STANDARDS FOR MATHEMATICAL PRACTICES OBSERVATION TOOL

Adapted from KATM/KSDE Summer Academy 2011…Developed by Melissa Hancock

Overall: The mathematics tasks focus on developing CONCEPTUAL UNDERSTANDING and encouraging ALL students to make sense of the mathematics and to exhibit higher-order thinking skills. As you observe lessons in the classroom, check to see if STUDENTS exhibited the following behaviors in solving mathematics problems and if TEACHERS facilitated these behaviors by providing cognitively demanding tasks and encouraging sense making for ALL students.

Mathematical Practice Standard	Task (Example)	Teacher: Actions/Responsibilities	Student: Actions/Responsibilities
MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM	Open-ended problem with no solution pathway evident. Non-routine problems with multiple solutions.	**Teacher:** • Provides time and facilitates discussion in problem solutions. • Facilitates discourse in the classroom so that students UNDERSTAND the approaches of others. • Provides opportunities for students to explain themselves, the meaning of a problem, etc. • Provides opportunities for students to connect concepts to "their" world. • Provides students TIME to think and become "patient" problem solvers. • Facilitates and encourages students to check their answers using different methods (not calculators). • Provides problems that focus on relationships and are "generalizable".	**Students:** • Are actively engaged in solving problems & thinking is visible (i.e., DOING MATHEMATICS vs. FOLLOWING STEPS OR PROCEDURES). • Are analyzing givens, constraints, relationships, and goals (NOT the teacher). • Are discussing with one another, making conjectures, planning a solution pathway, not jumping into a solution attempt or guessing at the direction to take. • Relate current "situation" to concept or skill previously learned and check answers using different methods. • Continually ask self, does this make sense?
Evidence & Comments:			

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Mathematical Practice Standard	Task (Example)	Teacher: Actions/Responsibilities	Student: Actions/Responsibilities
REASON ABSTRACTLY AND QUANTITATIVELY	Provide a context or situation for students that allows them to "abstract" the situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents. Tasks that allow for pausing during the manipulation process in order to probe into the referents for the symbols involved.	**Teacher:** • Provides a range of representations of math problem situations and encourages various solutions. • Provides opportunities for students to make sense of quantities and their relationships in problem situations. • Provides problems that require flexible use of properties of operations and objects. • Emphasizes quantitative reasoning which entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, **not just how to compute them and/or rules**; and knowing and flexibly using different properties of operations and objects.	**Students:** • Use varied representations and approaches when solving problems. • Make sense of quantities and their relationships in problem situations. • Are *decontextualizing* (abstract a given situation and represent it symbolically and manipulate the representing symbols), and *contextualizing* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved. • Use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities, NOT just how to compute them.
Evidence and Comments:			

