Paulsboro Schools

Curriculum

Trigonometry

Grade 10 - 12

June 2010

*** For adoption by all regular education programs as specified and for adoption or adaptation by all Special Education Programs in accordance with Board of Education Policy.**

Board Approved: <Month Year>

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Paulsboro Schools

Mission

The mission of the Paulsboro School District is to provide each student the educational opportunities to assist in attaining their full potential in a democratic society. Our instructional programs will take place in a responsive, community based school system that fosters respect among all people. Our expectation is that all students will achieve the New Jersey Core Curriculum Content Standards (NJCCCS) at every grade level.

INTRODUCTION, PHILOSOPHY OF EDUCATION, AND EDUCATIONAL GOALS

Introduction/philosophy: MATHEMATICS

Paulsboro Schools are committed to providing all students with a quality education resulting in life-long learners who can succeed in a global society. As students begin their mathematics education in Paulsboro, classroom instruction will reflect the best thinking of the day. Children will engage in a wide variety of learning activities designed to develop their ability to reason and solve complex problems. Calculators, computers, manipulatives, technology, and the Internet will be used as tools to enhance learning and assist in problem solving. Group work, projects, literature, and interdisciplinary activities will make mathematics more meaningful and aid understanding. Classroom instruction will be designed to meet the learning needs of all children and will reflect a variety of learning styles.

Paulsboro Schools are committed to providing all students with the opportunity and the support necessary to learn significant mathematics with depth and understanding. This curriculum guide is designed to be a resource for staff members and to provide guidance in the planning, delivery, and assessment of mathematics instruction.

Educational Goals: MATHEMATICS

(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.

(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.

(3) Investigate, research, and synthesize various information from a variety of media sources.

New Jersey State Department of Education
Core Curriculum Content Standards

A note about <Mathematics> Standards and Cumulative Progress Indicators:

The New Jersey Core Curriculum Content Standards for **<Mathematics>** were revised in **2009**. The Cumulative Progress Indicators (CPI's) referenced in this curriculum guide refer to these new standards and may be found in the Curriculum folder on the district servers. A complete copy of the new Core Curriculum Content Standards for Mathematics may also be found at:

http://www.nj.gov/education/aps/cccs/

Trigonometry

Scope and Sequence / Curriculum MAP

<table>
<tr><th colspan="2">Quarter I</th></tr>
<tr><td>Big Idea:
I. Right Triangle Trigonometry and Trigonometric Functions of any Angle
a. Identify the appropriate trigonometric function to solve a right triangle
b. Solve right triangular statement problems
c. Identify reference angles in any quadrant and apply them to right triangle definitions to solve problems.</td><td>Big Idea:
II. Radian and Degree Measure and The Unit Circle
a. Identify radian and degree measure
b. Convert radian to degree and vice versa using proportions.
c. Convert radian to degree and vice versa using a calculator.
d. Identify and graph equation of the unit circle
e. State the value of the 6 trigonometric functions as it pertains to a point on the circle and vice versa</td></tr>
<tr><td>Big Idea:
III. Graphing Trigonometric Functions
a. Use appropriate terminology in describing trigonometric function graphs
b. Determine the amplitude, period, and phase shift of a trigonometric function given the equation
c. Match trigonometric functions to graphs, determine trigonometric functions from given graphs and determine graphs from given equations.
d. Notice significant differences between the graphs of the 6 trigonometric functions</td><td></td></tr>
<tr><th colspan="2">Quarter II</th></tr>
<tr><td>Big Idea:
IV. Inverse Trigonometric Functions and Real Life Applications
a. Evaluate arc functions with and without a calculator
b. Solve application problems using trigonometry</td><td>Big Idea:
V. Trigonometric Identities
a. Use the fundamental identities to evaluate functions, simplify expressions, factor expressions, and verify identities
b. Identify appropriate equations from the list of the given task</td></tr>
</table>

Big Idea:

VI. Solving Trigonometric Equations

a. Solve any type of trigonometric equations

Quarter III

Big Idea:

VII. Law of Sine and Law of Cosines

a. Given an oblique triangle with AAS or ASA, calculate the missing part
b. Given an oblique triangle with SSA, determine the number of solutions and calculate the missing part
c. Given an oblique triangle with SAS or SSS, calculate the area

Big Idea:

VIII. Vectors in a Plane

a. Given a vector, determine its magnitude, direction, the unit vector and components
b. Understand the definitions of vector addition and scalar multiplication.
c. Given two vectors, find the angle between them as well as the resultant vector
d. Apply vector knowledge to statement problems

Big Idea:

IX. Vectors and Dot Products

a. Given two vectors, find their dot products and the angle between them
b. Determine if two vectors are orthogonal
c. Given two vectors, determine the projection of the resultant vector
d. Given rates in force and direction, calculate work.

