scale factor of 3. What are the perimeter and area of the image? ________________

ESSENTIAL QUESTION

6. How can you use dilations to solve real-world problems?

____

____

MODULE 13 MIXED REVIEW

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Selected Response

1. Quadrilateral *HIJK* has sides measuring 12 cm, 26 cm, 14 cm, and 30 cm. Which could be the side lengths of a dilation of *HIJK*?

Ⓐ 24 cm, 50 cm, 28 cm, 60 cm

Ⓑ 6 cm, 15 cm, 7 cm, 15 cm

Ⓒ 18 cm, 39 cm, 21 cm, 45 cm

Ⓓ 30 cm, 78 cm, 35 cm, 75 cm

2. A rectangle has vertices (6, 4), (2, 4), (6, –2), and (2, –2). What are the coordinates of the vertices of the image after a dilation with the origin as its center and a scale factor of 2.5?

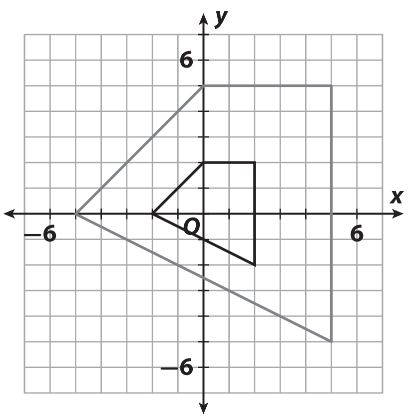
Ⓐ (9, 6), (3, 6), (9, –3), (3, –3)

Ⓑ (3, 2), (1, 2), (3, –1), (1, –1)

Ⓒ (12, 8), (4, 8), (12, –4), (4, –4)

Ⓓ (15, 10), (5, 10), (15, –5), (5, –5)

3. Which represents the dilation shown where the black figure is the preimage?



Ⓐ $(x, y) \rightarrow (1.5x, 1.5y)$

Ⓑ $(x, y) \rightarrow (2.5x, 2.5y)$

Ⓒ $(x, y) \rightarrow (3x, 3y)$

Ⓓ $(x, y) \rightarrow (6x, 6y)$

4. Solve $-a + 7 = 2a - 8$.

Ⓐ $a = -3$

Ⓑ $a = -\frac{1}{3}$

Ⓒ $a = 5$

Ⓓ $a = 15$

5. An equilateral triangle has a perimeter of 24 centimeters. If the triangle is dilated by a factor of 0.5, what is the length of each side of the new triangle?

Ⓐ 4 centimeters

Ⓑ 12 centimeters

Ⓒ 16 centimeters

Ⓓ 48 centimeters

6. Which equation does **not** represent a line with an *x*-intercept of 3?

Ⓐ $y = -2x + 6$

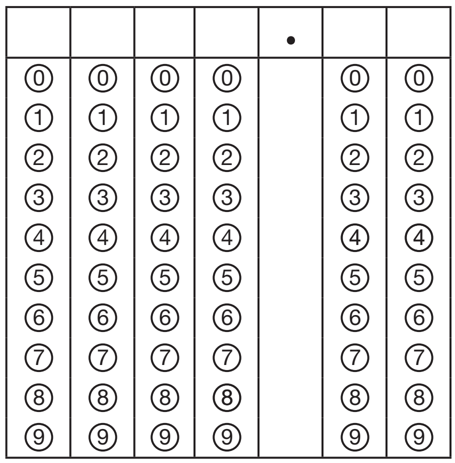
Ⓑ $y = -\frac{1}{3}x + 1$

Ⓒ $y = \frac{2}{3}x - 2$

Ⓓ $y = 3x - 1$

Gridded Response

7. A car is traveling at a constant speed. After 2.5 hours, the car has traveled 80 miles. If the car continues to travel at the same constant speed, how many hours will it take to travel 270 miles?



UNIT 5

Study Guide Review

MODULE 12 Transformations and Congruence

ESSENTIAL QUESTION

How can you use transformations and congruence to solve real-world problems?

