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Authors of Course Guide
Cortni Muir, NMPS Grade 4 -7 Math Coach
  Peggy Neal, CREC Math Specialist
  Tamara Gloster, CREC Math Specialist
New Milford’s Mission Statement

The mission of the New Milford Public Schools, a collaborative partnership of students, educators, family and community, is to prepare each and every student to compete and excel in an ever-changing world, embrace challenges with vigor, respect and appreciate the worth of every human being, and contribute to society by providing effective instruction and dynamic curriculum, offering a wide range of valuable experiences, and inspiring students to pursue their dreams and aspirations.
Course Overview

The Grade 7 Pre-Algebra course is an advanced math course addressing standards from grades 7 and 8. The resources for this course will include grade 7 and 8 Connected Mathematics Program (CMP3) materials. Mathematical tasks for students in class and in homework are the primary vehicle for student engagement with the mathematical concepts to be learned. Ideas are explored through these mathematical tasks in the depth necessary to allow students to make sense of them. The curriculum helps students grow in their ability to reason effectively with information represented in graphic, numeric, symbolic, and verbal forms and to move flexibly among these representations to produce fluency in both conceptual and procedural knowledge. Topics specific to grade 7 Pre-Algebra include: rational numbers and their operations, solving problems with ratios, rates, percents, and proportions, understanding slope and linear relationships, Pythagorean Theorem, symmetry and transformations, and systems of linear equations.
## Pacing Guide

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<thead>
<tr>
<th>Unit Title</th>
<th># of Weeks</th>
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<tbody>
<tr>
<td>Unit 1 – Integers and Rational Numbers</td>
<td>5 Weeks</td>
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<tr>
<td>Unit 2 – Understanding Similarity</td>
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<tr>
<td>Unit 3 – Ratios, Rates, Percents, and Proportions</td>
<td>4 Weeks</td>
</tr>
<tr>
<td>Unit 4 – Linear Relationships</td>
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<td>Unit 5 – Linear and Inverse Variation</td>
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<tr>
<td>Unit 6 – The Pythagorean Theorem</td>
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<td>Unit 7 – Symmetry and Transformations</td>
<td>5 Weeks</td>
</tr>
<tr>
<td>Unit 8 – System of Linear Equations</td>
<td>3 Weeks</td>
</tr>
</tbody>
</table>
## Identify Desired Results

### Common Core Standards

**Standards of Focus in Unit**

- **(7.NS.A.1)** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- **(7.NS.A.1.a)** Describe situations in which opposite quantities combine to make 0.
- **(7.NS.A.1.b)** Understand $p + q$ as the number located a distance $|q|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- **(7.NS.A.1.c)** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- **(7.NS.A.1.d)** Apply properties of operations as strategies to add and subtract rational numbers.
- **(7.NS.A.2)** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
- **(7.NS.A.2.a)** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- **(7.NS.A.2.b)** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $p$ and $q$ are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
- **(7.NS.A.2.c)** Apply properties of operations as strategies to multiply and divide rational numbers.
- **(7.NS.A.2.d)** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- **(7.NS.A.3)** Solve real-world and mathematical problems involving the four operations with rational numbers.
- **(7.EE.B.3)** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- (7.EE.B.4) Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- (7.EE.B.4.b) Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

**Standards for Mathematical Practices**
- MP 1 - Make sense of problems and persevere in solving them.
- MP 2 - Reason abstractly and quantitatively.
- MP 3 - Construct viable arguments and critique the reasoning of others.
- MP 4 - Model with mathematics.
- MP5 - Use appropriate tools strategically.
- MP6 - Attend to precision.
- MP7 - Look for and make use of structure.
- MP8 - Look for and express regularity in repeated reasoning.

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<tr>
<td>Generalizations of desired understanding via essential questions (Students will understand that …)</td>
<td>Inquiry used to explore generalizations</td>
</tr>
<tr>
<td>• Rational numbers consist of positive numbers, negative numbers and zero.</td>
<td>• How do negative and positive numbers and zero help me to describe real world situations?</td>
</tr>
<tr>
<td>• Rational numbers are an extension of integer models.</td>
<td>• What models for rational numbers help me show relationships when solving problems?</td>
</tr>
<tr>
<td>• Different models can be used to represent operations of rational numbers.</td>
<td>• How are the models of integer operations related to the rational number algorithm?</td>
</tr>
</tbody>
</table>

**Expected Performances**
What students should know and be able to do

**Students will know the following:**
- Integers are a subset of the set of rational numbers.
- Conceptual models and the algorithm can be used to perform operations with rational numbers.

**Students will be able to do the following:**
- Solve problems involving rational numbers using models and algorithms (including zero pairs and additive inverse).
- Compare and order rational numbers.
- Use properties to write equivalent rational number expressions.
- Apply the algorithm of rational operations to solve real world problems.

**Character Attributes**
- Cooperation
- Respect
- Honesty
- Responsibility

**Technology Competencies**
- None
### Develop Teaching and Learning Plan

#### Teaching Strategies:

**Use a problem-centered model with teacher-directed mini-lesson**
- Embed important mathematical ideas within contexts of interesting problems.
- Work with small groups of students to build conceptual understanding, fluency and application of key math focus areas.

**Use games to develop concepts and practice skills.**

**Use student-centered activities and worthwhile math tasks.**
- Arrange student seating so that learners can move freely to post, analyze, and discuss their work.

