Kindergarten Common Core State Standards

Flip Book

This document is intended to show the connections to the Standards of Mathematical Practices for the content standards and to get detailed information at each level. Resources used: CCSS, Arizona DOE, Ohio DOE and North Carolina DOE. This "flip book" is intended to help teachers understand what each standard means in terms of what students must know and be able to do. It provides only a *sample* of instructional strategies and examples. The goal of every teacher should be to guide students in understanding & making sense of the mathematics they are presented.

Construction directions:
Print on cardstock. Cut the tabs on each page starting with page 2. Cut the bottom off of this top cover to reveal the tabs for the subsequent pages. Staple or bind the top of all pages to complete your flip book.

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Mathematical Practice Standards (**MP**) summary of each standard

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students interpret and make meaning of the problem looking for starting points. In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
2. **Reason abstractly and quantitatively.**
   Mathematically proficient students make sense of quantities and their relationships. Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a **representation** of a problem while attending to the meanings of the quantities.
3. **Construct viable arguments and critique the reasoning of others.**
   Mathematically proficient students analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. They justify conclusions with mathematical ideas. They listen to the arguments of others and ask useful questions to determine if an argument makes sense or suggest ideas to improve/revise the argument. Younger students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. **Model with mathematics.**
   Mathematically proficient students understand that models are a way to reason quantitatively and abstractly (able to decontextualize and contextualize). In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
5. **Use appropriate tools strategically.**
   Mathematically proficient students use available tools recognizing the strengths and limitations of each. Younger students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergarteners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side. They use technological tools to deepen their understanding of mathematics.
6. **Attend to precision.** As K-1 students begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.
7. **Look for and make use of structure.** **(Deductive Reasoning)** Mathematically proficient students apply general mathematical rules to specific situations. They look for the overall structure and patterns in mathematics. For instance, younger students recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated. They also recognize that 3 + 2 = 5 and 2 + 3 = 5
8. **Look for and express regularity in repeated reasoning.** **(Inductive Reasoning)**
   Mathematically proficient students see repeated calculations and look for generalizations and shortcuts. In the early grades, students notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is one more. When counting by tens, the next number in the sequence is "ten more" (or one more group of ten). In addition, students continually check their work by asking themselves, "Does this make sense?"

Math Practice Standards					

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. Make sense of problems and persevere in solving them. • Interpret and make meaning of the problem to find a starting point. Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.	How would you describe the problem in your own words? How would you describe what you are trying to find? What do you notice about...? What information is given in the problem? Describe the relationship between the quantities. Describe what you have already tried. What might you change? Talk me through the steps you've used to this point. What steps in the process are you most confident about? What are some other strategies you might try? What are some other problems that are similar to this one? How might you use one of your previous problems to help you begin? How else might you organize...represent... show...?
2. Reason abstractly and quantitatively. • Make sense of quantities and their relationships. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	What do the numbers used in the problem represent? What is the relationship of the quantities? How is _______ related to ________? What is the relationship between ______and ______? What does_______mean to you? (e.g. symbol, quantity, diagram) What properties might we use to find a solution? How did you decide in this task that you needed to use...? Could we have used another operation or property to solve this task? Why or why not?
3. Construct viable arguments and critique the reasoning of others. • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	What mathematical evidence would support your solution? How can we be sure that...? / How could you prove that...? Will it still work if...? What were you considering when...? How did you decide to try that strategy? How did you test whether your approach worked? How did you decide what the problem was asking you to find? (What was unknown?) Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? What is the same and what is different about...? How could you demonstrate a counter-example?
4. Model with mathematics. • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the mathematics they know to solve everyday problems. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving/revising the model. • Ask themselves, "How can I represent this mathematically?"	What number model could you construct to represent the problem? What are some ways to represent the quantities? What is an equation or expression that matches the diagram, number line.., chart..., table..? Where did you see one of the quantities in the task in your equation or expression? How would it help to create a diagram, graph, table...? What are some ways to visually represent...? What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. Use appropriate tools strategically. • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	What mathematical tools could we use to visualize and represent the situation? What information do you have? What do you know that is not stated in the problem? What approach are you considering trying first? What estimate did you make for the solution? In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? What can using a ______ show us that _____may not? In what situations might it be more informative or helpful to use...?
6. Attend to precision. • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	What mathematical terms apply in this situation? How did you know your solution was reasonable? Explain how you might show that your solution answers the problem. What would be a more efficient strategy? How are you showing the meaning of the quantities? What symbols or mathematical notations are important in this problem? What mathematical language...,definitions..., properties can you use to explain...? How could you test your solution to see if it answers the problem?
7. Look for and make use of structure. • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	What observations do you make about...? What do you notice when...? What parts of the problem might you eliminate..., simplify...? What patterns do you find in...? How do you know if something is a pattern? What ideas that we have learned before were useful in solving this problem? What are some other problems that are similar to this one? How does this relate to...? In what ways does this problem connect to other mathematical concepts?
8. Look for and express regularity in repeated reasoning. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results	Explain how this strategy work in other situations? Is this always true, sometimes true or never true? How would we prove that...? What do you notice about...? What is happening in this situation? What would happen if...? Is there a mathematical rule for...? What predictions or generalizations can this pattern support? What mathematical consistencies do you notice ?

Critical Areas for Mathematics in Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Domain: **Counting and Cardinality (CC)**

Cluster: Know number names and the count sequence

Standards: **K.CC.1**

1. Count to 100 by ones and by tens.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.6 Attend to precision

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Explanations and Examples:

K.CC.1 The emphasis of this standard is on the counting sequence. When counting by ones, students need to understand that the next number in the sequence is one more. When counting by tens, the next number in the sequence is "ten more" (or one more group of ten). Students are to rote count (verbal saying of numbers in sequence) by starting at one and count to 100. (They are only expected to master counting on the decade (0, 10, 20, 30, 40 …). This objective does not require recognition of numerals. It is focused on the rote number sequence.

Provide settings that connect mathematical language and symbols to the everyday lives of kindergarteners. Support students' ability to make meaning and mathematize (*the process of seeing and focusing on the mathematical aspects and ignoring the nonmathematical aspects. Mathematizing in Kindergarten: Solving problems, Communicating or showing their thinking, Connecting and Representing Ideas)* the real world. Help them see patterns, make connections and provide repeated experiences that give students time and opportunities to develop understandings and increase fluency. Encourage students to explain their reasoning by asking probing questions such as "How do you know?" "How did you figure that out?"

Instruction on the counting sequence should be scaffolded (e.g. 1-10, then 1-20, etc.)

Counting should be reinforced throughout the day, not in isolation. (Meaningful Counting)
Examples:

- Count the number of chairs of the students who are absent
- Count the number of stairs, shoes, etc.
- Counting groups of ten such as "fingers in the classroom" (ten fingers per student).
- Count the number of students in a group.
- Count the number of specific object they have in their desk (e.g. crayons)

When counting orally, students should recognize the patterns that exist from 1 to 100. They should also recognize the patterns that exist when counting by 10s. Have students verbalize the patterns they see.

Accurate in counting depends on three things:

1. Knowing the patterns in the number-word list so that a correct number-word list can be said
2. Correctly assigning one number word to one object (one-to one-correspondence)
3. Keeping track of which objects have already been counted so that they are not counted more than once.

Continued next page

Keeping tract—differentiating counted from uncounted entities—is more easily done by moving objects into a counted set. Doing so I not possible with things that cannot be moved, such as pictures in a book. Strategies for keeping track of messy, large sets continue to develop for many years.

Regularity and rhythm are important aspects of counting. Activities that increase these aspects can be helpful to children making lots of correspondence errors.

Errors in Counting

Four factors strongly affect accuracy in counting correspondence:

- Amount of counting experiences (more experience leads to fewer errors)
- Size of set (children become accurate on small sets first)
- Arrangements of objects (objects in rows make it easier to keep track of what has been counted and what has not)
- Effort

NCTM Focus In, Kindergarten

Common Misconceptions:

Some students might not see zero as a number. Ask students to write 0 and say *zero* to represent the number of items left when all items have been taken away. Avoid using the word *none* to represent this situation. Find instances for which the response would be zero in real-world settings to provide experiences with the concept of zero.

As long as children *understand* that correct counting requires one point and one word for each object and are trying to do that, parents and teachers do not need to correct errors all the time. As with many physical activities, counting will improve with practice and does not need to be perfect each time. It is much more important for all children to get frequent counting practice and watch and help one another, with occasional help and corrections from the teacher.