Key Vocabulary

center of rotation *(centro de rotación)*

image *(imagen)*

line of reflection *(línea de reflexión)*

preimage *(imagen original)*

reflection *(reflexión)*

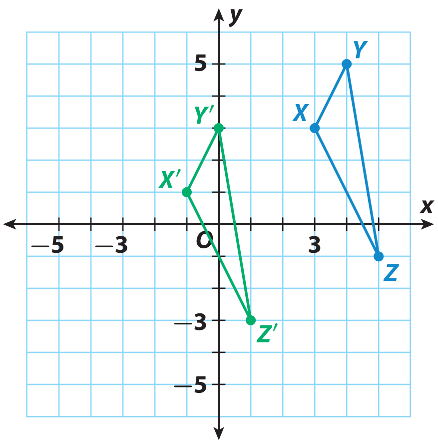
rotation *(rotación)*

transformation *(transformación)*

translation *(traslación)*

EXAMPLE

Translate triangle *XYZ* left 4 units and down 2 units. Graph the image and label the vertices.



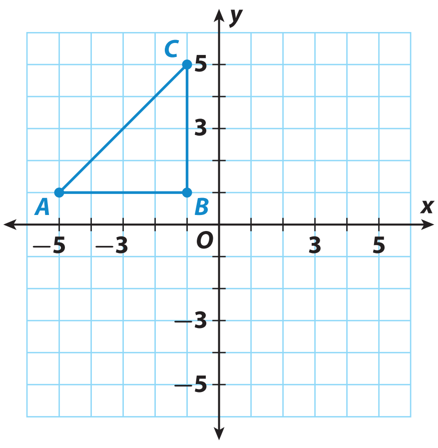
Translate the vertices by subtracting 4 from each x-coordinate and 2 from each y-coordinate. The new vertices are $X'(-1, 1)$, $Y'(0, 3)$, and $Z'(1, -3)$.

Connect the vertices to draw triangle $X'Y'Z'$.

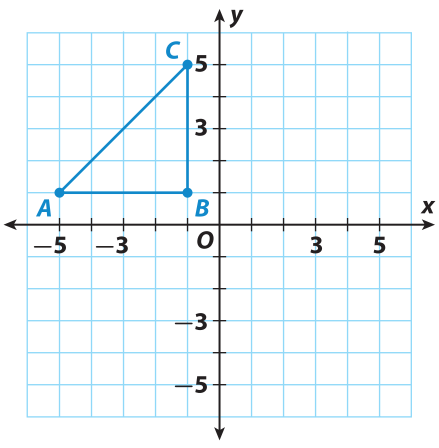
EXERCISES

Perform the transformation shown. (Lessons 12.1, 12.2, 12.3)

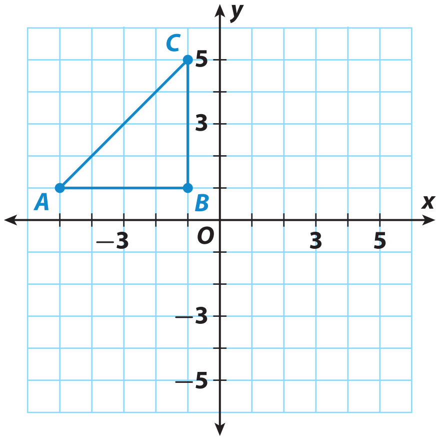
1. Reflection over the x-axis



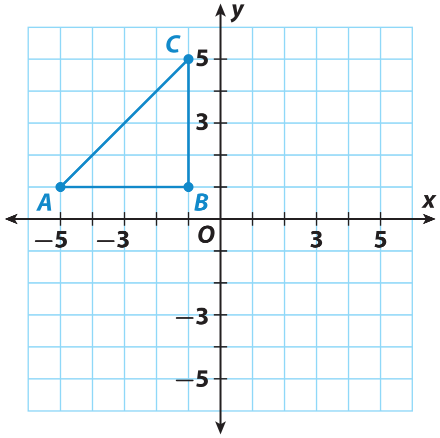
2. Translation 5 units right



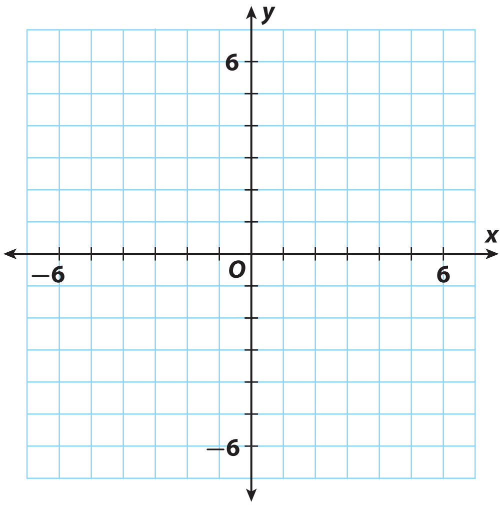
3. Rotation 90° counterclockwise about the origin