Quarter IV

Big Idea:

X. Complex Numbers and Complex Solutions of Equations

a. Understand imaginary numbers and their operations
b. Understand imaginary numbers and their standard form.
c. Given a polynomial equation, find the number of solutions and its zeros.

Big Idea:

XI. Complex Numbers in Trigonometric Form

a. Convert complex numbers in polar form to standard form and vice versa.
b. Given two complex numbers, find their product and quotient.
c. Raise a complex number to a power
d. Given a number, find its n^{th} root and state how many roots there are

<table>
<tr><td rowspan="5">Suggested days of Instruction</td><td rowspan="2">Curriculum Management System
<u>Subject/Grade Level</u>:
Trigonometry Grade 10-12</td><td colspan="2">Big Idea: Right Triangle Trigonometry and Trigonometric Functions of any Angle</td></tr>
<tr><td colspan="2">Topic: Solving Right Triangle Problems</td></tr>
<tr><td colspan="2"><u>Overarching Goals:</u>
(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.
(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.
(3) Investigate, research, and synthesize various information from a variety of media sources.</td></tr>
<tr><td colspan="2"><u>Goal 1:</u> The student will be able to use fundamental trigonometric identities to evaluate basic trigonometric functions.</td></tr>
<tr><td>Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's)
The student will be able to:</td><td>Essential Questions, Enduring Understandings, Sample Conceptual Understandings</td><td>Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model</td></tr>
<tr><td>15</td><td>G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.7. Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</td><td>Essential Questions:
What are the six trigonometric ratios and how do they apply to solving right triangles?
What is a reference angle and how is it applied to right triangle definitions to solve problems?
What does the acronym SOH-CAH-TOA stand for?
Enduring Understandings:
Redefine six trigonometric ratios as they apply to a right triangle
Use fundamental trigonometric identities to evaluate basic trigonometric functions
Evaluate trigonometric functions with a calculator
Solve a right triangle
Draw and label reference angles in any angle
Sample Conceptual Understandings:

Pythagorean Theorem: $a^2 + b^2 = c^2$</td><td>NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses).
Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.

Learning Activities:
Students will use Pythagorean theorem to find the missing side of a right triangle.
Students will use corresponding parts of pairs of triangles to solve problems.
Students will solve applied right- triangle problems.
Students will calculate the height of a tree given the length of its shadow and its angle of elevation.
Assessment Models:
Find the missing side and the missing angles of the given triangle where c = 15 and b = 7.</td></tr>
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<tr><td></td><td></td><td></td><td>2. The angle of elevation of a skyscraper from a point on the ground 1185 ft. from its base is 29º. Find the height of the building.
3. For each angle, state (a) its quadrant, (b) its reference angle, (c) a positive coterminal angle less than one revolution, and (d) a negative coterminal angle less than one revolution.

I. 521º
II. -421º
III. 670º
IV. -550º

Additional Resources:
• TI-84 Graphing Calculator
• Compass Odyssey
• <u>Essentials of Trigonometry, Fourth Edition</u>: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources.
• <u>Trigonometry</u> : Prentice Hall 1993
• <u>Trigonometry For Dummies:</u> Mary Jane Sterling; 2006
• <u>Trigonometry</u>: Scott Foreman; 1983</td></tr>
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Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Trigonometry Grade 10-12**	**Big Idea: Radian and Degree Measure and the Unit Circle** **Topic: Measures of Angles and The Unit Circle** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 2: The student will be able to interchange between degree and radian measure and evaluate any trigonometric function.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
15	F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F-TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for *x*, π + *x*, and 2π – *x* in terms of their values for *x*, where *x* is	**Essential Questions:** • What are radians and how are they related to degrees? • How do the values on the unit circle correlate to the rectangular graph of a trigonometric function? **Enduring Understandings:** • Recognize the difference between measurements made in degrees and measurements made in radians • Use the correct conversion ratio to convert radian measure to degree measure and vice versa • Use the correct conversion ratio to convert radian measure to degree measure and vice versa using a calculator • Draw and label the Unit Circle and write its equation • State the trigonomectic function definitions given the point (x,y) **Sample Conceptual Understandings:** Angle conversion: • degrees = radians $\cdot \frac{180^\circ}{\pi}$	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Students will use conversion ratios to change from degree measure to radian measure and vice versa. • Students will know the signs of the six trigonometric functions in each of the four quadrants. • Students will use trigonometric functions to model projectile motion. • Students will use the definition of the trigonometric functions to approximate their values for a given angle of for an angle passing through a given point.