Domain: **Counting and Cardinality (CC)**

Cluster: Know number names and the count sequence.

Standard: **K.CC.2**

2. Count forward beginning from a given number within known sequence (instead of having to begin at 1).

Standards for Mathematical Practice (SMP) to be emphasized:

MP.6 Attend to precision

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Explanations and Examples:

The emphasis of this standard is on the counting sequence to 100. **K.CC.2** includes numbers 0-100. This asks for students to begin rote counting forward counting in a sequence from a number other than one.. (e.g. Given the number 4, the student would count, "4, 5, 6 . . . .") This objective does

not require recognition of numerals. It is focused on the rote number sequence.

Games that require students to add on to a previous count to reach a goal number encourage developing this concept. Frequent and brief opportunities utilizing counting on and counting back are recommended. **These concepts emerge over time and cannot be forced**.

Examples:

http://www.edplus.canterbury.ac.nz/literacy_numeracy/maths/numdocuments/dot_card_and_ten_frame_package2005.pdf

Common Misconceptions:

Counting on or counting from a given number conflicts with the learned strategy of counting from the beginning. In order to be successful in counting on, students must understand **cardinality** (*the number that ends the counting sequence represents how many objects are in the collection*). Students often merge or separate two groups of objects and then re-count from the beginning to determine the final number of objects represented. For these students, counting is still a rote skill or the benefits of counting on have not been realized. Games that require students to add on to a previous count to reach a goal number encourage developing this concept. Frequent and brief opportunities utilizing counting on and counting back are recommended. These concepts emerge over time and cannot be forced.

See Progressions document: *K, Counting and Cardinality; K-5 Operations and Algebraic Thinking* at the website:

http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

Domain: **Counting and Cardinality (CC)**

Cluster: Know number names and the count sequence.

Standard: **K.CC.3**

3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Standards for Mathematical Practice (SMP) to be emphasized:

MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**.

This cluster is connected to the other clusters in the Counting and Cardinality Domain and to *Classify objects and count the number of objects in each category* in Kindergarten, and to *Add and subtract within 20* and *Extend the counting sequence* in Grade 1.

K.CC.4; KNBT.1; K.MD.3

Explanations and Examples:

K.CC.3 asks for students to represent a set of objects with a written numeral. The number of objects being recorded should not be greater than 20. Students can record the quantity of a set by selecting a number card/tile (numeral recognition) or writing the numeral. Students can also create a set of objects based on the numeral presented.

Students should be given multiple opportunities to count objects and recognize that a number represents a specific quantity. Once this is established, students begin to read and write numerals (numerals are the symbols for the quantities). The emphasis should first be on quantity and then connecting quantities to the written symbols.

- A sample unit sequence might include:
  1. Counting up to 20 objects in many settings and situations over several weeks.
  2. Beginning to recognize, identify, and read the written numerals, and match the numerals to given sets of objects.
  3. Writing the numerals to represent counted objects.
- Since the teen numbers are not written as they are said, teaching the teen numbers as one group of ten and extra ones is foundational to understanding both the concept and the symbol that represents each teen number. For example, when focusing on the number "14," students should count out fourteen objects using one-to-one correspondence and then use those objects to make one group of ten and four extra ones. Students should connect the representation to the symbol "14.

Common Misconceptions:

K.CC.3 addresses the writing of numbers and using the written numerals (0-20) to describe the amount of a set of objects. Due to varied development of fine motor and visual development, a reversal of numerals is anticipated for a majority of the students. While reversals should be pointed out to students, the emphasis is on the use of numerals to represent quantities rather than the correct handwriting formation of the actual numeral itself.

Some students might not see zero as a number. Ask students to write 0 and say *zero* to represent the number of items left when all items have been taken away. Avoid using the word *none* to represent this situation.

Kindergarten – CC.1-3
Extended Common Core State Standards Mathematics
North Carolina DOE

The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "…materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify,

Kindergarten Mathematics Counting and Cardinality		
Common Core State Standards	**Essence**	**Extended Common Core**
Know number names and the count sequence.	**Number names and counting sequence**	**Know number names and the count sequence.**
Cluster 1. Count to 100 by ones and by tens. 2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). 3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).		Cluster 1. Understand number words as representing a quantity. 2. Understand the concept of "one" and "more". 3. Count forward using the 1-10 sequence. 4. Write or use an alternative pencil to write numbers 0-10.

etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication.

Resources:

- NCTM, Focus in Kindergarten

Domain: **Counting and Cardinality (CC)**

Cluster: Count to tell the number of objects.

Standard: **K.CC.4**

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a) When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b) Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c) Understand that each successive number name refers to a quantity that is one larger.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**. This cluster is connected to the other clusters in the Counting and Cardinality Domain and to *Classify objects and count the number of objects in each category* in Kindergarten, and to *Add and subtract within 20* in Grade 1.

Explanations and Examples:

K.CC.4 asks students to count a set of objects and see sets and numerals in relationship to one another, rather than as isolated numbers or sets. These connections are higher-level skills that require students to analyze, to reason about, and to explain relationships between numbers and sets of objects. This standard should first be addressed using numbers 1-5 with teachers building to the numbers 1-10 later in the year. The expectation is that students are comfortable with these skills with the numbers 1-10 by the end of Kindergarten.

K.CC.4a reflects the ideas that students implement correct counting procedures by pointing to one object at a time (one-to-one correspondence) using one counting word for each object (one-to-one touching/synchrony), while keeping track of objects that have and have not been counted.. This is the foundation of counting.

K.CC.4b calls for students to answer the question "How many are there?" by counting objects in a set and understanding that the last number stated when counting a set (…8, 9, **10**) represents the total amount of objects: "There are **10** bears in this pile." (*cardinality*). It also requires students to understand that the same set counted three different times will end up being the same amount each time. The idea is to develop a purpose for counting as keeping track of objects is developed. Therefore, a student who moves each object as it is counted recognizes that there is a need to keep track in order to figure out the amount of objects present. Conservation of number, (regardless of the arrangement of objects, the quantity remains the same), conservation of number is a developmental milestone which some Kindergarten children will not have mastered. The goal of this objective is for students to be able to count a set of objects; regardless of the formation those objects are placed.

K.CC.4c represents the concept of "one more" while counting a set of objects. Students are to make the connection that if a set of objects was increased by one more object then the number name for that set is to be increased by one as well. Students are asked to understand this concept with and without objects. For example, after counting a set of 8 objects, students should be able to answer the question, "How many would there be if we added one more object?"; and answer a similar question when not using objects, by asking hypothetically, "What if we have 5 cubes and added one more. How many cubes would there be then?" This concept should be first taught with numbers 1-5 before building to numbers 1-10. Students should be expected to be comfortable with this skill with numbers to 10 by the end of Kindergarten.

K.CC.4					

Domain: **Counting and Cardinality (CC)**

Cluster: Count to tell the number of objects.

Standard: **K.CC.5**
Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Standards for Mathematical Practice (SMP) to be emphasized:
MP.2 Reason abstractly and quantitatively.
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:
See K.CC.4 above

Explanations and Examples:
K.CC.5 addresses various counting strategies. From the research in early childhood mathematics, (Kathy Richardson), students go through a progression of four general ways to count. These counting strategies progress from least difficult to most difficult: 1) students move objects and count them as they move them, 2) students line up the objects and count them, 3) students have a scattered arrangement and they touch each object as they count and 4) students have a scattered arrangement and count them by visually scanning without touching them. Since the scattered arrangements are the most challenging for students, K.CC.5 calls for students to only count 10 objects in a scattered arrangement, and count up to 20 objects in a line, rectangular array, or circle. Out of these 3 representations, a line is the easiest type of arrangement to count.

Students should develop counting strategies to help them organize the counting process to avoid re-counting or skipping objects.

Examples:
- If items are placed in a circle, the student may mark or identify the starting object.
- If items are in a scattered configuration, the student may move the objects into an organized pattern.
- Some students may choose to use grouping strategies such as placing objects in twos, fives, or tens (note: this is not a kindergarten expectation).
- Counting up to 20 objects should be reinforced when collecting data to create charts and graphs.
  (A student may use a clicker (electronic response system) to communicate his/her count to the teacher).

Arizona & NC DOE

Common Misconceptions:
Some students might think that the count word used to tag an item is permanently connected to that item. So when the item is used again for counting and should be tagged with a different count word, the student uses the original count word. For example, a student counts four geometric figures: triangle, square, circle and rectangle with the count words: one, two, three, four. If these items are rearranged as rectangle, triangle, circle and square and counted, the student says these count words: four, one, three, two.

