Overall: The mathematics tasks focus on developing **CONCEPTUAL UNDERSTANDING** and encouraging ALL students to make sense of the mathematics and to exhibit higher-order thinking skills. As you observe lessons in the classroom, check to see if **STUDENTS** exhibited the following behaviors in solving mathematics problems and if **TEACHERS** facilitated these behaviors by providing cognitively demanding tasks and encouraging sense making for ALL students.

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| **MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM** | Open-ended problem with no solution pathway evident. Non-routine problems with multiple solutions. | **Teacher:**  
- Provides time and facilitates discussion in problem solutions.  
- Facilitates discourse in the classroom so that students UNDERSTAND the approaches of others.  
- Provides opportunities for students to explain themselves, the meaning of a problem, etc.  
- Provides opportunities for students to connect concepts to “their” world.  
- Provides students TIME to think and become “patient” problem solvers.  
- Facilitates and encourages students to check their answers using different methods (not calculators).  
- Provides problems that focus on relationships and are “generalizable”. | **Students:**  
- Are actively engaged in solving problems & thinking is visible (i.e., DOING MATHEMATICS vs. FOLLOWING STEPS OR PROCEDURES).  
- Are analyzing givens, constraints, relationships, and goals (NOT the teacher).  
- Are discussing with one another, making conjectures, planning a solution pathway, not jumping into a solution attempt or guessing at the direction to take.  
- Relate current “situation” to concept or skill previously learned and check answers using different methods.  
- Continually ask self, does this make sense? |

**Evidence & Comments:**

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| **REASON ABSTRACTLY AND QUANTITATIVELY** | Provide a context or situation for students that allows them to “abstract” the situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents. Tasks that allow for pausing during the manipulation process in order to probe into the referents for the symbols involved. | Teacher:  
- Provides a range of representations of math problem situations and encourages various solutions.  
- Provides opportunities for students to make sense of quantities and their relationships in problem situations.  
- Provides problems that require flexible use of properties of operations and objects.  
- Emphasizes quantitative reasoning which entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, **not just how to compute them and/or rules**; and knowing and flexibly using different properties of operations and objects. | Students:  
- Use varied representations and approaches when solving problems.  
- Make sense of quantities and their relationships in problem situations.  
- Are **decontextualizing** (abstract a given situation and represent it symbolically and manipulate the representing symbols), and **contextualizing** (pause as needed during the manipulation process in order to probe into the referents for the symbols involved.  
- Use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities, **not just how to compute them**. |

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| CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE ARGUMENTS OF OTHERS | Tasks that allow students to analyze situations by breaking them into cases and then justify, defend/refute and communicate examples and counterexamples, etc. etc. | Teacher:  
- Provides ALL students opportunities to understand and use stated assumptions, definitions, and previously established results in constructing arguments.  
- Provides ample time for students to make conjectures and build a logical progression of statements to explore the truth of their conjectures.  
- Provides opportunities for students to construct arguments and critique arguments of peers.  
- Facilitates and guides students in recognizing and using counterexamples.  
- Encourages and facilitates students justifying their conclusions, communicating, and responding to the arguments of others.  
- Asks useful questions to clarify and/or improve students’ arguments. | Students:  
- Make conjectures and explore the truth of their conjectures.  
- Recognize and use counterexamples.  
- Justify and defend ALL conclusions and communicates them to others.  
- Recognize and explain flaws in arguments. (After listening or reading arguments of others, they respond by deciding whether or not they make sense. They ask useful questions to improve arguments.)  
- Elementary Students: construct arguments using concrete referents such as objects, drawings, diagrams, actions. Later, students learn to determine the domains to which an argument applies. |

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| **MODEL WITH MATHEMATICS**    | Problem solving situations such as: Elementary: this might be as simple as writing an addition equation to describe a situation. Middle grades: a student might apply proportional reasoning to plan a school event or analyze a problem in the community. High School: a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. | Teacher:  
• Provides problem situations that apply to everyday life.  
• Provides rich tasks that focus on conceptual understanding, relationships, etc. | Students:  
• Apply the mathematics they know to everyday life, society, and the workplace.  
• Write equations to describe situations.  
• Are comfortable in making assumptions and approximations to simplify complicated situations.  
• Analyze relationships to draw conclusions.  
• Improve their model if it has not served its purpose. |

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| **ATTENDS TO PRECISION**        | **Elementary:** students are solving problems and carefully formulating explanations to others. **High School:** students are examining claims and making explicit use of definitions. | **Teacher:**  
  - Facilitates, encourages and expects precision in communication.  
  - Provides opportunities for students to explain and/or write their reasoning to others. | **Students:**  
  - Use and clarify mathematical definitions in discussions and in their own reasoning (orally and in writing).  
  - Use, understand and state the meanings of symbols.  
  - Express numerical answers with a degree of precision. |

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| **USE APPROPRIATE TOOLS**     | **Elementary:** students are provided tasks that require a variety of tools to solve. **High School:** tasks might include students analyzing graphs of functions and solutions generated using a graphing calculator to detect possible errors by using estimations and other mathematical knowledge. | **Teacher:**  
- Provides a variety of tools and technology for students to explore to deepen their understanding of math concepts.  
- Provides problem solving tasks that require students to consider a variety of tools for solving. (Tools might include pencil/paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software, etc.) | **Students:**  
- Consider available tools when solving a mathematical problem.  
- Are familiar with a variety of mathematics tools and use them when appropriate to explore and deepen their understanding of concepts. |

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| **LOOK FOR AND MAKE USE OF STRUCTURE** | **Elementary:** task might require students to notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides they shapes have. Later, students will see 7x8=the well remembered 7x5+7x3, in preparation for the distributive property.  **High School:** in the expression $x^2+9x+14$, students see the 14 as 2x7 and the 9 as 2+7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. | Teacher:  
• Provides opportunities and time for students to explore patterns and relationships to solve problems.  
• Provides rich tasks and facilitates pattern seeking and understanding of relationships in numbers rather than following a set of steps and/or procedures. | Students:  
• Look closely to discern patterns or structure.  
• Associate patterns with properties of operations and their relationships.  
• Step back for an overview and can shift perspective.  
• See complicated things, such as algebraic expressions, as single objects or as composed of several objects. (Younger children decompose and compose numbers.) |

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| **LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING** | **Upper Elementary**: solving problems and noticing that when dividing 25 by 11 they are repeating the same calculations over and over again, and conclude they have a repeating decimal. **Middle School**: students might abstract the equation \((y-2)/3\) by paying attention to the calculation of slope as they repeatedly check whether the points are on the line through \((1,2)\) with a slope of 3. **High School**: Tasks that allow High School students to notice regularity in the way terms cancel when expanding \((x-1)(x+1)(x^2+1)\) and \((x-1)(x^3+x^2+x+1)\) which might lead to the general formula for the sum of a geometric series. | Teacher:  
- Provides problem situations that allow students to explore regularity and repeated reasoning.  
- Provides rich tasks that encourage students to use repeated reasoning to form generalizations and provides opportunities for students to communicate these generalizations. | Students:  
- Notice if calculations are repeated and look for both general methods and shortcuts.  
- Pay attention to regularity and use to solve problems.  
- Use regularity and use this to lead to a general formula and generalizations.  
- Maintain oversight of the process of solving a problem while attending to details and **continually** evaluates the reasonableness of immediate results. |

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