**Use a variety of grouping structures.**
- Collaborative groups, partners, individuals.
- Clearly communicate expectations about group work to students.

**Orchestrate class/student to student discourse.**
- Focus discussions on important mathematics and student strategies.
- Facilitate student-to-student discourse.
- Elicit participation from all students over the course of several discussions.

#### Learning Activities:

- Play Math Fever to find the total value of combinations of positive and negative integers (intuitively finding the sum).
- Extend the integer number line to include rational numbers.
- Use the chip model to represent addition and subtraction in the investigation In the Chips.
- Predict the result of change when adding and subtracting of two rational numbers to derive an algorithm.
- Discover the relationship when you change a subtraction expression to addition by adding the opposite by using the chip model.
- Extend fact families to include rational numbers.
- Use number lines and chip models to discover patterns when multiplying rational numbers.
- Use the algorithm to multiply and divide rational numbers.
- Play Dealing Up to apply the properties to the operations of rational numbers when solving problems.

### Assessments

<table>
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<tr>
<th>Performance Task(s)</th>
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<tr>
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#### Title: Football Night

**Goal:** Students will perform operations with integers to find averages, plot data on number lines, evaluate formulas, determine profit and money spent, and compare data relating to high school football.

**Role:** Statistician

#### Checkpoints

- Investigation Classwork and ACE (Applications, Connections, Extensions) exercises.
- Mathematical Reflection Questions at end of each Investigations
- Looking Back which can be used as review, helping students to stand back
<table>
<thead>
<tr>
<th>Audience: Athletic Director, Principal, Parents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Situation: Jenna is the statistician for Midtown High School. She records and compiles that statistics for the football team. So far this season she has recorded the final scores of the games for Midtown and its biggest rival, Bayshore High School.</td>
</tr>
<tr>
<td>Product or Performance: Detailed responses, showing all work.</td>
</tr>
<tr>
<td>Standards for Success: A scoring rubric is shared with students at the onset of the project.</td>
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and look at the big ideas and connections in the unit.

**Surveys of Knowledge**
- Quizzes: individual assessment instruments.
- Unit Tests: individual assessment that informs teachers about students’ ability to apply, refine, modify, and possibly extend the mathematical knowledge and skills acquired.
- Performance Task

**Suggested Resources**
- Connected Mathematics Project 3 (CMP3) Unit 1, Accentuate the Negative
- Teaching Student-Centered Mathematics, 6-8 by Van de Walle, et. al
- CT State Department of Education, Mathematics Units of Study, Grade 7
New Milford Public Schools

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<th>Committee Member(s): Cortni Muir, Peggy Neal, Tamara Gloster</th>
<th>Course/Subject: Mathematics Course/Subject: Mathematics</th>
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<tr>
<td>Unit Title: Unit 2 – Understanding Similarity</td>
<td>Grade Level: 7 Pre-Algebra # of Weeks: 4</td>
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</table>

### Identify Desired Results

#### Standards of Focus in Unit

- (7.RP.A.2) Recognize and represent proportional relationships between quantities.
- (7.RP.A.2.a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- (7.RP.A.2.b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- (7.G.A.1) Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- (7.G.A.2) Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- (7.G.A.6) Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

#### Additional Standards in Unit

- (7.RP.A.3) Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
- (7.EE.B.3) Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- (7.EE.B.4) Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

#### Standards for Mathematical Practices

- **MP 1** - Make sense of problems and persevere in solving them.
- **MP 2** - Reason abstractly and quantitatively.
- **MP 3** - Construct viable arguments and critique the reasoning of others.
- **MP 4** - Model with mathematics.
- **MP5** - Use appropriate tools strategically.
- MP6 - Attend to precision.
- MP7 - Look for and make use of structure.
- MP8 - Look for and express regularity in repeated reasoning.

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<tr>
<td>- Multiplication plays a role in similarity relationships.</td>
<td>- What is the same and what is different about similar figures?</td>
</tr>
<tr>
<td>- Properties exist among the angles, side lengths, areas, and perimeters of similar figures.</td>
<td>- How can I determine whether two shapes are similar?</td>
</tr>
<tr>
<td>- Similarity can be used to solve real world problems.</td>
<td>- How can I use similar figures to find missing measurements?</td>
</tr>
</tbody>
</table>

**Expected Performances**

**Students will know the following:**
- Scale factors and ratios are used to describe and create similar figures.
- Properties of similar figures exist among the angles, side lengths, areas, and perimeters.
- Similarity properties can be used to find distances and heights that cannot be measured directly.

**Students will be able to do the following:**
- Determine similarity among shapes.
- Use similarity properties to find distances and heights that cannot be measured directly.
- Use scale factors and ratios to describe and create similar figures.
- Predict the ways that stretching or shrinking a figure will affect its angles, side lengths, areas, and perimeters.

**Character Attributes**
- Cooperation
- Respect
- Honesty
- Responsibility

**Technology Competencies**
- *** All available on CMP3 Dashboard
- Expression Calculator
- Mug Wumps
- Coordinate Grapher
- Pattern Blocks

**Develop Teaching and Learning Plan**

**Teaching Strategies:**
**Use a problem-centered model with teacher-directed mini-lesson**
- Embed important mathematical ideas within contexts of interesting problems.
- Work with small groups of students

**Learning Activities:**
- Define similarity by describing corresponding sides and angles.
- Draw figures by changing at least one attribute, determine if the result is similar, and write algebraic rule to describe change.
Use games to develop concepts and practice skills.
Use student-centered activities and worthwhile math tasks.
- Arrange student seating so that learners can move freely to post, analyze, and discuss their work.