Also, Number lines are not appropriate for pre-K, Kindergarten, or grade 1. See explanation and research in Flip Book 1st grade, pages 8 and 9. (NCTM, 2011)

	K.CC.5				

Domain: **Counting and Cardinality (CC)**

Cluster: Compare Numbers

Standard: **K.CC.6**
Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects)

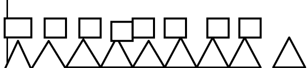
Standards for Mathematical Practice (SMP) to be emphasized:

MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:
See K.CC.

Explanations and Examples:

K.CC.6 expects mastery of up to ten objects. Students can use matching strategies (Student 1), counting strategies or equal shares (Student 3) to determine whether one group **is greater than**, **less than**, or **equal to** the number of objects in another group (Student 2).

Student 1	Student 2	Student 3
I lined up one square and one triangle. Since there is one extra triangle, there are more triangles than squares. 	I counted the squares and I got 8. Then, I counted the triangles and got 9. Since 9 is greater than 8, there are more triangles than squares.	I put them in a pile. I then took away objects. Every time I took a square, I also took a triangle. When I had taken almost all of the shapes away, there was still a triangle left. That means that there are more triangles than squares.

As children develop meaning for numerals, they also compare these numerals to the quantities represented and their number words. Modeling numbers with manipulatives such as dot cards and five- and ten-frames are tools for such comparisons. Children can look for similarities and differences in these different representations of numbers. They begin to "see" the relationship of one more, one less, two more and two less, leading to the concept that successive numbers name quantities where one is larger. In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10. Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will greatly impact children's ability to break numbers into parts.
Children demonstrate their understanding of the meaning of numbers when they can justify why their answer represents a quantity just counted. This justification could merely be the expression that the number said is the total because it was just counted, or a "proof" by demonstrating a one to-one match, by counting again or other similar means (concretely or pictorially) that makes sense. An ultimate level of understanding is reached when children can compare two numbers from 1 to10 represented as written numerals without counting.

Students need to explain their reasoning when they determine whether a number is greater than, less than, or equal to another number. Teachers need to ask probing questions such as "How do you know?" to elicit their thinking. For students, these comparisons increase in difficulty, from greater than to less than to equal. It is easier for students to identify differences than to find similarities.

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Students should develop a strong sense of the relationship between quantities and numerals before they begin comparing numbers.

Other strategies:

- Matching: Students use one-to-one correspondence, repeatedly matching one object from one set with one object from the other set to determine which set has more objects.
- Counting: Students count the objects in each set, and then identify which set has more, less, or an equal number of objects.
- Observation: Students may use observation to compare two quantities (e.g., by looking at two sets of objects, they may be able to tell which set has more or less without counting).
- Observations in comparing two quantities can be accomplished through daily routines of collecting and organizing data in displays. Students create object graphs and pictographs using data relevant to their lives (e.g., favorite ice cream, eye color, pets, etc.). Graphs may be constructed by groups of students as well as by individual students.
- Benchmark Numbers: This would be the appropriate time to introduce the use of 0, 5 and 10 as benchmark numbers to help students further develop their sense of quantity as well as their ability to compare numbers.

Students state whether the number of objects in a set is more, less, or equal to a set that has 0, 5, or 10 objects.

Domain: **Counting and Cardinality (CC)**

Cluster: Compare numbers

Standard: **K.CC.7** Compare two numbers between 1 and 10 presented as written numerals.

Standards for Mathematical Practice (SMP) to be emphasized:

MP. 2 Reason abstractly and quantitatively.
MP. 6 Attend to precision
MP. 7 Look for and make use of structure.
MP. 8 Look for and express regularity in repeated reasoning

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**. More information about this critical area of focus can be found by clicking here. This cluster is also connected to *Work with numbers 11-19 to gain foundations for place value* in Kindergarten, and to all clusters in the Operations and Algebraic Thinking Domain in Grade 1.

Explanations and Examples:

K.CC.7 calls for students to apply their understanding of numerals 1-10 to compare one from another. Thus, looking at the numerals 8 and 10, a student must be able to recognize that the numeral 10 represents a larger quantity than the numeral 8. Students should begin this standard by having ample experiences with sets of objects (K.CC.3 and K.CC.6) before completing this standard with just numerals. Based on early childhood research, students should not be expected to be comfortable with this skill until the end of Kindergarten.

Strategies: As children develop meaning for numerals, they also compare these numerals to the quantities represented and their number words. Modeling numbers with manipulatives such as dot cards and five- and ten-frames are tools for such comparisons. Children can look for similarities and differences in these different representations of numbers. They begin to "see" the relationship of one more, one less, two more and two less, thus landing on the concept that successive numbers name quantities where one is larger. In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10. **Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will impact children's ability to break numbers into parts.**

Children demonstrate their understanding of the meaning of numbers when they can justify why their answer represents a quantity just counted. This justification could merely be the expression that the number said is the total because it was just counted, or a "proof" by demonstrating a one to-one match, by counting again or other similar means (concretely or pictorially) that makes sense. An ultimate level of understanding is reached when children can compare two numbers from 1 to10 represented as written numerals without counting.

Students need to explain their reasoning when they determine whether a number is greater than, less than, or equal to another number. Teachers need to ask probing questions such as "How do you know?" to elicit their thinking. For students, these comparisons increase in difficulty, from greater than to less than to equal. It is easier for students to identify differences than to find similarities.

Arizona, Ohio & NC DOE

K.CC.7

Kindergarten – Counting and Cardinality (CC.4-7)
Extended Common Core State Standards Mathematics
North Carolina DOE

Count to tell the number of objects.	Count to tell "how many"/ quantity	Count to tell the number of objects.
Cluster 4. Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. 5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle; or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.		**Cluster** 5. Understand the relationship between numbers and quantities (0-10); connect counting to cardinality. a. When counting objects, indicate the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. 6. Count to answer "how many?" questions about as many as 10 things arranged in a line or a rectangular array; given a number from 1-10, count out that many objects or indicate the number of objects.
Compare numbers.	**Compare numbers to determine more, less or equal**	**Compare numbers.**
6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. 7. Compare two numbers between 1 and 10 presented as written numerals.		7. Identify whether the number of objects in one group is more, less, or equal to the number of objects in another group, e.g., by using matching and counting strategies.

Resources:

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. All standards in this cluster should only include numbers through 10 Students will model simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations. Student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

Standard: **K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problems. This applies wherever drawings are mentioned in the Standards.)

Standards for Mathematical Practice (SMP) to be emphasized:

MP. 1 Make sense of problems and persevere in solving them.
MP. 2 Reason abstractly and quantitatively.
MP. 4 Model with mathematics.
MP. 5 Use appropriate tools strategically.

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**.

Explanations and Examples:

K.OA.1 asks students to demonstrate the understanding of how objects can be joined (addition) and separated (subtraction) by representing addition and subtraction situations in various ways. This objective is primarily focused on understanding the concept of addition and subtraction, rather than merely reading and solving addition and subtraction number sentences (equations).

Instructional Strategies: Using addition and subtraction in a word problem context allows students to develop their understanding of what it means to add and subtract.

Students should use objects, fingers, mental images, drawing, sounds, acting out situations and verbal explanations in order to develop the concepts of addition and subtraction. Then, they should be introduced to writing expressions and equations using appropriate terminology and symbols which include **+**, **–**, and **=.**

- Addition terminology: *add, join, put together, plus, combine, total*
- Subtraction terminology: *minus, take away, separate, difference, compare*

Have students decompose numbers less than or equal to 5 during a variety of experiences to promote their fluency with sums and differences less than or equal to 5 that result from using the numbers 0 to 5. For example, ask students to use different models to decompose 5 and record their work with drawings or equations. Next, have students decompose 6, 7, 8, 9, and 10 in a similar fashion. As students begin to understand the role and meaning of arithmetic operations in number systems, they will gain computational fluency, and using efficient and accurate methods for computing.
The teacher can use backmapping and scaffolding (Explain these) to teach students who show a need for more help with counting. For instance, ask students to build a tower of 5 using 2 green and 3 blue linking cubes while you discuss composing and decomposing 5. Have them identify and compare other ways to make a tower of 5. Repeat the activity for towers of 7 and 9. Help students use counting as they explore ways to compose 7 and 9.