4. Translation 4 units right and 4 units down



5. Quadrilateral *ABCD* with vertices *A*(4, 4), *B*(5, 1), *C*(5, −1) and *D*(4, −2) is translated left 2 units and down 3 units. Graph the preimage and the image. (Lesson 12.4)



6. Triangle *RST* has vertices at (−8, 2), (−4, 0), and (−12, 8). Find the vertices after the triangle has been reflected over the *y*-axis. (Lesson 12.4)

____

7. Triangle *XYZ* has vertices at (3, 7), (9, 14), and (12, −1). Find the vertices after the triangle has been rotated 180° about the origin. (Lesson 12.4)

____

MODULE 13

Dilations, Similarity, and Proportionality

ESSENTIAL QUESTION

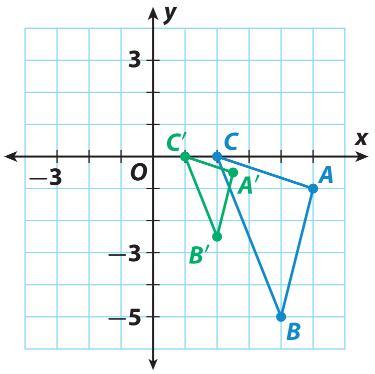
How can you use dilations, similarity, and proportionality to solve real-world problems?

Key Vocabulary

center of dilation *(centro de dilatación)*
dilation *(dilatación)*
enlargement *(agrandamiento)*
reduction *(reducción)*
scale factor *(factor de escala)*

EXAMPLE

Dilate triangle *ABC* with the origin as the center of dilation and scale factor $\frac{1}{2}$. Graph the dilated image.



Multiply each coordinate of the vertices of *ABC* by $\frac{1}{2}$ to find the vertices of the dilated image.

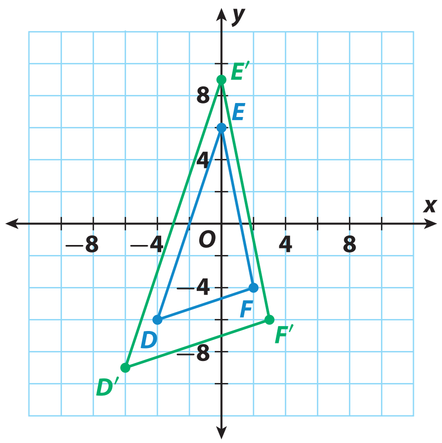
$A(5, -1) \rightarrow A'\left(5 \cdot \frac{1}{2}, -1 \cdot \frac{1}{2}\right) \rightarrow A'\left(2\frac{1}{2}, -\frac{1}{2}\right)$

$B(4, -5) \rightarrow B'\left(4 \cdot \frac{1}{2}, -5 \cdot \frac{1}{2}\right) \rightarrow B'\left(2, -2\frac{1}{2}\right)$

$C(2, 0) \rightarrow C'\left(2 \cdot \frac{1}{2}, 0 \cdot \frac{1}{2}\right) \rightarrow C'(1, 0)$

EXERCISES

1. For each pair of corresponding vertices, find the ratio of the *x*-coordinates and the ratio of the *y*-coordinates. (Lesson 13.1)