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Mathematical Practice Standard	Task (Example)	Teacher: Actions/Responsibilities	Student: Actions/Responsibilities
CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE ARGUMENTS OF OTHERS	Tasks that allow students to analyze situations by breaking them into cases and then justify, defend/refute and communicate examples and counterexamples, etc. etc.	**Teacher**: • Provides **ALL** students opportunities to understand and use stated assumptions, definitions, and previously established results in constructing arguments. • Provides ample time for students to make conjectures and build a logical progression of statements to explore the truth of their conjectures. • Provides opportunities for students to construct arguments and critique arguments of peers. • Facilitates and guides students in recognizing and using counterexamples. • Encourages and facilitates students justifying their conclusions, communicating, and responding to the arguments of others. • Asks useful questions to clarify and/or improve students' arguments.	**Students:** • Make conjectures and explore the truth of their conjectures. • Recognize and use counterexamples. • Justify and defend ALL conclusions and communicates them to others. • Recognize and explain flaws in arguments. (After listening or reading arguments of others, they respond by deciding whether or not they make sense. They ask useful questions to improve arguments.) • Elementary Students: construct arguments using concrete referents such as objects, drawings, diagrams, actions. Later, students learn to determine the domains to which an argument applies.
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MODEL WITH MATHEMATICS	Problem solving situations such as: **Elementary**: this might be as simple as writing an addition equation to describe a situation. **Middle grades**: a student might apply proportional reasoning to plan a school event or analyze a problem in the community. **High School**: a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.	**Teacher:** • Provides problem situations that apply to everyday life. • Provides rich tasks that focus on conceptual understanding, relationships, etc.	**Students:** • Apply the mathematics they know to everyday life, society, and the workplace. • Write equations to describe situations. • Are comfortable in making assumptions and approximations to simplify complicated situations. • Analyze relationships to draw conclusions. • Improve their model if it has not served its purpose.
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Mathematical Practice Standard	Task (Example)	Teacher: Actions/Responsibilities	Student: Actions/Responsibilities
ATTENDS TO PRECISION	**Elementary**: students are solving problems and carefully formulating explanations to others. **High School**: students are examining claims and making explicit use of definitions.	**Teacher:** • Facilitates, encourages and expects precision in communication. • Provides opportunities for students to explain and/or write their reasoning to others.	**Students:** • Use and clarify mathematical definitions in discussions and in their own reasoning (orally and in writing). • Use, understand and state the meanings of symbols. • Express numerical answers with a degree of precision.
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Mathematical Practice Standard	Task (Example)	Teacher: Actions/Responsibilities	Student: Actions/Responsibilities
USE APPROPRIATE TOOLS	**Elementary**: students are provided tasks that require a variety of tools to solve. **High School**: tasks might include students analyzing graphs of functions and solutions generated using a graphing calculator to detect possible errors by using estimations and other mathematical knowledge.	**Teacher:** • Provides a variety of tools and technology for students to explore to deepen their understanding of math concepts. • Provides problem solving tasks that require students to consider a variety of tools for solving. (Tools might include pencil/paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software, etc.)	**Students:** • Consider available tools when solving a mathematical problem. • Are familiar with a variety of mathematics tools and use them when appropriate to explore and deepen their understanding of concepts.
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Mathematical Practice Standard	Task (Example)	Teacher: Actions/Responsibilities	Student: Actions/Responsibilities
LOOK FOR AND MAKE USE OF STRUCTURE	**Elementary:** task might require students to notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides they shapes have. Later, students will see 7x8=the well remembered 7x5+7x3, in preparation for the distributive property. **High School**: in the expression $x^2+9x+14$, students see the 14 as 2x7 and the 9 as 2+7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.	**Teacher:** • Provides opportunities and time for students to explore patterns and relationships to solve problems. • Provides rich tasks and facilitates pattern seeking and understanding of relationships in numbers rather than following a set of steps and/or procedures.	**Students:** • Look closely to discern patterns or structure. • Associate patterns with properties of operations and their relationships. • Step back for an overview and can shift perspective. • See complicated things, such as algebraic expressions, as single objects or as composed of several objects. (Younger children decompose and compose numbers.)
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Mathematical Practice Standard	Task (Example)	Teacher: Actions/Responsibilities	Students: Actions/Responsibilities
LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING	**Upper Elementary**: solving problems and noticing that when dividing 25 by 11 they are repeating the same calculations over and over again, and conclude they have a repeating decimal. **Middle School:** students might abstract the equation (y-2)/)=3 by paying attention to the calculation of slope as they repeatedly check whether the points are on the line through (1,2) with a slope of 3. **High School**: Tasks that allow High School students to notice regularity in the way terms cancel when expanding (x-1)(x+1()x^2+1) and (x-1)(x^3+x^2+x+1) which might lead to the general formula for the sum of a geometric series.	**Teacher:** • Provides problem situations that allow students to explore regularity and repeated reasoning. • Provides rich tasks that encourage students to use repeated reasoning to form generalizations and provides opportunities for students to communicate these generalizations.	**Students:** • Notice if calculations are repeated and look for both general methods and shortcuts. • Pay attention to regularity and use to solve problems. • Use regularity and use this to lead to a general formula and generalizations. • Maintain oversight of the process of solving a problem while attending to details and **continually** evaluates the reasonableness of immediate results.
Evidence and Comments:			

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