Use a variety of grouping structures.
- Collaborative groups, partners, individuals.
- Clearly communicate expectations about group work to students.

Orchestrates class/student to student discourse.
- Focus discussions on important mathematics and student strategies.
- Facilitate student-to-student discourse.
- Elicit participation from all students over the course of several discussions.

- Sketch similar triangles/quadrilaterals and examine the effects on perimeter and area.
- Use scale factors to draw similar figures and to find missing side lengths in similar figures.
- Use similar figures to find distance that is difficult to measure directly.

Assessments

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N/A

Checkpoints
- Investigation Classwork and ACE (Applications, Connections, Extensions) exercises.
- Mathematical Reflection Questions at end of each Investigations
- Looking Back which can be used as review, helping students to stand back and look at the big ideas and connections in the unit.

Surveys of Knowledge
- Quizzes: individual assessment instruments.
- Unit Tests: individual assessment that informs teachers about students’ ability to apply, refine, modify, and possibly
extend the mathematical knowledge and skills acquired.

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<tr>
<td>• Connected Mathematics Project 3 (CMP3) Unit 2, Stretching and Shrinking</td>
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<tr>
<td>• Teaching Student-Centered Mathematics, 6-8 by Van de Walle, et. al</td>
</tr>
<tr>
<td>• CT State Department of Education, Mathematics Units of Study, Grade 7</td>
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</table>
New Milford Public Schools

Committee Member(s): Cortni Muir, Peggy Neal, Tamara Gloster
Unit Title: Unit 3 – Rates, Ratios, Percents and Proportions
Course/Subject: Mathematics
Grade Level: 7 Pre-Algebra
# of Weeks: 4

Identify Desired Results

Common Core Standards

Standards of Focus in Unit
- (7.RP.A.1) Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units
- (7.RP.A.2) Recognize and represent proportional relationships between quantities.
- (7.RP.A.2.a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- (7.RP.A.2.b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- (7.RP.A.2.c) Represent proportional relationships by equations.
- (7.RP.A.2.d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
- (7.RP.A.3) Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
- (7.EE.B.4) Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- (7.EE.B.4.a) Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Additional Standards in Unit
- (7.NS.A.3) Solve real-world and mathematical problems involving the four operations with rational numbers.
- (7.EE.B.3) Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Standards for Mathematical Practices
- MP 1 - Make sense of problems and persevere in solving them.
- MP 2 - Reason abstractly and quantitatively.
- MP 3 - Construct viable arguments and critique the reasoning of others.
- MP 4 - Model with mathematics.
- MP5 - Use appropriate tools strategically.
- MP6 - Attend to precision.
- **MP7 - Look for and make use of structure.**
- **MP8 - Look for and express regularity in repeated reasoning.**

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- Ratios, rates, differences and percents can be used to write comparison statements.
- Rate is a special ratio that compares two measurements with different units.
- Proportional situations are represented by constant growth in tables, graphs and/or equations.
- Unit rate and constant of proportionality are interchangeable.

- What are some strategies for comparing quantities?
- How do I know a relationship is proportional?
- How might ratios, rates, or proportions be used to solve problems?
- What does unit rate or constant of proportionality tell me about a situation?

**Expected Performances**
What students should know and be able to do

**Students will know the following:**
- Ratios, rates, differences and percents can be used to write comparison statements.
- Rate is a special ratio that compares two measurements with different units.
- Proportional situations are represented by constant growth in tables, graphs and/or equations.
- Unit rate and constant of proportionality are interchangeable.

**Students will be able to do the following:**
- Interpret, analyze and/or write comparison statements using ratio, rates, differences, and/or percents.
- Solve involving part to part and part to whole ratios.
- Write and use proportions to solve a problem.
- Create rate tables to solve problems.
- Compare rates given in different formats (table, graph, equation, words).
- Find and describe unit rate/constant of proportionality from a table, graph or equation.
- Solve multi-step percent problems (tax, tip, commissions, markup, discount).

**Character Attributes**
- Cooperation
- Respect
- Honesty
- Responsibility

**Technology Competencies**
*** All available on CMP3 Dashboard
- Expression Calculator
Develop Teaching and Learning Plan

Teaching Strategies:
- **Use a problem-centered model with teacher-directed mini-lesson**
  - Embed important mathematical ideas within contexts of interesting problems.
  - Work with small groups of students to build conceptual understanding, fluency and application of key math focus areas.

- **Use games to develop concepts and practice skills.**

- **Use student-centered activities and worthwhile math tasks.**
  - Arrange student seating so that learners can move freely to post, analyze, and discuss their work.

- **Use a variety of grouping structures.**
  - Collaborative groups, partners, individuals.
  - Clearly communicate expectations about group work to students.

- **Orchestrate class/student to student discourse.**
  - Focus discussions on important mathematics and student strategies.
  - Facilitate student-to-student discourse.
  - Elicit participation from all students over the course of several discussions.

Learning Activities:
- Describe the accuracy of comparison statements.
- Write comparison statements in Surveying Opinions.
- Compare ratios to make decisions.
- Write and use proportions to solve real world problems.
- Compare unit rates within a problem to decide which option is best.
- Find unit rate in words, tables, graphs, and/or equations.
- Use ratios, rates, rate tables, proportions, and equations to convert quantities to different measurement units.
- Solve multi-step percent problems (tax, tip, commissions, markup, and discount).