Ratio of *x*-coordinates: ____________________

Ratio of *y*-coordinates: ____________________

What is the scale factor of the dilation? __________

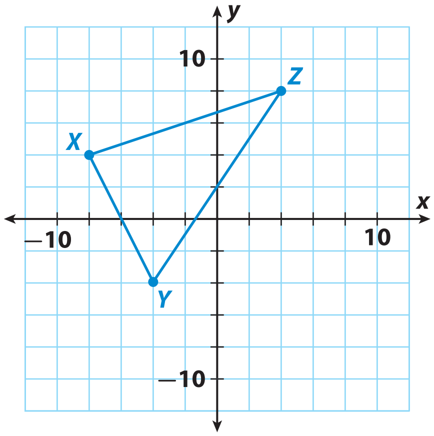
2. Andrew's old television had a width of 32 inches and a height of 18 inches. His new television is larger by a scale factor of 2.5. Find the perimeter and area of Andrew's old television and his new television. (Lesson 13.3)

Perimeter of old TV: ______________ Perimeter of new TV: ______________

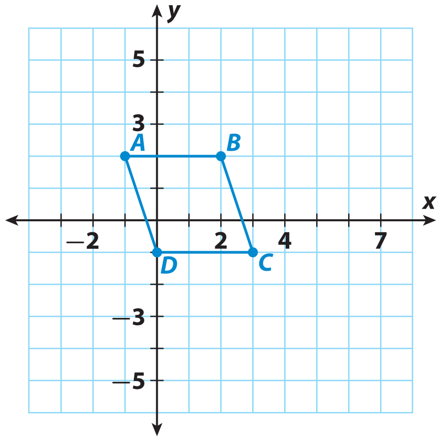
Area of old TV: ______________ Area of new TV: ______________

Dilate each figure with the origin as the center of the dilation. List the vertices of the dilated figure then graph the figure. (Lesson 13.2)

3. $(x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)$

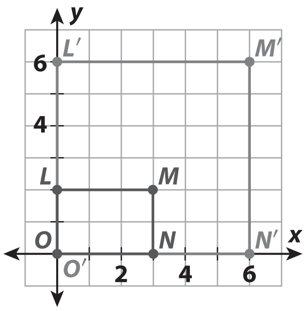


4. $(x, y) \rightarrow (2x, 2y)$



Unit 5 Performance Tasks

1. **CAREERS IN MATH** **Contractor** Fernando is expanding his dog's play yard. The original yard has a fence represented by rectangle *LMNO* on the coordinate plane. Fernando hires a contractor to construct a new fence that should enclose 6 times as much area as the current fence. The shape of the fence must remain the same. The contractor constructs the fence shown by rectangle *L′M′N′O′*.



a. Did the contractor increase the area by the amount Fernando wanted? Explain.

b. Does the new fence maintain the shape of the old fence? How do you know?

2. A sail for a sailboat is represented by a triangle on the coordinate plane with vertices (0, 0), (5, 0), and (5, 4). The triangle is dilated by a scale factor of 1.5 with the origin as the center of dilation. Find the coordinates of the dilated triangle. Are the triangles similar? Explain.

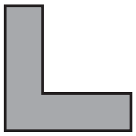
UNIT 5 **MIXED REVIEW**

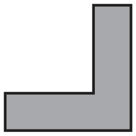
Texas Test Prep

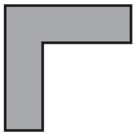
Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

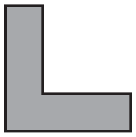
Selected Response

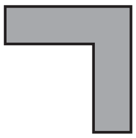
1. What would be the orientation of the figure below after a reflection over the *x*-axis?



Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

2. A triangle with coordinates (4, 2), (0, −3), and (−5, 3) is translated 5 units right and rotated 180° about the origin. What are the coordinates of its image?

Ⓐ (9, 2), (−1, −2), (5, −7)
Ⓑ (−10, 3), (−1, 2), (−5, −3)
Ⓒ (2, −1), (−3, −5), (3, −10)
Ⓓ (−9, −2), (−5, 3), (0, −3)

3. Quadrilateral *LMNP* has sides measuring 16, 28, 12, and 32. Which could be the side lengths of a dilation of *LMNP*?

Ⓐ 24, 40, 18, 90
Ⓑ 32, 60, 24, 65
Ⓒ 20, 35, 15, 40
Ⓓ 40, 70, 30, 75

4. The table below represents which equation?

x	−1	0	1	2
y	1	−2	−5	−8

Ⓐ $y = x + 2$
Ⓑ $y = -x$
Ⓒ $y = 3x + 6$
Ⓓ $y = -3x - 2$

5. Which of the following is **not** true of a trapezoid that has been translated 8 units down?

Ⓐ The new trapezoid is the same size as the original trapezoid.
Ⓑ The new trapezoid is the same shape as the original trapezoid.
Ⓒ The new trapezoid is in the same orientation as the original trapezoid.
Ⓓ The *y*-coordinates of the new trapezoid are the same as the *y*-coordinates of the original trapezoid.