Common Misconceptions:

Students may over-generalize the vocabulary in word problems and think that certain words indicate solution strategies that must be used to find an answer. They might think that the word *more* always means to add and the words *take away* or *left* always means to subtract. When students use the words *take away* to refer to subtraction and its symbol, teachers need to repeat students' ideas using the words *minus* or *subtract*. For example, students use addition to solve this Take from/Start Unknown problem: Melisa took the 8 stickers she no longer wanted and gave them to Anna. Now Melisa has 11 stickers *left*. How many stickers did Melisa have to begin with?

Note on vocabulary: The term "total" should be used instead of the term "sum". "Sum" sounds the same as "some", but has the opposite meaning. "Some" is used to describe problem situations with one or both addends unknown, so it is better in the earlier grades to use "total" rather than "sum". Formal vocabulary for subtraction ("minuend" and "subtrahend") is not needed in Kindergarten. ("tota"l and "addend" are sufficient for classroom discussion).

Students should be encouraged to use create drawings /pictorial representations of the problems and/or situation.

If students progress from working with manipulatives to writing numerical expressions and equations, and they skip using pictorial thinking—students will then be more likely to use finger counting and rote memorization for work with addition and subtraction.

Counting forward builds to the concept of addition while counting back leads to the concept of subtraction. However, counting is an inefficient strategy. Teachers need to provide instructional experiences so that students progress from the concrete level, to the pictorial level, then to the abstract level when learning mathematical concepts. (**C**oncrete, **R**epresentational, **A**bstract **CRA**)

Just knowing the basic facts is not enough. We need to help students develop the ability to quickly and accurately understand the **relationships** between numbers. They need to make sense of numbers as they find and make strategies for joining and separating quantities

Arizona & NC DOE

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Standard**: K.OA.2.** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.5 Use appropriate tools strategically.
MP.6 Attend to precision

Connections:

See K.OA.1

Explanations and Examples:

K.OA.2 asks students to solve problems presented in a story format (context) with a specific emphasis on using objects or drawings to determine the solution. This builds upon the students understanding of addition and subtraction from K.OA.1, to solve problems. Once again, numbers should not exceed 10.
Teachers should focus on three types of problems during instruction. There are three types of addition and subtraction problems are: **Result Unknown, Change Unknown, and Start Unknown**. These types of problems become increasingly difficult for students. Research has found that Result Unknown problems are easier than Change and Start Unknown problems. Kindergarten students should have experiences with all three types of problems. The level of difficulty can be decreased by using smaller numbers (up to 5) or increased by using larger numbers (up to 10). (See Table 1 page 40)

Using a word problem context allows students to develop their understanding about what it means to add and subtract. (*Addition is putting together and adding to. Subtraction is taking apart and taking from)*. Kindergarteners develop the concept of addition/subtraction by modeling the actions in word problem using objects, fingers, mental images, drawings, sounds, acting out situations, and/or verbal explanations. Students may use different representations based on their experiences, preferences, etc. They may connect their conceptual representations of the situation using symbols, expressions, and/or equations. Students should experience the following addition and subtraction problem types (see Table 1 page 40).

- Add To word problems, such as, "Mia had 3 apples. Her friend gave her 2 more. How many does she have now?"
    - A student's "think aloud" of this problem might be, "I know that Mia has some apples and she's getting some more. So she's going to end up with more apples than she started with."
- Take From problems such as:
    - José had 8 markers and he gave 2 away. How many does he have now? When modeled, a student would begin with 8 objects and remove two to get the result.
- Put Together/Take Apart problems with Total Unknown gives students opportunities to work with addition in another context such as:
    - There are 2 red apples on the counter and 3 green apples on the counter. How many apples are on the counter?
- Solving Put Together/Take Apart problems with Both Addends Unknown provides students with experiences with finding all the decompositions of a number and investigating the patterns involved.
    - There are 10 apples on the counter. Some are red and some are green. How many apples could be green? How many apples could be red?

Arizona, Ohio & NC DOE

K.OA.2					

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Standard**: K.OA.3.** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

Standards for Mathematical Practice (SMP) to be emphasized:

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.4 Model with mathematics.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**.

Explanations and Examples:

K.OA.3 asks students to understand that a set of (5) object can be broken into two sets (3 and 2) and still be the same total amount (5). The focus is on number pairs which add to a specified total, 1-10. In addition, this standard asks students to understand that a set of objects (5) can be broken in multiple ways (3 and 2; 4 and 1). Thus, when breaking apart a set (decomposing), students develop the understanding that a smaller set of objects exists within that larger set (inclusion). This should be developed in context before moving into how to represent decomposition with symbols (+, -, =).

Example:
"Bobby Bear is missing 5 buttons on his jacket. How many ways can you use blue and red buttons to finish his jacket? Draw a picture of all your ideas.
Students could draw pictures of:
4 blue and 1 red button
3 blue and 2 red buttons
2 blue and 3 red buttons
1 blue and 4 red buttons

After the students have had numerous experiences with decomposing sets of objects and recording with pictures and numbers, the teacher eventually makes connections between the drawings and symbols*:5=4+1, 5=3+2, 5=2+3, and 5=1+4*
The number sentence only comes after pictures or work with manipulatives, and students should never give the number sentence without a mathematical representation.

Students may use objects such as cubes, two-color counters, square tiles, etc. to show different number pairs for a given number. For example, for the number 5, students may split a set of 5 objects into 1 and 4, 2 and 3, etc.

Examples continue next page

Students may also use drawings to show different number pairs for a given number. For example, students may draw 5 objects, showing how to decompose in several ways.

x x x x x 5 objects

[x x | x x x] 5 = 2 + 3

[x x x x | x] 5 = 4 + 1

Sample unit sequence:

- A contextual problem (word problem) is presented to the students such as, "Melisa goes to Debbie's house. Debbie tells her she may have 5 pieces of fruit to take home. There are lots of apples and bananas. How many of each can she take?"
- Students find related number pairs using objects (such as cubes or two-color counters), drawings, and/or equations. Students may use different representations based on their experiences, preferences, etc.
- Students may write equations that equal 5 such as:

  * 5=4+1
  * 3+2=5
  * 2+3=4+1
  * 5+ 0 = 5

This is a good opportunity for students to systematically list all the possible number pairs for a given number. For example, all the number pairs for 5 could be listed as 0+5, 1+4, 2+3, 3+2, 4+1, and 5+0. Students should describe the pattern that they see in the addends, e.g., each number is one less or one than the previous addend. (Continue to make sure students include the number plus zero as a possible solution).

Arizona, Ohio & NC DOE

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Standard**: K.OA.4.** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.4 Model with mathematics.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:

See K.OA.1

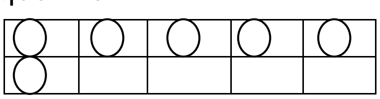
Explanations and Examples:

KOA.4 builds upon the understanding that a number can be decomposed into parts (K.OA.3). The number pairs that total ten are foundational for students' ability to work fluently within numbers and operations. Different models, such as ten-frames, cubes, two-color counters, etc., assist students in visualizing these number pairs for ten.

Once students have had experiences breaking apart ten into various combinations, this asks students to find a missing part of 10.

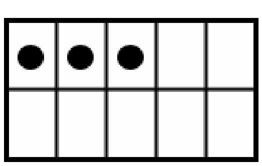
Example 1:

"A full case of juice boxes has 10 boxes. There are only 6 boxes in this case. How many juice boxes are missing?

Student 1	**Student 2**	**Student 3**
Using a Ten Frame "I used 6 counters for the 6 boxes of juice still in the case. There are 4 blank spaces so 4 boxes have been removed. This makes sense since 6 and 4 more equal 10". 	Think addition. "I counted out 10 cubes because I knew there needed to be ten. I Pushed these 6 over here because they were in the container. These are left over. So there's 4 missing"	Basic Fact. " I know that it's 4 because 6 and 4 is the same amount as 10".

_**Example 2:**

Students place three objects on a ten frame and then determine how many more are needed to "make a ten."
Students may use electronic versions of ten frames to develop this skill.



Example 3:
The student snaps ten cubes together to make a "train."

- Student breaks the "train" into two parts. S/he counts how many are in each part and record the associated equation (10 = ___ + ___).
- Student breaks the "train into two parts. S/he counts how many are in one part and determines how many are in the other part without directly counting that part. Then s/he records the associated equation (if the counted part has 4 cubes, the equation would be 10 = 4 + ___).
- Student covers up part of the train, without counting the covered part. S/he counts the cubes that are showing and determines how many are covered up. Then s/he records the associated equation (if the counted part has 7 cubes, the equation would be 10 = 7 + ___).