Assessments

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**Title:** Kyle Prices the Inventory  
**Goal:** Students will price inventory items, calculate profit on particular items,

**Checkpoints**
- Investigation Classwork and ACE (Applications, Connections, Extensions) exercises.
calculate discount prices based on sale percentages, and find the percent increase of certain items.

Role: Assistant Manager

Audience: Manager and customers

Situation: Kyle is the assistant manager at Digital Age. He is in charge of inventory and pricing for products such as TVs, tablets, and MP3 players. The manager has informed Kyle that a recent shipment has 120 items. Three fifths of the items are tablets, 15% of the items are TVs and the rest are MP3 players.

Product or Performance: Detailed answers to each scenario.

Standards for Success: A scoring rubric is shared with students at the onset of the project.

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- Mathematical Reflection Questions at end of each Investigations
- Looking Back which can be used as review, helping students to stand back and look at the big ideas and connections in the unit.

Surveys of Knowledge
- Quizzes: individual assessment instruments.
- Unit Tests: individual assessment that informs teachers about students' ability to apply, refine, modify, and possibly extend the mathematical knowledge and skills acquired.
- Performance Task
Identify Desired Results

Common Core Standards

Standards of Focus in Unit

- (7.RP.A.2.b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- (7.RP.A.2.c) Represent proportional relationships by equations.
- (7.RP.A.2.d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
- (7.EE.A.1) Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- (7.EE.A.2) Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05.
- (7.EE.A.3) Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- (7.EE.A.4) Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- (7.EE.B.4.a) Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Additional Standards in Unit

- (7.RP.A.2) Recognize and represent proportional relationships between quantities.
- (7.RP.A.2.a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- (7.EE.A.4.b) Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.
Standards for Mathematical Practices

- **MP 1** - Make sense of problems and persevere in solving them.
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- The pattern of constant change between two variables forms a linear relationship.
- A linear relationship can be presented in various formats (words, equations, tables, graphs).
- Slope is the ratio of the vertical distance and the horizontal distance between two points on a line and the rate of change between two variables that have a linear relationship.
- Compare and analyze linear relationships in various situations, presented in different formats, within the context of a problem.
- Decisions and solutions can be made about linear relationships using information given in words, tables, graphs, and equations.

- What patterns in the problem suggest the relationship is linear?
- Under what circumstances is a relationship linear and proportional?
- How do changes in one variable affect changes in a related variable?
- How does technology help me to see and analyze a linear relationship?
- What elements do I use to compare and analyze linear relationships in various situations, presented in different formats, within the context of a problem?

<table>
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<tr>
<th>Expected Performances</th>
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<tbody>
<tr>
<td>What students should know and be able to do</td>
</tr>
</tbody>
</table>

**Students will know the following:**

- Slope is the ratio of the vertical distance and the horizontal distance between two points on a line and the rate of change between two variables that have a linear relationship and how to interpret its meaning and context.
- A linear relationship can be presented in various formats (words, equations, tables, graphs).
- \( y = mx + b \) is the slope-intercept form of an equation and is proportional when \( b \) is equal to zero.

**Students will be able to do the following:**

- Move from one linear relationship format to another fluently.
- Interpret what is happening in the table, graph, equation, or word.
- Solve problems and make decisions about linear relationships using information given in words, tables, graphs, and equations.
- Write equations that represent linear relationships given specific pieces of information, and describe what information the variables and numbers represent.
- Solve multi-step equations presented conceptually, in context, or algebraically.
- Find the slope of a linear relationship.

<table>
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<td>ideas within contexts of</td>
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<td>interesting problems.</td>
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<td>- Work with small groups of students</td>
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<tr>
<td>to build conceptual understanding,</td>
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<tr>
<td>fluency and application of key math</td>
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<td>focus areas.</td>
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<td>Use games to develop concepts and</td>
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<td>practice skills.</td>
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<td>Use student-centered activities and</td>
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<td>- Arrange student seating so that</td>
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<td>learners can move freely to post,</td>
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<td>analyze, and discuss their work.</td>
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<td>Use a variety of grouping structures.</td>
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<td>- Collaborative groups, partners,</td>
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<td>individuals.</td>
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<td>Learning Activities:</td>
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<td>Identify and describe patterns of</td>
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<tr>
<td>change between the independent and</td>
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<td>dependent variables for linear</td>
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<tr>
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<td>graphs, equations, or contextual</td>
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<td>Write equations that represent</td>
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<td>what information the variables and</td>
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<td>numbers represent.</td>
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<tr>
<td>Find the rate of change or slope, y-</td>
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<tr>
<td>intercept, x-intercept, and equation</td>
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<tr>
<td>for a linear relationship.</td>
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<tr>
<td>Solve multi-step equations</td>
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<tr>
<td>conceptually and algebraically.</td>
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<tr>
<td>Discover situations where linear</td>
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<tr>
<td>relationships have a positive,</td>
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<td>negative, no slope, same slope, or</td>
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<tr>
<td>negative reciprocals given tables,</td>
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- Elicit participation from all students over the course of several discussions.

### Assessments

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<td>Application that is functional in a classroom context to evaluate student achievement of desired results</td>
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N/A

#### Checkpoints
- Investigation Classwork and ACE (Applications, Connections, Extensions) exercises.
- Mathematical Reflection Questions at end of each Investigations.
- Looking Back which can be used as review, helping students to stand back and look at the big ideas and connections in the unit.