Suggested days of Instruction	Curriculum Management System **Subject/Grade Level:** **Trigonometry Grade 10-12**	**Big Idea: Radian and Degree Measure and the Unit Circle** **Topic: Measures of Angles and The Unit Circle** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 2: The student will be able to interchange between degree and radian measure and evaluate any trigonometric function.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	any real number. • radians = degrees $\cdot \frac{\pi}{180^\circ}$		**Assessment Models:** 1) Convert each angle to radians in terms of π. a) 60° b) 140° c) 180° d) 315° 2) Convert each angle to degrees. a) 2π b) $\frac{2\pi}{3}$ c) $\frac{11\pi}{6}$ d) $\frac{5\pi}{4}$ **Additional Resources:** • TI-84 Graphing Calculator • Compass Odyssey • Essentials of Trigonometry, Fourth Edition: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources. • Trigonometry : Prentice Hall 1993 • Trigonometry For Dummies: Mary Jane Sterling; 2006 • Trigonometry: Scott Foreman; 1983

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<tr><td colspan="2">Topic: Trigonometric Functions and their Graphs</td></tr>
<tr><td colspan="2"><u>Overarching Goals:</u>
(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.
(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.
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<tr><td colspan="2"><u>Goal 3:</u> The student will be able to create graphs given various trigonometric equations.</td></tr>
<tr><td>Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's)
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<tr><td>15</td><td>F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context</td><td>Essential Questions:
What are the graphs of trigonometric functions and what do they look like?
Can you tell what the graph of a function looks like by analyzing its equation?
How does a change in amplitude or period affect the graph of a Sine or Cosine curve?
Explain the effect of A, B, h, and k on the graph of a sine or Cosine curve using the equations
$y = A\sin B(x-h)+k$
$y = A\cos B(x-h)+k$ Enduring Understandings: Define period, amplitude, intercept, cycle, maximum and minimum. Use the standard form of a trigonometric function (y = a cos (bx – c) to find the amplitude ($|a|$), the period (2π/b), and phase shift (c/b) Match sine and cosine curves to their respective graphs</td><td>NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses).
Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.

Learning Activities:
Students will graph the trigonometric functions by plotting points.
Students will graph the general sine, cosine, and tangent functions.
Assessment Models:
Graph y + 2 = 2 sin (x – 3)
Find (h, k) for y + 6 = f(x – π)
Additional Resources:
TI-84 Graphing Calculator
Compass Odyssey</td></tr>
</table>

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Trigonometry Grade 10-12**	**Big Idea: Graphing Trigonometric Functions** **Topic: Trigonometric Functions and their Graphs** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 3: The student will be able to create graphs given various trigonometric equations.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
			• Essentials of Trigonometry, Fourth Edition: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources. • Trigonometry : Prentice Hall 1993 • Trigonometry For Dummies: Mary Jane Sterling; 2006 • Trigonometry: Scott Foreman; 1983

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Trigonometry Grade 10-12**	**Big Idea: Inverse Trigonometric Functions and Real- Life Applications** **Topic: Inverse Trigonometric Functions and their Applications** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 4: The student will be able to calculate and graph the six inverse trigonometric functions.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
15	F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F-BF.4. Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x–1) for x ≠ 1.* (+) Verify by composition that one function is the inverse of	**Essential Questions:** • What is an inverse trigonometric function and how does it relate to its parent function? • What do the graphs of inverse trigonometric functions look like? **Enduring Understandings:** • Write and sketch a graph of the six inverse trigonometric functions. • Write specific as well as general arc functions. • Solve problems concerning right angles, bearings, and harmonic motion. **Sample Conceptual Understandings:** • $\csc^{-1} x = (1/ \sin^{-1} x)$ • $\sec^{-1} x = (1/ \cos^{-1} x)$ • $\cot^{-1} x = (1/ \tan^{-1} x)$	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Students will evaluate inverse trigonometric functions. • Students will draw a quick sketch of each inverse trigonometric function. **Assessment Models:** • Evaluate $\cot^{-1}(\cot 35°)$ without using a calculator. • Graph y – 1 = arcsin x **Additional Resources:**

Suggested days of Instruction	Curriculum Management System **Subject/Grade Level:** **Trigonometry Grade 10-12**	**Big Idea: Inverse Trigonometric Functions and Real- Life Applications** **Topic: Inverse Trigonometric Functions and their Applications** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 4: The student will be able to calculate and graph the six inverse trigonometric functions.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	another. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (+) Produce an invertible function from a non-invertible function by restricting the domain F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.		• TI-84 Graphing Calculator • Compass Odyssey • Essentials of Trigonometry, Fourth Edition: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources. • Trigonometry : Prentice Hall 1993 • Trigonometry For Dummies: Mary Jane Sterling; 2006 • Trigonometry: Scott Foreman; 1983