Example 4:
The student tosses ten two-color counters on the table and records how many of each color are facing up.

Instructional Strategies for K.OA. 1-5:
It is essential to provide contextual situations for addition and subtraction that relate to the everyday lives of kindergarteners. A variety of situations can be found in children's literature books. Students then model the addition and subtraction using a variety of representations such as drawings, sounds, acting out situations, verbal explanations and numerical expressions. Manipulatives, like two-color counters, clothespins on hangers, connecting cubes and stickers can also be used for modeling these operations. Kindergarten students should see addition and subtraction equations written by the teacher. Although students might struggle at first, teachers should encourage them to try writing the equations. Students' writing of equations in Kindergarten is encouraged, but it is not required.
Create written addition or subtraction problems with sums and differences less than or equal to 10 using the numbers 0 to 10 and Table 1 on page 39 of this document for guidance. It is important to use a problem context that is relevant to kindergarteners. After the teacher reads the problem, students choose their own method to model the problem and find a solution. Students discuss their solution strategies while the teacher represents the situation with an equation written under the problem. The equation should be written by listing the numbers and symbols for the unknown quantities in the order that follows the meaning of the situation. The teacher and students should use the words *equal* and *is the same as* interchangeably.
Have students decompose numbers less than or equal to 5 during a variety of experiences to promote their fluency with sums and differences less than or equal to 5 that result from using the numbers 0 to 5. For example, ask students to use different models to decompose 5 and record their work with drawings or equations. Next, have students decompose 6, 7, 8, 9, and 10 in a similar fashion. As they come to understand the role and meaning of arithmetic operations in number systems, students gain computational fluency, using efficient and accurate methods for computing.
The teacher can use backmapping and scaffolding EXPLAIN to teach students who show a need for more help with counting. For instance, ask students to build a tower of 5 using 2 green and 3 blue linking cubes while you discuss composing and decomposing 5. Have them identify and compare other ways to make a tower of 5. Repeat the activity for towers of 7 and 9. Help students use counting as they explore ways to compose 7 and 9.

Arizona, Ohio & NC DOE

		K.OA.4			

Domain: **Operations and Algebraic Thinking (OA)**

Cluster:

Standard**: K.OA.5** Fluently add and subtract within 5.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:

See K.OA .1

Explanations and Examples:

K.OA.5 uses the word fluently, which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property and/or those shown below). Fluency is developed by working with many different kinds of objects over an extended amount of time. This objective does not require students to instantly know the answer. ***Traditional flash cards or timed tests have not been proven as effective instructional strategies for developing fluency.***

This standard focuses on students being able to add and subtract numbers within 5. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Strategies students may use to attain fluency include:

- Counting on (e.g., for 3+2, students will state, "3," and then count on two more, "4, 5," and state the solution is "5")
- Counting back (e.g., for 4-3, students will state, "4," and then count back three, "3, 2, 1" and state the solution is "1")
- Counting up to subtract (e.g., for 5-3, students will say, "3," and then count up until they get to 5, keeping track of how many they counted up, stating that the solution is "2")
- Using doubles (e.g., for 2+3, students may say, "I know that 2+2 is 4, and 1 more is 5")
- Using commutative property (e.g., students may say, "I know that 2+1=3, so 1+2=3")
- Using fact families (e.g., students may say, "I know that 2+3=5, so 5-3=2")

Students may use electronic versions of five frames to develop fluency of these facts.

Arizona,, Ohio & NC DOE

K.OA.5

Domain: **Number and Operations in Base Ten (NBT)**

Cluster: Work with numbers 11-19 to gain foundations for place value.

Standard**: K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (MP.4 *model*) (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.4 Model with mathematics.
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:

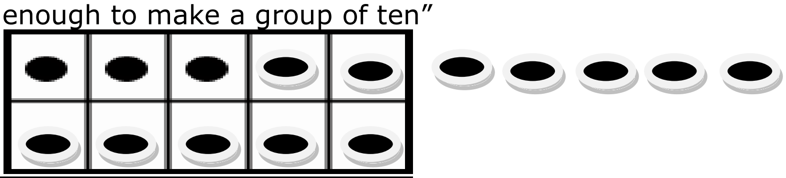
This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**. K.CC.3; K.RI.3; K.W.2

Explanations and Examples:

K.NBT.1 is the first time that students move beyond the number 10 with representations, such as objects (manipulatives) or drawings. The spirit of this standard is that students separate out a set of 11-19 objects into a group of ten objects with leftovers. This ability is a pre-cursor to later grades when they need to understand the complex concept that a group of 10 objects is also one ten (*unitizing*). Ample experiences with ten frames will help solidify this concept. Research states that students are not ready to unitize until the end of first grade. Therefore, this work in Kindergarten lays the foundation of composing tens and recognizing leftovers.

Example:

Teacher: "Please count out 15 chips."
Student: Student counts 15 counters (chips or cubes).
Teacher: "Do you think there is enough to make a group of ten chips? Do you think there might be some chips leftover?"
Student: Student answers.
Teacher: "Use your counters to find out."
Student: Student can either fill a ten frame or make a stick of ten connecting cubes. They answer, "There is enough to make a group of ten"



Teacher: How many leftovers do you have?
Student: Students say, "I have 5 left over."
Teacher: How could we use words and/or numbers to show this?
Student: Students might say "Ten and five is the same amount as 15", "15 = 10 + 5"

Special attention needs to be paid to this set of numbers as they do not follow a consistent pattern in the verbal counting sequence.

- Eleven and twelve are special number words.
- "Teen" means one "ten" plus ones.
- The verbal counting sequence for teen numbers is backwards – we say the ones digit before the tens digit. For example "27" reads tens to ones (twenty-seven), but 17 reads ones to tens (seven-teen).
- In order for students to interpret the meaning of written teen numbers, they should read the number as well as describe the quantity. For example, for 15, the students should read "fifteen" and state that it is one group of ten *and* five ones and record that 15 = 10 + 5.

Teaching the teen numbers as one group of ten and extra ones is foundational to understanding both the concept and the symbol that represent each teen number. For example, when focusing on the number "14," students should count out fourteen objects using one-to-one correspondence and then use those objects to make one group of ten ones and four additional ones. Students should connect the representation to the symbol "14." Students should recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated.

Instructional Strategies:

Kindergarteners need to understand the idea of *a ten* so they can develop the strategy of adding onto 10 to add within 20 in Grade 1. Students need to construct their own base-ten ideas about quantities and their symbols by connecting to counting by ones. They should use a variety of manipulatives to model and connect equivalent representations for the numbers 11 to19. For instance, to represent 13, students can count by ones and show 13 beans. They can anchor to five and show one group of 5 beans and 8 beans or anchor to ten and show one group of 10 beans and 3 beans. Students need to eventually see *a ten* as different from 10 ones.

After the students are familiar with counting up to 19 objects by ones, have them explore different ways to group the objects that will make counting easier. Have them estimate before they count and group. Discuss their groupings and lead students to conclude that grouping by ten is desirable. *10 ones make 1 ten* makes students wonder how something that means a lot of things can be one thing. Students need to first use *groupable* materials to represent numbers 11 and 19 because a group of ten such as a bundle of 10 straws or a cup of 10 beans makes more sense than *a ten* in pre-grouped materials. They need to see that there are 10 single objects represented on the item for ten in pre-grouped materials, such as the rod in base-ten blocks. Students need to learn to attach words to materials and groups and understand what they represent. Eventually, they need to see the rod as *a ten* that they did not group themselves.

Students should impose their base-ten concepts on a model made from groupable and pre-groupable materials. Students can transition from groupable to pre-groupable materials by leaving a group of ten intact to be reused as a pre-grouped item. When using pre-grouped materials, students should reflect on the ten-to-one relationships in the materials, such as the "tenness" of the rod in base-ten blocks. After many experiences with pre-grouped materials, students can use dots and a stick (one tally mark) to record singles and a ten. Kindergartners should use proportional base-gen models, where a group often is physically 10 times larger than the model for one. Non-proportional models such as an abacus and money should not be used at this grade level. (Exception: penny can be used to for counting, grouping, etc. to represent one, the nickel can be used when working with 5-frames as a representation of five, and the dime as a representation of the 10 frame).