#### Surveys of Knowledge
- Quizzes: individual assessment instruments.
- Unit Tests: individual assessment that informs teachers about students’ ability to apply, refine, modify, and possibly extend the mathematical knowledge and skills acquired.

### Suggested Resources
- Connected Mathematics Project 3 (CMP3) Unit 4, Moving Straight Ahead
- Teaching Student-Centered Mathematics, 6-8 by Van de Walle, et. al
- CT State Department of Education, Mathematics Units of Study, Grade 7
New Milford Public Schools

Committee Member(s): Cortni Muir, Peggy Neal, Tamara Gloster
Unit Title: Unit 5 – Linear and Inverse Variations

### Identify Desired Results

**Course/Subject:** Mathematics  
**Grade Level:** 7 Pre-Algebra  
**# of Weeks:** 5

<table>
<thead>
<tr>
<th>Standards of Focus in Unit</th>
<th>Common Core Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>• (8.EE.B.5)</td>
<td>Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.</td>
</tr>
<tr>
<td>• (8.F.A.1)</td>
<td>Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</td>
</tr>
<tr>
<td>• (8.F.A.2)</td>
<td>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</td>
</tr>
<tr>
<td>• (8.F.A.3)</td>
<td>Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.</td>
</tr>
<tr>
<td>• (8.F.B.4)</td>
<td>Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
</tr>
<tr>
<td>• (8.F.B.5)</td>
<td>Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</td>
</tr>
<tr>
<td>• (8.SP.A.1)</td>
<td>Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</td>
</tr>
<tr>
<td>• (8.SP.A.2)</td>
<td>Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</td>
</tr>
<tr>
<td>• (8.SP.A.3)</td>
<td>Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.</td>
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<tr>
<td>• (8.SP.A.4)</td>
<td>Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.</td>
</tr>
</tbody>
</table>

### Additional Standards in Unit
• (8.EE.C.7) Solve linear equations in one variable.
• (8.EE.C.7.b) Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
• (8.EE.C.8) Analyze and solve pairs of simultaneous linear equations.
• (8.EE.C.8.a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
• (8.EE.C.8.c) Solve real-world and mathematical problems leading to two linear equations in two variables.

Standards for Mathematical Practices
• MP 1 - Make sense of problems and persevere in solving them.
• MP 2 - Reason abstractly and quantitatively.
• MP 3 - Construct viable arguments and critique the reasoning of others.
• MP 4 - Model with mathematics.
• MP5 - Use appropriate tools strategically.
• MP6 - Attend to precision.
• MP7 - Look for and make use of structure.
• MP8 - Look for and express regularity in repeated reasoning.

Enduring Understandings
Generalizations of desired understanding via essential questions
(Students will understand that …)

Essential Questions
Inquiry used to explore generalizations

• Linear and nonlinear data patterns can be represented using graphs, tables, word descriptions, and algebraic expressions.
• Linear and inverse variation equations model bivariate data and can be used to solve real world problems.
• Scatterplots, two-way tables and correlation coefficients are used to describe patterns of association in pairs of variables.
• Functions model relationships between variables and are described using slope and y-intercepts.

• Is there a pattern relating the variables and is it strong enough to allow me to make predictions?
• What strategies can I use to write equations for linear functions?
• How can I tell if the pattern of change can be modeled by a linear or inverse variation function?
• How can I write an equation for a linear equation when I am given the graph, table or two points?

Expected Performances
What students should know and be able to do

Students will know the following:
• Tables, graphs, word descriptions, and algebraic expressions can model linear and nonlinear relationships.
• Bivariate data can be modeled with linear and inverse equations.
• Equation $y = mx + b$ defines a linear function whose graph is a straight line.

Students will be able to do the following:
• Write equations for a linear equation when given the graph, table or two points.
• Recognize a linear or nonlinear function from word description, graph, or table.
• Model data with inverse variation characteristics.
• Compare inverse variation relationships with linear relationships.
• Construct and interpret scatterplots for bivariate measurement data.
• Describe qualitatively the functional relationship between two variables and sketch a graph that exhibits the features of the function.
• Analyze and solve linear equations.

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<td>Climbing Monkeys</td>
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### Develop Teaching and Learning Plan

#### Teaching Strategies:
**Use a problem-centered model with teacher-directed mini-lesson**
- Embed important mathematical ideas within contexts of interesting problems.
- Work with small groups of students to build conceptual understanding, fluency and application of key math focus areas.

**Use games to develop concepts and practice skills.**

**Use student-centered activities and worthwhile math tasks.**
- Arrange student seating so that learners can move freely to post, analyze, and discuss their work.

**Use a variety of grouping structures.**
- Collaborative groups, partners, individuals.
- Clearly communicate expectations about group work to students.

**Orchestrate class/student to student discourse.**
- Focus discussions on important mathematics and student

#### Learning Activities:
- Perform the Bridge Thickness experiment to determine the relationship between thickness and penny-breaking weight.
- Perform the Bridge Length experiment to determine the relationship between length and penny-breaking weight.
- Write an equation for a linear function given a graph, table or two points and use the equation to find a solution to a real world problem.
- Determine if a linear function is a good model for a set of data and measure the accuracy of the model.
- Identify the patterns in data tables and graphs that can be modeled by inverse variation functions.
- In the investigation Planning a Field Trip, explore the effect of doubling the independent variable and halving the dependent variable in an inverse variation function.
- Analyze data using the statistical concepts of correlation and
strategies.
• Facilitate student-to-student discourse.
• Elicit participation from all students over the course of several discussions.

standard deviation.
• Describe what the coordinates of the intersection points mean when two linear functions intersect.