6. Which represents a reduction?

Ⓐ $(x, y) \rightarrow (0.9x, 0.9y)$
Ⓑ $(x, y) \rightarrow (1.4x, 1.4y)$
Ⓒ $(x, y) \rightarrow (0.7x, 0.3y)$
Ⓓ $(x, y) \rightarrow (2.5x, 2.5y)$

7. Grain is stored in cylindrical structures called silos. Which is the best estimate for the volume of a silo with a diameter of 12.3 feet and a height of 25 feet?

Ⓐ 450 cubic feet
Ⓑ 900 cubic feet
Ⓒ 2970 cubic feet
Ⓓ 10,800 cubic feet

8. A rectangle has vertices (8, 6), (4, 6), (8, −4), and (4, −4). What are the coordinates after dilating from the origin by a scale factor of 1.5?

Ⓐ (9, 6), (3, 6), (9, −3), (3, −3)

Ⓑ (10, 8), (5, 8), (10, −5), (5, −5)

Ⓒ (16, 12), (8, 12), (16, −8), (8, −8)

Ⓓ (12, 9), (6, 9), (12, −6), (6, −6)

9. Two sides of a right triangle have lengths of 56 centimeters and 65 centimeters. The third side is **not** the hypotenuse. How long is the third side?

Ⓐ 9 centimeters

Ⓑ 27 centimeters

Ⓒ 33 centimeters

Ⓓ 86 centimeters

10. Which statement is false?

Ⓐ No integers are irrational numbers.

Ⓑ All whole numbers are integers.

Ⓒ No real numbers are rational numbers.

Ⓓ All integers greater than or equal to 0 are whole numbers.

Hot Tip!

Make sure you look at all answer choices before making your decision. Try substituting each answer choice into the problem if you are unsure of the answer.

11. Which inequality represents the solution to $1.5x + 4.5 < 2.75x - 5.5$?

Ⓐ $x > 8$

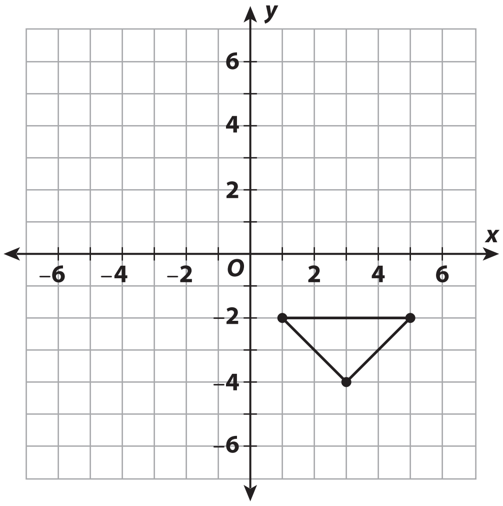
Ⓑ $x < 8$

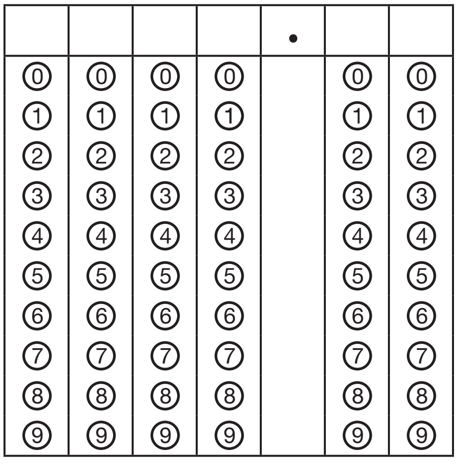
Ⓒ $x > 12.5$

Ⓓ $x < 12.5$

Gridded Response

12. In what quadrant would the triangle below be located after a rotation of 90° counterclockwise?





13. An equilateral triangle has a perimeter of 48 centimeters. If the triangle is dilated by a factor of 0.75, what is the length of each side of the new triangle?