Encourage students to use base-ten language to describe quantities between 11 and 19. At the beginning, students do not need to use *ones* for the singles. Some of the base-ten language that is acceptable for describing quantities such as18 includes *one ten and eight, a bundle and eight*, *a rod and 8 singles* and *ten and eight more.* Write the horizontal equation 18 = 10 + 8 and connect it to base-ten language. Encourage, but do not require, students to write equations to represent quantities. Students have difficulty with *ten* as a singular word that means 10 things. For many students, the understanding that a group of 10 things can be replaced by a single object and they both represent 10 is confusing.

Common Misconceptions:

Students have difficulty with *ten* as a singular word that means 10 things. For many students, the understanding that a group of 10 things can be replaced by a single object and they both represent 10 is confusing. Help students develop the sense of *ten* by first using groupable materials then replacing the group with an object or representing 10, such as a rod or 10 Frame. Watch for and address the issue of attaching words to materials and groups without knowing what they represent. If this misconception is not addressed early on it can cause additional issues when working with numbers 11-19 and beyond.

Arizona, Ohio and NC DOE

Domain: **Measure and Data (MD)**

Cluster: Describe and compare measureable attributes.

Standard**: K. MD.1** Describe measureable attributes of objects, such as **length** or **weight.** Describe several measurable attributes of a single object.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.4 Model with mathematics
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #1**, Representing and comparing whole numbers, initially with sets of objects**.

Explanations and Examples:

K.MD.1 calls for students to describe measurable attributes of objects, such as **length** and **weight**, In order to describe attributes such as length and weight, students must have many opportunities to informally explore these attributes.

- Students should state comparisons of objects verbally and then focus on specific attributes when making verbal comparisons for K.MD.2. They may identify measurable attributes such as length, width, height, and weight. For example, when describing a soda can, a student may talk about how tall, how wide, how heavy, or how much liquid can fit inside. These are all measurable attributes. Non-measurable attributes include: words on the object, colors, pictures, etc.

This standard focuses on using descriptive words and does not mean that students should sort objects based on attributes. (Sorting appears later).

Instructional Strategies:

It is critical for students to be able to identify and describe measureable attributes of objects. An object has different attributes that can be measured, like the height and weight of a can of food. Students should be given many opportunities to compare directly where the attribute becomes the focus. For example, when comparing the volume of two different boxes, ask students to discuss and justify their answers to these questions: Which box will hold the most? Which box will hold least? Will they hold the same amount? "How could you find out?" Students can decide to fill one box with dried beans then pour the beans into the other box to determine the answers to these questions.

Have students work in pairs to compare their arm spans. As they stand back-to-back with outstretched arms, compare the lengths of their spans, then determine who has the shortest arm span. Ask students to explain their reasoning. Then ask students to suggest other measureable attributes of their bodies that they could directly compare, such as their height or the length of their feet.

Domain: **Measure and Data (MD)**

Cluster: Describe and compare measureable attributes.

Standard**: K. MD.2** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Standards for Mathematical Practice (SMP) to be emphasized:

MP.2 Reason abstractly and quantitatively
MP.4 Model with mathematics
MP.6 Attend to precision
MP.7 Look for and make use of structure.

Connections:

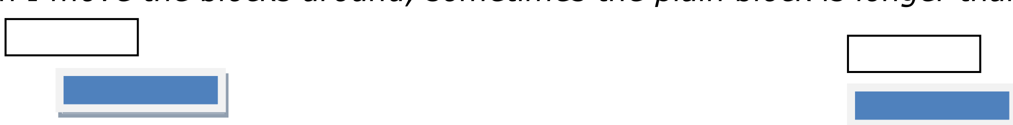
This cluster is connected to the Kindergarten Critical Area of Focus #1**, Representing and comparing whole numbers, initially with sets of objects**.
This cluster is connected to *Measure lengths indirectly and by iterating length units* in Grade 1.

Explanations and Examples:

K.MD.2 asks for direct comparisons of objects. Direct comparisons are made when objects are put next to each other, such as two children, two books, two pencils. For example, a student may line up two blocks and say, "This block is a lot longer than this one." Students are not comparing objects that cannot be moved and lined up next to each other.

When making direct comparisons for length, students must attend to the "starting point" of each object and recognize that objects should be matched up at the end of objects to get accurate measurements. For example, the ends need to be lined up at the same point, or students need to compensate when the starting points are not lined up (conservation of length includes understanding that if an object is moved, its length does not change; an important concept when comparing the lengths of two objects). Since this understanding requires conservation of length, a developmental milestone for young children, children need multiple experiences to move beyond the idea that …. *"Sometimes this block is **longer than** this one and sometimes it's **shorter** (depending on how I lay them side by side) and that's okay." "This block is always longer than this block (with each end lined up appropriately)."*

Before conservation of length*: The blue block is longer than the plain block when they are lined up like this. But when I move the blocks around, sometimes the plain block is longer than the blue block.*



After conservation of length*: I have to line up the blocks to measure them.*

Language plays an important role in this standard as students describe the similarities and differences of measurable attributes of objects (e.g., shorter than, taller than, lighter than, the same as, etc.).

Students should have many opportunities to compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third objects.

Common Misconceptions:

Many students have difficulty understanding that when an object is moved away from the object they are comparing it with, the length does not change. With multiple opportunities, students learn that they have to line up the items they are comparing and/or measuring. *(Conservation of Length: includes understanding that if an object is moved, its length does not change; an important concept when comparing the lengths of two objects).*

Arizona, Ohio & NC DOE

K.MD.2

Domain: **Measure and Data (MD)**
Cluster: Classify objects and count the number of objects in each category.
Standard**: K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. *(Limit category counts to be less than or equal to 10)*
<u>Standards for Mathematical Practice (SMP) to be emphasized:</u> MP.2 Reason abstractly and quantitatively MP.6 Attend to precision MP.7 Look for and make use of structure.
<u>Connections:</u> This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**. This cluster is connected to *Know number names and the count sequence* and *Count to tell the number of objects* in Kindergarten, and to *Represent and interpret data* in Grade 1.

<u>Explanations and Examples</u>:

K.MD.3 asks students to identify similarities and differences between objects (e.g., size, color, shape) and use the identified attributes to **sort** a collection of objects. Once the objects are sorted, the student counts the amount in each set. Once each set is counted, then the student is asked to sort (or group) each of the sets by the amount in each set.

For example, when given a collection of buttons, the student separates the buttons into different piles based on color (all the blue buttons are in one pile, all the orange buttons are in a different pile, etc.). Then the student counts the number of buttons in each pile: blue (5), green (4), orange (3), purple (4). Finally, the student organizes the groups by the quantity in each group (Orange buttons (3), Green buttons (4), Purple buttons with the green buttons because purple also had (4), Blue buttons last (5).

Other possible objects to sort include: shells, shapes, beans, small toys, coins, rocks, etc. After sorting and counting, it is important for students to:

- *explain how they sorted the objects;*
- *label each set with a category;*
- *answer a variety of counting questions that ask, "How many …"; and*
- *compare sorted groups using words such as, "most", "least", "alike" and "different".*

This objective helps to build a foundation for data collection in future grades. In later grade, students will transfer these skills to creating and analyzing various graphical representations.

<u>Instructional Strategies</u>:

Provide categories for students to use to sort a collection of objects. Each category can relate to only one attribute, like *Red* and *Not Red* or *Hexagon* and *Not Hexagon,* and contain up to 10 objects. Students count how many objects are in each category and then order the categories by the number of objects they contain.

Ask questions to initiate discussion about the attributes of shapes. Then have students sort a collection of two-dimensional and three-dimensional shapes by their attributes. Provide categories like *Circles* and *Not Circles* or *Flat* and *Not Flat*. <u>Have students count the objects in each category and order the categories by the number of objects they contain.</u>

Have students infer the classification of objects by guessing the rule for a sort. First, the teacher uses one attribute to sort objects into two loops or regions without labels. Then the students determine how the objects were sorted, suggest labels for the two categories and explain their reasoning.

Arizona, Ohio & NC DOE

	K.MD.3				

Kindergarten – Measurement and Data
Extended Common Core State Standards Mathematics
North Carolina DOE

Common Core State Standards		Essence	Extended Common Core	
Describe and compare measurable attributes.		**Measureable attributes of length**	**Describe and compare measurable attributes.**	
Cluster	1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. 2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter*.		Cluster	1. Compare the length of two objects using direct comparison. 2. Use appropriate vocabulary to describe differences in length (e.g., longer/ shorter). *Concepts added at this grade to begin development of background knowledge for concepts developed in later grades.* 3. Use the words, before/after, now/later, soon/never to refer to personal activities and events (time concepts). 4. Understand first-then schedule (time concepts).
Classify objects and count the number of objects in each category.		**Sort objects by attribute and count "how many" in set.**	**Sort objects and count the number of objects in each category.**	
Cluster	3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.		Cluster	5. Identify objects as "same" or "different." 6. Recognize similarities and differences between objects (attribute). 7. Sort objects according to attribute and count "how many" in sets (1-5 objects per set).