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**Title:** Apple Festival

**Goal:** Students will write and solve equations and inequalities to convert temperatures between different scales, computer travel distances, calculate pay per item, and determine the number of apples picked based on expressions.

**Role:** Students on field trip

**Audience:** Principal, Orchard Owner

**Situation:** A select group of Midtown High students travel to an Apple Festival. They plan to take advantage of all the opportunities the trip offers.

**Product or Performance:** Detailed responses and work for each part.

**Standards for Success:** A scoring rubric is shared with students at the onset of the project.

**Surveys of Knowledge**

**Checkpoints**
• Investigation Classwork and ACE (Applications, Connections, Extensions) exercises.
• Mathematical Reflection Questions at end of each Investigations
• Looking Back which can be used as review, helping students to stand back and look at the big ideas and connections in the unit.

**Suggested Resources**

• Connected Mathematics Project 3 (CMP3) Unit 5, Thinking with Mathematical Models
• Teaching Student-Centered Mathematics, 6-8 by Van de Walle, et. al
• CT State Department of Education, Mathematics Units of Study, Grade 7
Identify Desired Results

Common Core Standards

Standards of Focus in Unit

- (8.NS.A.1) Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
- (8.NS.A.2) Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$).
- (8.EE.A.2) Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- (8.G.B.6) Explain a proof of the Pythagorean Theorem and its converse.
- (8.G.B.7) Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- (8.G.B.8) Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Additional Standards in Unit

- (8.EE.C.7) Solve linear equations in one variable.
- (8.G.A.4) Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Standards for Mathematical Practices

- MP 1 - Make sense of problems and persevere in solving them.
- MP 2 - Reason abstractly and quantitatively.
- MP 3 - Construct viable arguments and critique the reasoning of others.
- MP 4 - Model with mathematics.
- MP5 - Use appropriate tools strategically.
- MP6 - Attend to precision.
- MP7 - Look for and make use of structure.
- MP8 - Look for and express regularity in repeated reasoning.

Enduring Understandings

Generalizations of desired understanding via essential questions

Essential Questions

Inquiry used to explore generalizations
Students will understand that …

• Irrational numbers cannot be represented by fractions and are nonterminating, nonrepeating decimals.
• The square root of a whole number that is not a square is irrational.
• The Pythagorean Theorem and its converse can be used to solve a variety of problems.

• How do I know I can use the Pythagorean Theorem to solve a problem?
• How can I find the distance between two points?
• How do I know if a number is irrational?

Expected Performances
What students should know and be able to do

Students will know the following:
• Pythagorean Theorem and its converse can be used to solve a variety of problems.
• The real number system is comprised of rational (integers, whole numbers, natural numbers) and irrational numbers.

Students will be able to do the following:
• Explain a proof of the Pythagorean Theorem.
• Apply the Pythagorean Theorem to find the distance between two points on the coordinate grid and solve real-world problems.
• Locate irrational numbers, square and cube roots on a number line.
• Represent rational numbers as fractions and as terminating or repeating decimals.

Character Attributes
• Integrity
• Respect
• Honesty
• Responsibility
• Perseverance

Technology Competencies
• None

Develop Teaching and Learning Plan
Teaching Strategies:
Use a problem-centered model with teacher-directed mini-lesson
• Embed important mathematical ideas within contexts of interesting problems.
• Work with small groups of students to build conceptual understanding, fluency, and application of key math focus areas.

Use games to develop concepts and practice skills.
Use student-centered activities and worthwhile math tasks.

Learning Activities:
• Discover from the activity Driving Around Euclid, the hypotenuse of a right triangle is always the longest side and is shorter than the sum of the legs.
• Draw parallel or perpendicular lines to a given line segment on the coordinate grid to create a triangle or rectangle and find its area.
• Understand from the activity Square Roots, how $x^2$ relates to $\sqrt{x}$.
• Using dot paper, conceptually find the distance between two points.
- Arrange student seating so that learners can move freely to post, analyze, and discuss their work.

**Use a variety of grouping structures.**
- Collaborative groups, partners, individuals.
- Clearly communicate expectations about group work to students.

**Orchestrate class/student to student discourse.**
- Focus discussions on important mathematics and student strategies.
- Facilitate student-to-student discourse.
- Elicit participation from all students over the course of several discussions.

and relate this to the distance formula.
- Find the square and cube root of a number and order them on a number line.
- Discover and explain a proof of the Pythagorean Theorem.
- Investigate the converse of the Pythagorean Theorem.
- Represent fractions as repeating or terminating decimals.
- Represent repeating and terminating decimals as fractions.
- Identify numbers as rational or irrational.
- Use the Pythagorean Theorem to find distances in a geometric shape.
- In the activity, Analyzing Triangles, investigate properties of equilateral and 30/60/90 triangles by applying the Pythagorean Theorem.

### Assessments

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**Checkpoints**
- Investigation Classwork and ACE (Applications, Connections, Extensions) exercises.
- Mathematical Reflection Questions at end of each Investigations
- Looking Back which can be used as review, helping students to stand back and look at the big ideas and connections in the unit.

**Surveys of Knowledge**
- Quizzes: individual assessment instruments.
- Unit Tests: individual assessment that informs teachers about students’ ability to apply, refine, modify, and possibly extend the mathematical knowledge and skills acquired.