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Trigonometry Grade 10-12**	**Big Idea: Trigonometric Identities** **Topic: Using Trigonometric Identities** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 5: The student will be able to use the fundamental trigonometric identities to evaluate, simplify expressions, factor expressions, and verify other identities.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
15	F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. F-TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems	**Essential Questions:** • What are the fundamental trigonometric identities and what are they used for? • What methods can be used to prove identities? **Enduring Understandings:** • Perform trigonometric function evaluations using a list of fundamental identities • Use the list of basic trigonometric identities and algebra models to manipulate one of an identity into the other sides. **Sample Conceptual Understandings:** • $\sin^2\theta + \cos^2\theta = 1$ • $1 + \tan^2\theta = \sec^2\theta$	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Students will solve trigonometric equations in linear form, by factoring, or by using quadratic formula. • Students will prove identities using algebraic simplification and the eight fundamental identities. **Assessment Models:** Solve the following equations. 1. $2 \cos 3x - 1 = 0$ 2. $4 \cos^2 x - 1 = 0$ 3. $2 \sin^2 x + 3 \sin x + 1 = 0$

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Trigonometry Grade 10-12**	**Big Idea: Solving Trigonometric Equations** **Topic: Solving equations by using laws and identities** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 6: The student will be able to solve trigonometric equations using numerous methods.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
15	F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	**Essential Questions:** • How would I solve an equation involving half angles and double angles? • What does the term "half angle" mean? • What does the term "double angle" mean? **Enduring Understandings:** • Solve trigonometric functions using algebraic concepts, factoring, or quadratic formula. • Simplifying trigonometric equations given a list of sum and difference formulas. • Simplifying trigonometric functions using double and half angle formulas. **Sample Conceptual Understandings:** • $\cos 2\theta = \cos^2\theta - \sin^2\theta$ • $\sin 2\theta = 2 \sin\theta \cos\theta$	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Students will find exact values by using addition laws and identities. • Students will find angles by using double and half angle identities. • Students will use product and sum identities to simplify certain trigonometric equations. **Additional Resources:** • TI-84 Graphing Calculator • Compass Odyssey • Essentials of Trigonometry, Fourth Edition: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources.

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<u>Subject/Grade Level</u>:
Trigonometry Grade 10-12</td><td colspan="2">Big Idea: Law of Sine and Law of Cosines</td></tr>
<tr><td colspan="2">Topic: Using Law of Sines and Cosines to solve oblique triangles.</td></tr>
<tr><td colspan="2"><u>Overarching Goals:</u>
(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.
(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.
(3) Investigate, research, and synthesize various information from a variety of media sources.</td></tr>
<tr><td colspan="2"><u>Goal 7:</u> The student will be able to solve oblique triangles using various techniques and methods.</td></tr>
<tr><td>Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's)
The student will be able to:</td><td>Essential Questions, Enduring Understandings, Sample Conceptual Understandings</td><td>Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model</td></tr>
<tr><td>15</td><td>G-SRT.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</td><td>Essential Questions:
How do I solve for missing parts if the triangle is not a right triangle?
How can I find the area of a triangle using trigonometry?
Enduring Understandings:
Given an oblique triangle with AAS or ASA, find the missing parts.
Given an oblique triangle with AAS, determine the number of solutions and solve for the missing parts using the law of sines
Given an oblique triangle with SAS or SSS, calculate the area of a triangle
Sample Conceptual Understandings:
$K = \frac{1}{2}bc\sin\alpha$
$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$</td><td>NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses).
Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.

Learning Activities/Assessment Models:
Students will solve triangles given various sides and angles using law of cosines.
Students will solve triangles given various sides and angles using law of sines.
Find the area of a triangle given SAS, ASA, and SSS.
Additional Resources:
TI-84 Graphing Calculator
Compass Odyssey
<u>Essentials of Trigonometry, Fourth Edition</u>: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources.</td></tr>
</table>

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Trigonometry Grade 10-12**	**Big Idea: Vectors in a Plane** **Topic: Vectors and their operations** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 8: The student will be able to solve and construct vectors based on missing sides or angles.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
15	N-VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v). N-VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N-VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.	**Essential Questions:** • What are vectors and what are they used for? • Where in the real world are vectors used? **Enduring Understandings:** • Define a vector, components of a vector, and the length of a vector • Identify the properties of vectors and vector multiplication • Identify and find direction angles when given a vector. • Convert data of an applied problem to vectors to obtain a solution **Sample Conceptual Understandings:** • $\lvert v\rvert = \sqrt{(a^2 + b^2)}$ • $\frac{\sin\theta}{b} = \frac{\sin 180 - \phi}{\lvert v\rvert}$	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities/Assessment Models:** • Students will solve applied problems by finding the resultant vector. • Students will resolve a vector into horizontal and vertical components. **Additional Resources:** • TI-84 Graphing Calculator • Compass Odyssey • Essentials of Trigonometry, Fourth Edition: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources.