Resources:

Domain: **Geometry (G)**

Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Standard**: K.G.1.** Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.6 Attend to precision

MP.7 Look for and make use of structure.

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #2, **Describing shapes and space**.

This cluster is connected to *Analyze, compare, create and compose shape*s in Kindergarten, and to *Reason with shapes and their attributes* in Grade 1.

Explanations and Examples:

K.G.1 expects students to use positional words (such as those italicized above) to describe objects in the environment. Kindergarten students need to focus first on location and position of two-and-three-dimensional objects in their classroom prior to describing location and position of two-and-three-dimension representations on paper.

Examples of environments in which students would be encouraged to identify shapes would include nature, buildings, and the classroom using positional words in their descriptions.

Teachers should work with children and pose four mathematical questions: Which way? How far? Where? And what objects? To answer these questions, children develop a variety of important skills contributing to their spatial thinking.

Examples:

- Teacher holds up an object such as an ice cream cone, a number cube, ball, etc. and asks students to identify the shape. Teacher holds up a can of soup and asks," What shape is this can?" Students respond "cylinder!"
- Teacher places an object next to, behind, above, below, beside, or in front of another object and asks positional questions. Where is the water bottle? (water bottle is placed behind a book) Students say "The water bottle is behind the book."

Students should have multiple opportunities to identify shapes; these may be displayed as photographs, or pictures using the document camera or interactive whiteboard.

Instructional Strategies:

Develop spatial sense by connecting geometric shapes to students' everyday lives. Initiate natural conversations about shapes in the environment. Have students identify and name two- and three-dimensional shapes in and outside of the classroom and describe their relative position.

Ask students to find rectangles in the classroom and describe the relative positions of the rectangles they see, e.g. *This rectangle (a poster) is over the sphere (globe)*. Teachers can use a digital camera to record these relationships.

Continued next page

Hide shapes around the room. Have students say where they found the shape using positional words, e.g*. I found a triangle UNDER the chair*.

Have students create drawings involving shapes and positional words: *Draw a window ON the door* or *Draw an apple UNDER a tree*. Some students may be able to follow two- or three-step instructions to create their drawings.

Use a shape in different orientations and sizes along with non-examples of the shape so students can learn to focus on defining attributes of the shape.

Manipulatives used for shape identification actually have three dimensions. However, Kindergartners need to think of these shapes as two-dimensional or "flat" and typical three-dimensional shapes as "solid." Students will identify two-dimensional shapes that form surfaces on three-dimensional objects. Students need to focus on noticing two and three dimensions, not on the words *two-dimensional* and *three-dimensional.*

You may create an activity where an object is identified and whispered into the teacher's ear. Students then ask question of the first student---"is it in front of", etc.

Common Misconception

Students many times use incorrect terminology when describing shapes. For example students may say a *cube* is a *square* or that a *sphere* is a *circle*. The use of the two-dimensional shape that appears to be part of a three-dimensional shape to name the three-dimensional shape is a common misconception. Work with students to help them understand that the two-dimensional shape is a part of the object but it has a different name.

Arizona, Ohio & NC DOE

Domain: **Geometry (G)**

Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Standard**: K.G.2.** Correctly name shapes regardless of their orientations or overall size.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.6 Attend to precision

MP.7 Look for and make use of structure.

Connections:

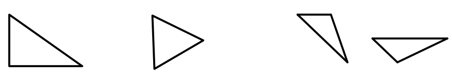
This cluster is connected to the Kindergarten Critical Area of Focus #2, **Describing shapes and space**.

This cluster is connected to *Analyze, compare, create and compose shapes* in Kindergarten, and to *Reason with shapes and their attributes* in Grade 1.

Explanations and Examples:

K.G.2 addresses students' identification of shapes based on known examples. Students at this level do not yet recognize triangles that are turned upside down as triangles, since they do not "look like" triangles. Students need many experiences looking at and manipulating shapes with various typical and atypical orientations. Through these experiences, students will begin to move beyond what a shape "looks like" to identifying particular geometric attributes that define a shape.

Students should be exposed to many types of triangles in many different orientations in order to eliminate the misconception that a triangle is always right-side-up and equilateral.



Students should also be exposed to many shapes in many different sizes.

Examples:

- Teacher makes pairs of paper shapes that are different sizes. Each student is given one shape and the objective is to find the partner who has the same shape.

Teacher brings in a variety of spheres (tennis ball, basketball, globe, ping pong ball, etc) to demonstrate that size doesn't change the name of a shape.

Instructional Strategies:

Kindergartners form visual templates, or models of shape categories. For example, children recognize a shape as rectangle because "it looks like a door." Because children base their understanding of shapes on examples, they need to experience a rich variety of shapes in each shape category so that their mental models are not overly restricted. For example, children without good experiences often reject both triangles and rectangles that are "too skinny" or "not wide enough".

Children should see examples of rectangles that are long and skinny, and they should contrast rectangles with non-rectangles that appear similar but do not have an important defining attribute. Similarly, they should see examples of triangles that have sides of three different lengths, and they should contrast triangles with non-triangles.

Continued next page

Children also need to see examples of shapes beyond circles, squares, rectangles, and triangles. Without these, children develop limited notions. (eg. many children come to believe incorrectly that a trapezoid "is not a shape" because it is not a shape for which they know a name)

Kindergartners should also learn to recognize these shapes whether they are in "standard position" or rotated so that their bases are not horizontal.

Kindergartners can begin to develop explicit and sophisticated levels of thinking and communication. They can learn to describe, and even define, shapes in terms of their *parts* or *attributes* (properties). For example, they can build accurate representations of shapes from physical models of line segments, such as sticks. As they discuss what they have built, attributes of the shapes will arise naturally.

Example:
Student: "I built a rectangle"
Teacher: "How do you know it is a rectangle?"
Student: **"Because** the two opposite sides are the same length and all the angles are the same—they are right angles like in a square."

The experience of discussing attributes of rectangles (or any shape they build) helps children begin to understand the *geometric structure* of all rectangles at an explicit level of thinking.

Another valuable activity is the tactile-kinesthetic exploration of shapes—feeling shapes hidden in a box. Kindergartners can name the shape they are feeling rather than just match shapes. After this, they can extend the activity further as they **describe** the shape without using its name, so that their friends can name the shape. In this way, children learn the properties of the shape, moving from intuitive to explicit, verbalized knowledge. All these variations can be repeated with less familiar shapes.

Such activities help children learn to identify and describe shapes by the number of their sides or corners. Such descriptions build geometric concepts but also reasoning skills and language. They encourage children to view shapes analytically. Children begin to describe some shapes in terms of their properties, such as saying that squares have four side of *equal length*. They informally describe properties of blocks in functional contexts, such as that some blocks roll and other do not.

Focus In Kindergarten, NCTM 2011

Domain: **Geometry (G)**

Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Standard**: K.G.3.** Identify shapes as two-dimensional (lying in a plane, "**flat**") or three dimensional ("**solid**").

Standards for Mathematical Practice (SMP) to be emphasized:

MP.6 Attend to precision
MP.7 Look for and make use of structure.

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #2, **Describing shapes and space**.
This cluster is connected to *Analyze, compare, create and compose shape*s in Kindergarten, and to *Reason with shapes and their attributes* in Grade 1.

Explanations and Examples:

K.G.3. asks students to identify two-dimensional (flat objects) and three-dimensional (solid objects). This standard can be done by having students sort 2-dimensional and 3-dimensional objects, or by having students describe the appearance or thickness of shapes.

A final type of relationship between shapes that is very important is the difference between two-dimensional (flat) and three-dimensional shapes.

Student should be able to differentiate between two dimensional and three dimensional shapes.

- Student names a picture of a shape as two dimensional because it is flat and can be measured in only ***two*** ways (length and width).
- Student names an object as three dimensional because it is not flat (it is a solid object/shape) and can be measured in ***three*** different ways (length, width, height/depth).

Faces of three-dimensional shapes can be identified as specific two-dimensional shapes.

Arizona, Ohio & NC DOE

Domain: **Geometry (G)**

Cluster: Analyze, compare, create, and compose shapes.