### Suggested Resources
- Connected Mathematics Project 3 (CMP3) Unit 6, Looking for Pythagoras
- Teaching Student-Centered Mathematics, 6-8 by Van de Walle, et. al
- CT State Department of Education, Mathematics Units of Study, Grade 7
# Identify Desired Results

**Standards of Focus in Unit**

- (8.EE.B.6) Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation \( y = mx \) for a line through the origin and the equation \( y = mx + b \) for a line intercepting the vertical axis at \( b \).
- (8.G.A.1) Verify experimentally the properties of rotations, reflections, and translations:
  - (8.G.A.1.A) Lines are taken to lines, and line segments to line segments of the same length.
  - (8.G.A.1.B) Angles are taken to angles of the same measure.
  - (8.G.A.1.C) Parallel lines are taken to parallel lines.
- (8.G.A.2) Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- (8.G.A.3) Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- (8.G.A.4) Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- (8.G.A.5) Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

**Standards for Mathematical Practices**

- MP 1 - Make sense of problems and persevere in solving them.
- MP 2 - Reason abstractly and quantitatively.
- MP 3 - Construct viable arguments and critique the reasoning of others.
- MP 4 - Model with mathematics.
- MP5 - Use appropriate tools strategically.
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## Enduring Understandings

Generalizations of desired understanding via essential questions
(Students will understand that …)

## Essential Questions

Inquiry used to explore generalizations
Rigid motion transformations relate points by the motions of reflections, rotations, and translations, and describe methods for identifying and creating symmetric plane figures.

Two figures are congruent if one is derived from the other by a sequence of reflection, rotation, and/or translation transformations.

Two figures are similar if one is derived from the other by a sequence of reflection, rotation, translation, and/or dilations.

Properties of congruent and similar triangles are used to solve problems about shapes and measurements.

Properties of angles are formed by parallel lines and transversals, and the angle sum in any triangle, to properties of transformations.

What does it mean to say that a figure has:
- Flip or reflectional symmetry?
- Turn or rotational symmetry?
- Slide or translational symmetry?

How is each point related to its image under each transformation?

How will the shape, size and position of a figure change after each of the transformations-reflection, rotation, or translation?

What does it mean to say that two geometric shapes are congruent to each other? How can I show congruence with movable copies of the figures? How do I plan the transformations that “move” one triangle onto another?

What is the smallest number of side and/or angle measurements needed to conclude that two triangles are congruent?

How can you describe how points “move” under a reflection with the coordinate rules in the form of (x,y) -->(? ,?) when the reflection line is:
- (1) the x-axis? (2) the y-axis? (3) the line y = x?

What kind of coordinate rule (x, y) → (?, ?) tells how to “move” any point to its image under a translation?

What are the coordinate rules that describe “motion” of points on a grid under turns of 90 degrees and 180 degrees?

How are lines and their images under translations and half-turns related to each other?

What coordinate rules model dilations? How do dilations change or preserve characteristics of the original figure?

When two parallel lines are cut by a transversal, what can be said about the angles formed?

How can you use transformations to
<table>
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<th>check whether two figures are similar or not?</th>
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<td>What students should know and be able to do</td>
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**Students will know the following:**
- Rigid motion transformations relate points by the motions of reflections, rotations, and translations, and describe methods for identifying and creating symmetric plane figures.
- Two figures are congruent if one is derived from the other by a sequence of reflection, rotation, and/or translation transformations.
- Two figures are similar if one is derived from the other by a sequence of reflection, rotation, translation, and/or dilations.
- Properties of congruent and similar triangles are used to solve problems about shapes and measurements.
- Properties of angles are formed by parallel lines and transversals, and the angle sum in any triangle, and the properties of angles are related to the properties of transformations.

**Students will be able to do the following:**
- Recognize properties of reflection, rotation, and translation transformations.
- Use rigid motions transformations to create symmetric designs.
- Use coordinate rules for basic rigid motion transformations.
- Recognize that two figures are congruent and similar by a sequence of transformations.
- Use transformations to describe a sequence that exhibits the congruence between figures.
- Use transformations to establish similarity of triangles.
- Relate properties of angles formed by parallel lines and transversals, and the angle sum in any triangle, to properties of transformations.
- Check similarity without transformations.
- Use properties of congruent and similar triangles to solve problems about shapes and measurements.

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<td>*** The following are all available on CMP3 Dashboard</td>
</tr>
<tr>
<td>Hubcap Maker</td>
</tr>
<tr>
<td>Transformations</td>
</tr>
<tr>
<td>Tessellations</td>
</tr>
</tbody>
</table>

**Develop Teaching and Learning Plan**

**Teaching Strategies:**
- Use a problem-centered model with teacher-directed mini-lesson
  - Embed important mathematical

- Use drawing tools to make a symmetric figure by using the line of symmetry to reflect an image.
- Explore the pinwheel design to
ideass within contexts of interesting problems.
• Work with small groups of students to build conceptual understanding, fluency and application of key math focus areas.

Use games to develop concepts and practice skills.
Use student-centered activities and worthwhile math tasks.
• Arrange student seating so that learners can move freely to post, analyze, and discuss their work.

Use a variety of grouping structures.
• Collaborative groups, partners, individuals.
• Clearly communicate expectations about group work to students.

Orchestrate class/student to student discourse.
• Focus discussions on important mathematics and student strategies.
• Facilitate student-to-student discourse.
• Elicit participation from all students over the course of several discussions.