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Trigonometry Grade 10-12**	**Big Idea: Vectors and Dot Products** **Topic: Vector Operations and their Applications** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 9: The student will be able to find the components of vectors and the angle between those vectors.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
15	N-VM.4. (+) Add and subtract vectors. N-VM.5. (+) Multiply a vector by a scalar. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|\|c\boldsymbol{v}\|\| = \|c\|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).	**Essential Questions:** • What does the term "orthogonal vectors" mean? • When would you need to know the angle between 2 vectors? **Enduring Understandings:** • Define and use properties of dot products • Define the angle between 2 vectors by using the dot product • Define orthogonal vectors **Sample Conceptual Understandings:** • v·w = ac + bd • v + w = (a + c)i + (b + d)j	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities/Assessment Models:** • Students will find the angle between vectors. • Students will write the algebraic representation of a vector. • Find the scalar product of two vectors. **Additional Resources:** • TI-84 Graphing Calculator • Compass Odyssey • Essentials of Trigonometry, Fourth Edition: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources.

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Trigonometry Grade 10-12**	**Big Idea: Complex Numbers and Complex Solutions of Equations** **Topic: Complex Numbers and their Operations** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 10: The student will be able to comprehend the imaginary unit and the operations associated with the imaginary unit.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
20	N-CN.1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real. N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. N-CN.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	**Essential Questions:** • What is a "complex number?" • Why is De Moivre's theorem important? **Enduring Understandings:** • Add, subtract, multiply, and divide complex numbers. • Given a complex number, determine its conjugate. • Convert complex numbers to standard form **Sample Conceptual Understandings:** • $i^2 = -1$ • $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities/Assessment Models:** • Students will define the imaginary unit as well as complex number. • Students will carry out operations with complex number. **Additional Resources:** • TI-84 Graphing Calculator • Compass Odyssey • Essentials of Trigonometry, Fourth Edition: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources.

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Trigonometry Grade 10-12**	**Big Idea: Complex Numbers in Trigonometric Form** **Topic: Complex Numbers and Polar Coordinates** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Goal 11: The student will be able to interchange between polar and rectangular coordinates.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions, Enduring Understandings, Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
20	N-CN.4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. N-CN.5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. N-CN.6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	**Essential Questions:** • Why is De Moivre's theorem important? • What are polar coordinates? **Enduring Understandings:** • Convert standard form to polar form • Multiply and divide complex numbers in standard form • Use De Moivre's Theorem to determine the power of a complex number. • **Sample Conceptual Understandings:** • $x = r \cos \theta$ • $y = r \sin \theta$	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities/Assessment Models:** • Students will plot points in polar form. • Students will write the two primary representations in polar form. **Additional Resources:** • TI-84 Graphing Calculator • Compass Odyssey • Essentials of Trigonometry, Fourth Edition: Karl J. Smith; Thomsom Brooks/Cole 2006, including Teachers Resources. • Trigonometry : Prentice Hall 1993 • Trigonometry For Dummies: Mary Jane Sterling; 2006

Trigonometry

COURSE BENCHMARKS

1. The student will be able to use fundamental trigonometric identities to evaluate basic trigonometric functions.

2. The student will be able to interchange between degree and radian measure and evaluate any trigonometric function.

3. The student will be able to create graphs given various trigonometric equations.

4. The student will be able to calculate and graph the six inverse trigonometric functions.

5. The student will be able to use the fundamental trigonometric identities to evaluate, simplify expressions, factor expressions, and verify other identities.

6. The student will be able to solve trigonometric equations using numerous methods.

7. The student will be able to solve oblique triangles using various techniques and methods.

8. The student will be able to solve and construct vectors based on missing sides and/or angles.

9. The student will be able to find the components of vectors and the angle between those vectors.

10. The student will be able to comprehend the imaginary unit *i* and the operations associated with the imaginary unit *i*.

11. The student will be able to interchange between polar and rectangular coordinates.