Standard**: K.G.4** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

Standards for Mathematical Practice (SMP) to be emphasized:

MP.4 Model with mathematics
MP.6 Attend to precision
MP.7 Look for and make use of structure.

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #2**, Describing shapes and space**.

This cluster is connected *to Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)* in Kindergarten, and to *Reason with shapes and their attributes* in Grade 1.

Explanations and Examples:

K.G.4 asks students to note similarities and differences between and among 2-D and 3-D shapes using informal language. These experiences help young students begin to understand how 3-dimensional shapes are composed of 2-dimensional shapes (e.g.., The base and the top of a cylinder is a circle; a circle is formed when tracing a sphere).

Students analyze and compare two- and three-dimensional shapes by observations. Their visual thinking enables them to determine if things are alike or different based on the appearance of the shape. Students sort objects based on appearance. Even in early explorations of geometric properties, they are introduced to how categories of shapes are subsume (*contained*) within other categories. For instance, they will recognize that a square is a special type of rectangle.

Students should be exposed to triangles, rectangles, and hexagons whose sides are not all congruent. They first begin to describe these shapes using everyday language and then refine their vocabulary to include sides and vertices/corners. Opportunities to work with pictorial representations, concrete objects, as well as technology helps student develop their understanding and descriptive vocabulary for both two- and three- dimensional shapes.

Instructional Strategies:

The abilities involved in composing and decomposing shapes are important for many reasons. These geometric competencies are at the foundation of geometry, but also arithmetic (e.g., composing and decomposing numbers and arrays in multiplication), measurement, and higher-order geometric work. Creating and then iterating units and higher-order units in the context of construction patterns, measuring, and computing, are established bases for mathematical understanding and analysis.

It is important to allow students to explore and build geometric understanding themselves. One important step to take is to switch from making assertions and generalizations to framing ideas as questions. Rather than saying, "Every time you put two triangles together, you get a square"—a mathematically incorrect statement. Ask the following:

"How many different ways can you put these two triangles together to make a new shape?"
"What shapes will you get?"

Continued next page

This allows children to see that even with two right triangles made from a square, they can put these together to make a triangle or a parallelogram .

Kindergartners can develop the ability to intentionally and systematically combine shapes to make new shapes and complete puzzles. They do so with increasing anticipation, on the basis of the shapes' attributes, and thus, children developmental imagery of the component shapes. They move from using shapes separately to putting them together to make pictures.

A significant advance is that they can combine shapes with different properties, extending the pattern block shapes (whose angles are multiples of 30 degrees) common at early levels to such shapes as tangrams (with angles that are multiples of 45 degrees), and with sets of various shapes that include angles that are multiples of 15 degrees, as well as sections of circles. Combining these shape sets should be done after children have worked with the pattern-block shapes separately from the square/rectangle/right triangle shapes based on 90 degrees and 45 degrees because many compositions are possible when the angles are consistent.
Focus in Kindergarten, NCTM 2011

Use shapes collected from students to begin the investigation into basic properties and characteristics of two- and three-dimensional shapes. Have students analyze and compare each shape with other objects in the classroom and describe the similarities and differences between the shapes. Ask students to describe the shapes while the teacher records key descriptive words in common student language. Students may use the word *flat* to describe two-dimensional shapes and the word *solid* to describe three-dimensional shapes.

Use the sides, faces and vertices of shapes to practice counting and reinforce the concept of one-to-one correspondence.

The teacher and students orally describe and name the shapes found on a Shape Hunt. Students draw a shape and build it using materials regularly kept in the classroom such as construction paper, clay, wooden sticks or straws.

Students can use a variety of manipulatives and real-world objects to build larger shapes with these and other smaller shapes: squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. Kindergarteners can manipulate cardboard shapes, paper plates, pattern blocks, tiles, canned food, and other common items.

Have students compose (build) a larger shape using only smaller shapes that have the same size and shape. The sides of the smaller shapes should touch and there should be no gaps or overlaps within the larger shape. For example, use one-inch squares to build a larger square with no gaps or overlaps. Have students also use different shapes to form a larger shape where the sides of the smaller shapes are touching and there are no gaps or overlaps. Ask students to describe the larger shape and the shapes that formed it.

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Common Misconceptions:

One of the most common misconceptions in geometry is the belief that orientation is tied to shape. A student may see the first of the figures below as a triangle, but claim to not know the name of the second.

Students need to have many experiences with shapes in different orientations. For example, in the *Just Two Triangles* activity, ask students to form larger triangles with the two triangles in different orientations.

Another misconception is confusing the name of a two-dimensional shape with a related three-dimensional shape or the shape of its face. For example, students might call a *cube* a *square* because the student sees the face of the cube.

It is important when students are exploring 2-dimensional shapes (flat) that the shapes they are working with are on paper or other "FLAT" material.

Arizona, Ohio & NC DOE

Domain: **Geometry (G)**
Cluster: Analyze, compare, create, and compose shapes.
Standard**: K.G.5** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
<u>Standards for Mathematical Practice (SMP) to be emphasized:</u> MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.7 Look for and make use of structure.
<u>Connections:</u> This cluster is connected to the Kindergarten Critical Area of Focus #2**, Describing shapes and space**. This cluster is connected *to Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)* in Kindergarten, and to *Reason with shapes and their attributes* in Grade 1.
<u>Explanations and Examples</u>: **K.G.5** asks students to apply their understanding of geometric attributes of shapes in order to create given shapes. For example, a student may roll a clump of play-doh into a sphere or use their finger to draw a triangle in the sand table, recalling various attributes in order to create that particular shape. Because two-dimensional shapes are flat and three-dimensional shapes are solid, students should draw two-dimensional shapes and build three-dimensional shapes. Shapes may be built using materials such as clay, toothpicks, marshmallows, gumdrops, straws, pipe cleaners, etc. **<u>Instructional Strategies</u>**: See strategies in K.G.4. Focus in Kindergarten, NCTM 2011

Cluster: Analyze, compare, create, and compose shapes.

Standard**: K.G.6** Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"

Standards for Mathematical Practice (SMP) to be emphasized:

MP.1 Make sense of problems and persevere in solving them.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.7 Look for and make use of structure.

Connections:

(See K.G.1 earlier)

Explanations and Examples:

K .G.6 moves beyond identifying and classifying simple shapes to manipulating two or more shapes to create a new shape. This concept begins to develop as students" first move, rotate, flip, and arrange puzzle pieces. Next, students use their experiences with puzzles to move given shapes to make a design (e.g., "Use the 7 tangram pieces to make a fox."). Finally, using these previous foundational experiences, students manipulate simple shapes to make a new shape.

Instructional Strategies:

Students use pattern blocks, tiles, or paper shapes and technology to make new two- and three-dimensional shapes. Their investigations allow them to determine what kinds of shapes they can join to create new shapes. They answer questions such as "What shapes can you use to make a square, rectangle, circle, triangle? …etc."

This is an opportunity to use blocks from a play center to create shapes composed of a series of blocks. Laying several rectangular prisms can make other identifiable shapes.

Students may use a document camera to display shapes they have composed from other shapes. They may also use an interactive whiteboard to copy shapes and compose new shapes. They should describe and name the new shape.

Arizona, Ohio, & NC DOE

K.G.6

<table>
<tr><th colspan="5">Kindergarten – Geometry
Extended Common Core State Standards Mathematics
North Carolina DOE</th></tr>
<tr><th colspan="2">Common Core State Standards</th><th>Essence</th><th colspan="2">Extended Common Core</th></tr>
<tr><td colspan="2">Identify and describe shapes (such as squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).</td><td rowspan="2">Identify shapes and describe positions</td><td colspan="2">Identify and describe shapes (squares and circles).</td></tr>
<tr><td>Cluster</td><td>1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).</td><td>Cluster</td><td>1. Describe objects in the environment using names of shapes.
2. Describe the relative position of objects using terms such as in, on, out, under, off to locate objects.</td></tr>
<tr><td colspan="2">Analyze, compare, create, and compose shapes.</td><td rowspan="2">Understand shapes can be different sizes</td><td colspan="2">Compare shapes.</td></tr>
<tr><td>Cluster</td><td>4. Analyze and compare a variety of two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes.</td><td>Cluster</td><td>3. Compare a variety of two-dimensional shapes, in different sizes to describe differences (big/little, small/medium/large).</td></tr>
</table>

Resources:

TABLE 1. Common addition and subtraction situations.[34]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown[35]
Put Together/ Take Apart[36]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[37]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

[34] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).
[35] These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.
[36] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.
[37] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.