Discover rotational symmetry using the center of rotation
• Analyze an image under two translations to discover how each point is related to its image after a translation.
• Analyze transformations of trapezoids from several diagrams to describe the shape, size and position of the trapezoid affected.
• Match the vertices to pair sides and angles of the same size.
• Inspect and measure parts of figures to determine whether they are congruent.
• Determine minimum measurements needed to tell when two triangles are congruent.
• Reflect and translate an image on a coordinate grid indicating the coordinates of the images.
• Discover and prove the special relationship among corresponding sides of pentagon ABCDE and its images after a translation and after a 180 degree rotation.
• Explain why the angles formed by parallel lines cut by a transversal are congruent, and use the transformations to explain why the sum of the measures of the interior angles of a triangle is 180 degrees.
• Observe how dilations affect the size and shape of the figures they transform.
• Explore strategies that can be used to check for similarity of two figures in Return of Super Sleuth.
• Check for similarity without using transformations.
• Use similar triangles to solve measurement problems.

Assessments

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<tr>
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<td>Authentic application to evaluate student achievement of desired results designed according to GRASPS</td>
<td>Application that is functional in a classroom context to evaluate student achievement of desired results</td>
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<tr>
<td>Title: Geometry at the Gym</td>
<td></td>
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<td>---------------------------</td>
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<tr>
<td>Goal: Students will use what they know about translations, reflections, and rotations to describe the effects of transformations; draw images on the coordinate plane; and explain that these transformations preserve segment length, angle measure, and distances between segments. Students will also use dilations to enlarge figures.</td>
<td></td>
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<tr>
<td>Role: Gym Owner or Coach</td>
<td></td>
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<tr>
<td>Audience: Gymnasts</td>
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<tr>
<td>Situation: Kazakov's Gym is a gymnastic center. Coach Kasakov, the gym's owner, says that an understanding of geometry and spatial relationships is key to being a strong gymnast. He also uses geometry to rearrange his gym equipment.</td>
<td></td>
</tr>
<tr>
<td>Product or Performance: Detailed responses and work for each part.</td>
<td></td>
</tr>
<tr>
<td>Standards for Success: A scoring rubric is shared with students at the onset of the project.</td>
<td></td>
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**Checkpoints**
- Investigation Classwork and ACE (Applications, Connections, Extensions) exercises.
- Mathematical Reflection Questions at end of each Investigations
- Looking Back which can be used as review, helping students to stand back and look at the big ideas and connections in the unit.

**Surveys of Knowledge**
- Quizzes: individual assessment instruments.
- Unit Tests: individual assessment that informs teachers about students' ability to apply, refine, modify, and possibly extend the mathematical knowledge and skills acquired.
- Performance Task

**Suggested Resources**
- Connected Mathematics Project 3 (CMP3) Unit 7, Butterflies, Pinwheels and Wallpaper
- Teaching Student-Centered Mathematics, 6-8 by Van de Walle, et. al
- CT State Department of Education, Mathematics Units of Study, Grade 7
Identify Desired Results

Standards of Focus in Unit

- (8.EE.C.8) Analyze and solve pairs of simultaneous linear equations.
- (8.EE.C.8.a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- (8.EE.C.8.b) Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
- (8.EE.C.8.c) Solve real-world and mathematical problems leading to two linear equations in two variables.
- (8.F.A.3) Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Standards for Mathematical Practices

- MP 1 - Make sense of problems and persevere in solving them.
- MP 2 - Reason abstractly and quantitatively.
- MP 3 - Construct viable arguments and critique the reasoning of others.
- MP 4 - Model with mathematics.
- MP5 - Use appropriate tools strategically.
- MP6 - Attend to precision.
- MP7 - Look for and make use of structure.
- MP8 - Look for and express regularity in repeated reasoning.

Enduring Understandings

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<th>Essential Questions Inquiry used to explore generalizations</th>
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<td>- Solving a system of linear equations, using algebraic or graphic reasoning, is equivalent to find values for the variables that will satisfy all equations in the system.</td>
<td>- How do I know the most efficient way to solve a problem is by writing a system of linear equations?</td>
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<tr>
<td>- Solutions to systems of linear equations may have one, infinitely many or no solutions.</td>
<td>- What strategies are used when solving systems of linear equations?</td>
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Expected Performances

What students should know and be able to do

Students will know the following:
- The solution to a problem is best written as a system of linear equations.
- The solution methods for solving a system of linear equations are by graphing or
algebraically.

- Solutions to systems of linear equations may have one, infinitely many or no solutions.

**Students will be able to do the following:**
- Explain what the solution(s) or no solution means in the context of the problem.
- Determine how many solutions a system of linear equations has.
- Write a system of linear equations to solve a real world problem.
- Recognize that the form $Ax + By = C$ of linear equations is equivalent to $y = mx + b$ for linear equations.
- Choose a method to solve systems of linear equations.

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<td>Recognize linear equations in two variables in standard form ($Ax + By = C$).</td>
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<td>Solve linear equations and graph their solutions.</td>
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<td>Write linear equations in equivalent forms (standard vs y-intercept).</td>
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<td>Find a common solution to two linear equations with two variables.</td>
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<tr>
<td>Write a system of linear equations to solve a real world problem.</td>
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<tr>
<td>Solve a system of linear equations with two variables using various methods: graphing, substitution, elimination.</td>
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- Facilitate student-to-student discourse.
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